



**UNIVERSITÀ DEGLI STUDI DI CATANIA
INFN-LNS**



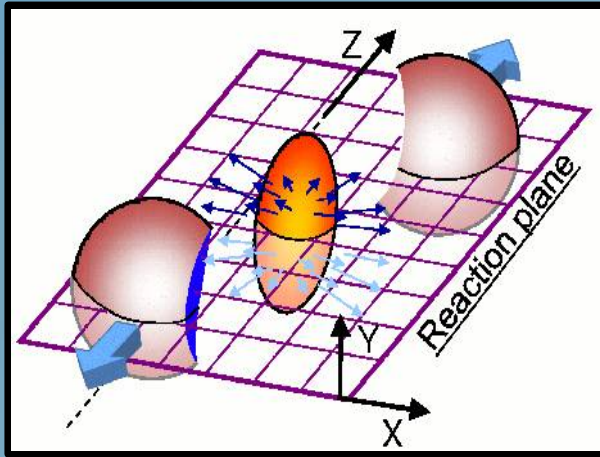
**ANISOTROPIC FLOWS AND SHEAR VISCOSITY
FROM A BEAM ENERGY SCAN**

S. Plumari, A. Puglisi, F. Scardina,

V. Greco, L.P. Csernai

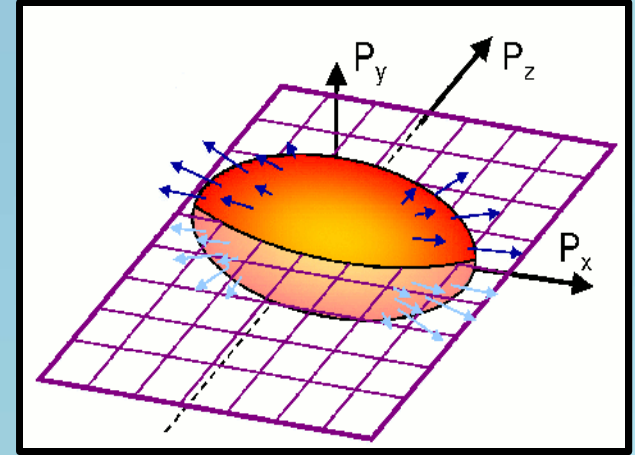
- **Transport approach at fixed η/s :**
 - $\eta \leftrightarrow \sigma(\theta)$, M , T comparing Green-Kubo and Chapman-Enskog approach.
- $\eta/s(T)$ and generation of v_2 : from RHIC to LHC.
- **Conclusions**

Information from non-equilibrium: elliptic flow



$$\lambda = (\sigma\rho)^{-1} \text{ or } \eta/s \text{ viscosity}$$

$$c_s^2 = dP/d\varepsilon, \text{ EoS-IQCD}$$



$$\varepsilon_x = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$$

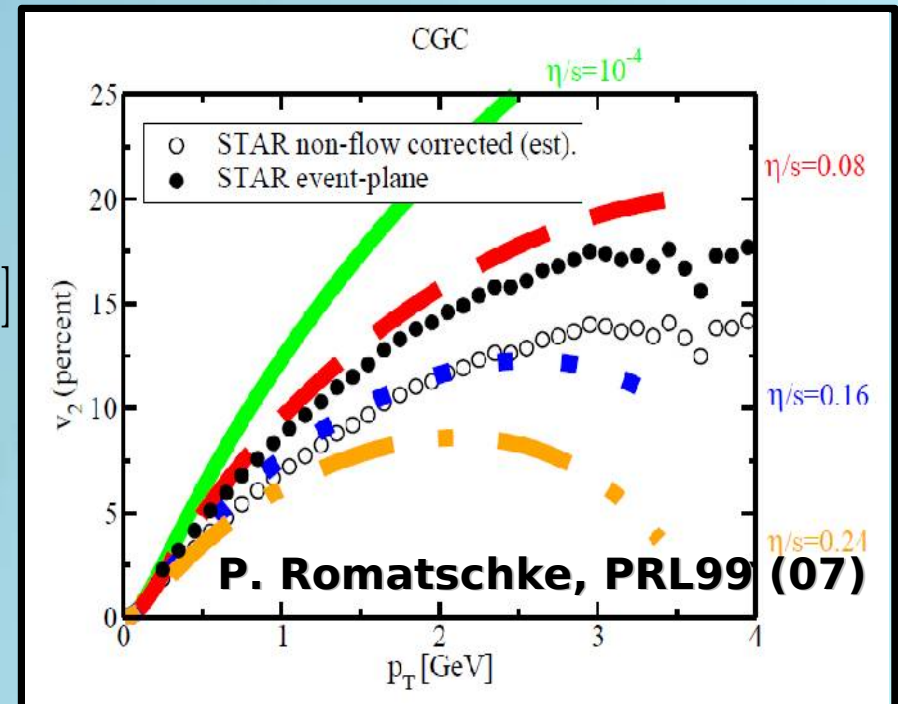
The v_2/ε measures efficiency in converting the eccentricity from Coordinate to Momentum space
J.Y. Ollitrault, PRD 46 (1992).

$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Can be seen also as Fourier expansion

$$\frac{dN}{dp_T d\phi} = \frac{dN}{dp_T} [1 + 2v_2 \cos(2\phi) + 2v_4 \cos(4\phi) + \dots]$$

by symmetry v_n with n odd expected to be zero ... (but event by event fluctuations)



Viscosity η/s

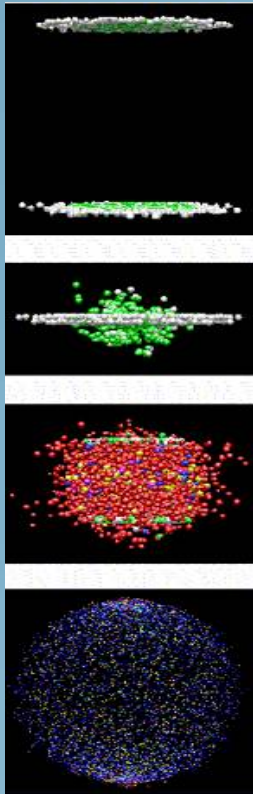
Motivation for a kinetic approach:

$$\left\{ p^\mu \partial_\mu + \left[p_\nu F^{\mu\nu} + M \partial^\mu M \right] \partial_\mu^p \right\} f(x, p) = C_{22} + C_{23} + \dots$$

Free
streaming

Field Interaction $\rightarrow \epsilon \neq 0$

Collisions $\rightarrow \eta \neq 0$



- It is not a gradient expansion in η/s .
- Valid at intermediate p_T out of equilibrium.
- Valid at high η/s (cross over region).
- Include hadronization by coalescence + fragmentation.
- Allows to study the jet-bulk talk.

Parton Cascade model

$$p^\mu \partial_\mu f(X, p) = C = C_{22} + C_{23} + \dots$$

Collisions

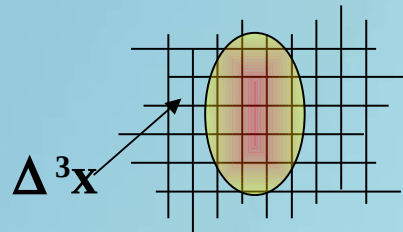


$$\left\{ \begin{array}{l} \varepsilon - 3p = 0, \\ \eta \neq 0 \end{array} \right.$$

$$C_{22} = \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{1}{v} \int \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \frac{d^3 p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |M_{1'2' \rightarrow 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

For the numerical implementation of the collision integral we use the stochastic algorithm. (Z. Xu and C. Greiner, PRC 71 064901 (2005))

$$P_{22} = \frac{\Delta N_{coll}^{2 \rightarrow 2}}{\Delta N_1 \Delta N_2} = v_{rel} \sigma_{22} \frac{\Delta t}{\Delta^3 x}$$



$\Delta t \rightarrow 0$

$\Delta^3 x \rightarrow 0$



right solution

Passed several numerical test on the box.

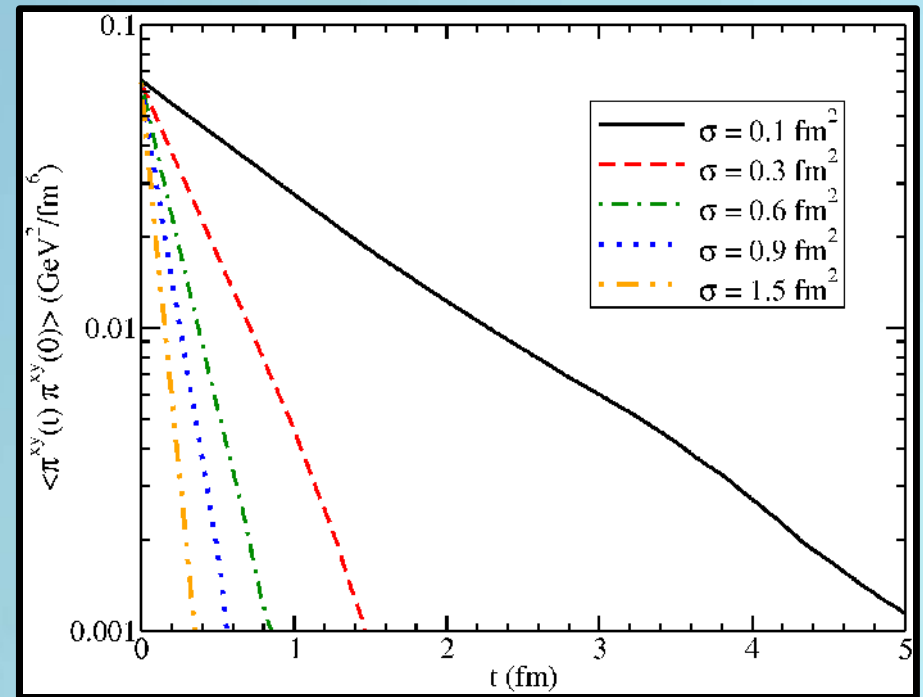
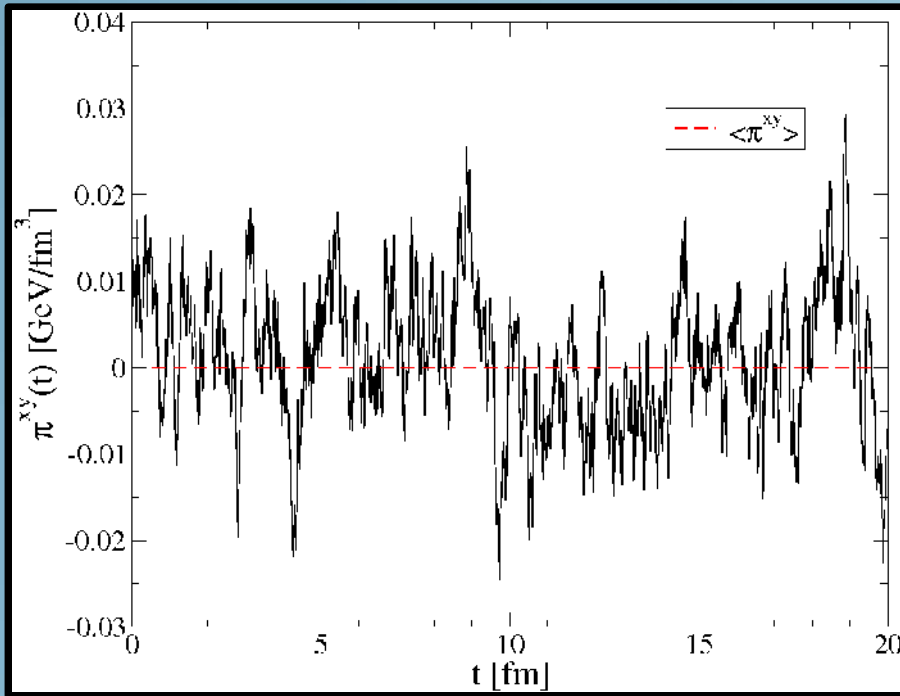
Extraction of the Shear Viscosity: Box calculation

Green – Kubo relation

$$\eta = \frac{1}{T} \int_0^\infty dt \int_V d^3x \langle \pi^{xy}(x,t) \pi^{xy}(0,t) \rangle$$

$$\langle \pi^{xy}(\vec{x},t) \pi^{xy}(\vec{0},t) \rangle = \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \cdot e^{-t/\tau}$$

$$\eta = \frac{V}{T} \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \cdot \tau$$



S. Plumari et al., Phys. Rev. C 86 (2012) 054902.

C. Wesp et al., Phys. Rev. C 84, 054911 (2011).

J. Fuini III et al. J. Phys. G38, 015004 (2011).

Extraction of the Shear Viscosity: Box calculation

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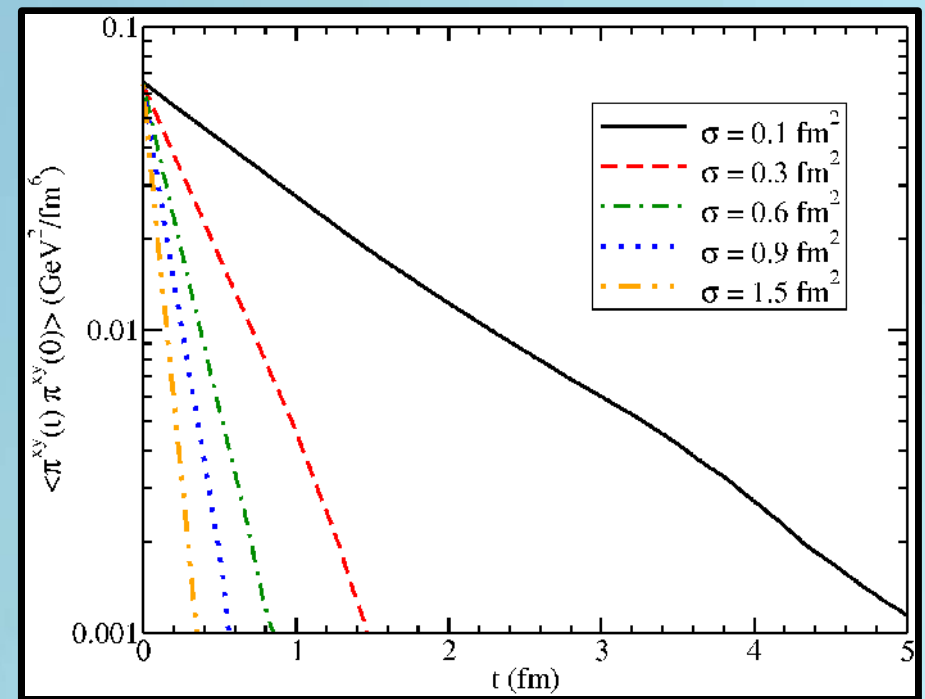
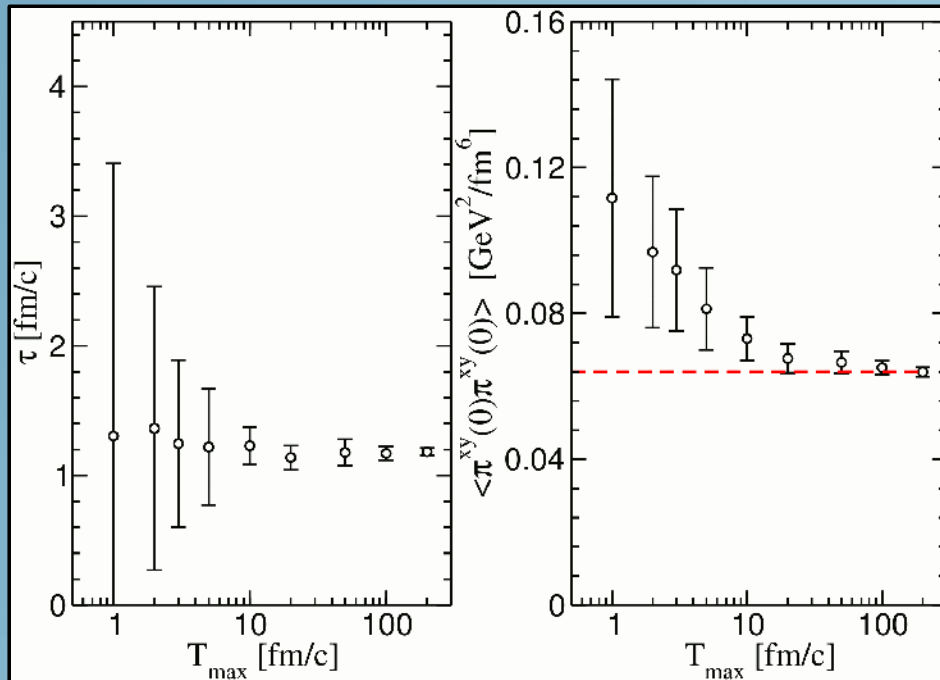
$$\langle \pi^{xy}(\vec{x},t) \pi^{xy}(\vec{0},t) \rangle = \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \cdot e^{-t/\tau}$$

Depends on microscopical details: $\tau(\sigma)$

$$\eta = \frac{V}{T} \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \cdot \tau$$

Depends on macroscopical details:

$$= \frac{4}{15} \frac{eT}{V}$$



S. Plumari et al., Phys. Rev. C 86 (2012) 054902.

C. Wesp et al., Phys. Rev. C 84, 054911 (2011).

J. Fuini III et al. J. Phys. G38, 015004 (2011).

Extraction of the Shear Viscosity: Box calculation

Relaxation Time Approximation

Kapusta, PRC(2010); Gavin NPA(1985);

$$\eta = \frac{1}{15T} \int_0^\infty \frac{d^3 p}{(2\pi)^3} \frac{p^4}{E^2} \tau(E) f^{eq}(E) \quad \tau^{-1}(E) = \rho \langle \sigma_{tot} v_{rel} \rangle$$

$$\eta_{relax} = 0.8 \frac{T}{\sigma_{tot}} \quad \longrightarrow \quad \eta \sim \frac{1}{\sigma_{tot}}$$

Usual as Relax. Time Approx. - Israel Stewart $\sigma_{tot} \rightarrow \sigma_{tr} = (2/3) \sigma_{tot}$

$$\eta_{relax}^{IS} = 0.8 \frac{T}{\sigma_{tr}} = 1.2 \frac{T}{\sigma_{tot}}$$

Molnar-Huovinen PRC(2009),
G. Ferini PLB(2009),
Khvorostukhin PRC (2010)
....

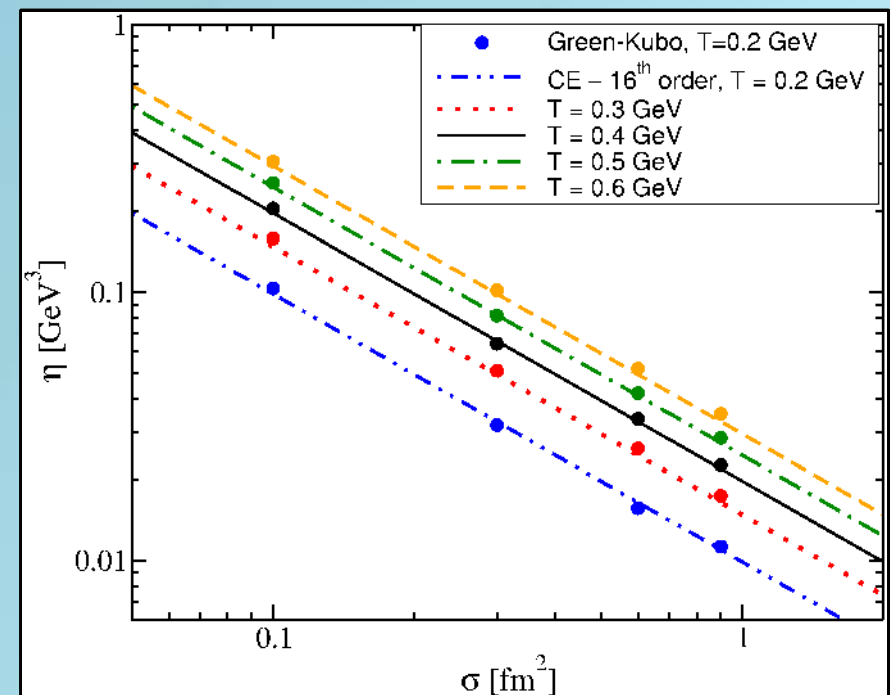
S. Plumari et al., PRC86 (2012) 054902.

Isotropic cross section: massless case

• At 1st order of approx. in the Chapman-Enskog: $[\eta]_{1st}^{CE} = 1.2 \frac{T}{\sigma_{tot}}$

• successive approx. up to 16 order: $[\eta]_{CE}^{16th} = 1.267 \frac{T}{\sigma_{tot}}$

A. Wiranata, M. Prakash, arXiv:1203.0281 [nucl-th].
O. N. Moroz, arXiv:1112.0277 [hep-ph].



Extraction of the Shear Viscosity: Box calculation

$$\eta_{relax}^{IS}/s = \frac{1}{15} \langle p \rangle \tau_r = \frac{1}{15} \frac{\langle p \rangle}{\sigma_{tot} \langle f(a) v_{rel} \rangle \rho}$$

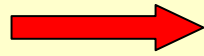
Employed also for non-isotropic cross section:

G.Ferini, PLB(2009); D. Molnar, JPG35(2008);
V.Greco, PPNP(2009);

$$\sigma_{tr} = \int d\Omega \sin^2(\theta_{cm}) \frac{d\sigma}{d\Omega_{cm}} = \sigma_{tot} f(a) \leq \frac{2}{3} \sigma_{tot}$$

For the standard pQCD-like cross section

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{9\pi\alpha_s^2}{2} \frac{1}{(q^2 + m_D^2)^2} \left(1 + \frac{m_D^2}{s}\right)$$



m_D regulates the anisotropy of collision
 $m_D \rightarrow \infty$ we recover the isotropic limit

$$f(a) = 4a(1+a)[(2a+1)\ln(1+a^{-1}) - 2], \quad a = m_D^2/s$$

1st Chapman-Enskog approximation

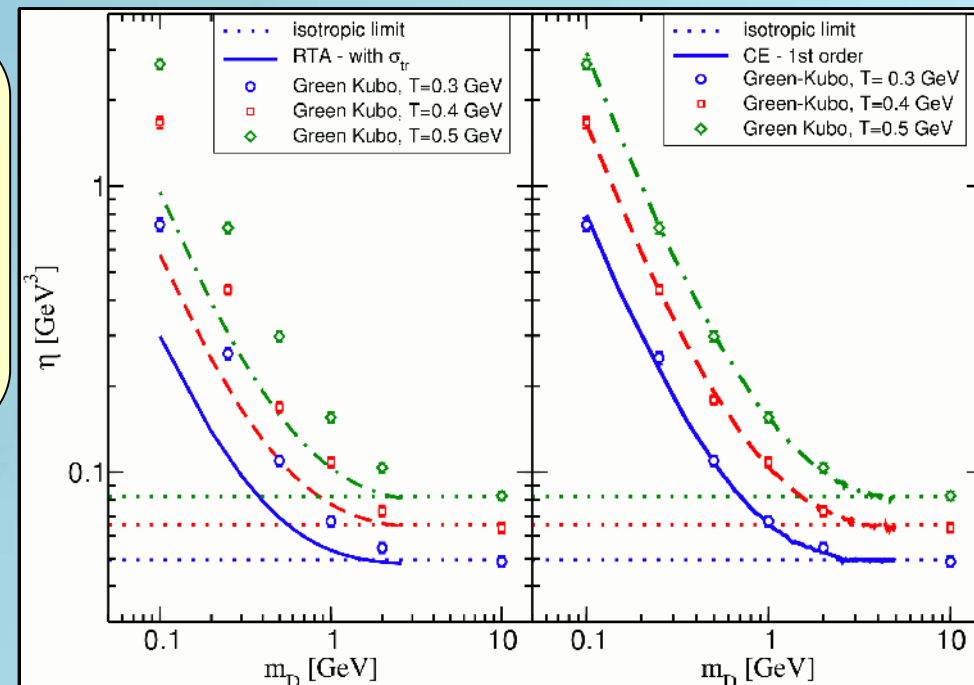
$$[\eta]_{1st}/s = \frac{1}{15} \langle p \rangle \tau_\eta = \frac{1}{15} \frac{\langle p \rangle}{\sigma_{tot} g(a) \rho}$$

$$g(a) = \frac{1}{50} \int_0^\infty dy y^6 \left[\left(y^2 + \frac{1}{3}\right) K_3(2y) - y K_2(2y) \right] f(a), \quad a = \frac{m_D}{2T}$$

- CE and RTA can differ by a factor of 2
- Green-Kubo agree with CE (< 3%)

A. Wiranata, M. Prakash, arXiv:1203.0281 [nucl-th].
O. N. Moroz, arXiv:1112.0277 [hep-ph].

S. Plumari et al., PRC86 (2012) 054902.



Extraction of the Shear Viscosity: Box calculation

Isotropic cross section: massive case

Massive case is relevant in quasi-particle models where $M(T)$.
 Good agreement with CE 1st order for isotropic cross section
 and massive particles.

1st Chapman-Enskog approximation

$$[\eta]_{1st} = 10 T \left[\frac{K_3(z)}{K_2(z)} \right]^2 \frac{1}{c_{00}} = g(m_D, T) \frac{T}{\sigma_{tot}}$$

$$c_{00} = 16 \left[\omega_2^{(2)} - z^{-1} \omega_1^{(2)} + (3z^2)^{-1} \omega_0^{(2)} \right] \quad \text{for } s=2 \propto \sigma_{tr}$$

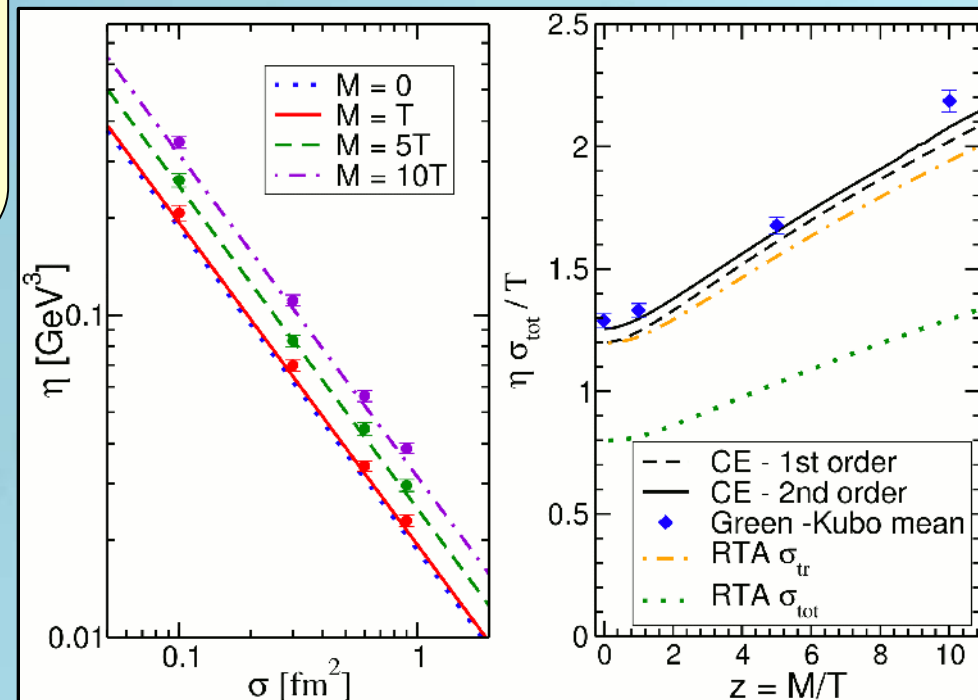
$$\omega_i^{(s)} = \frac{2\pi z^3}{[K_2(z)]^2} \int_1^\infty dy (y^2 - 1)^3 y^i K_j(2zy) \int_0^\pi d\Theta \sin\Theta \frac{d\sigma}{d\Omega} (1 - \cos^s \Theta)$$

$$[\eta]_{1st}^{CE} = f(z) \frac{T}{\sigma_{tot}}$$

$$f(z) = \frac{15}{16} \frac{z^4 K_3^2(z)}{(15z^2 + 2)K_2(2z) + (3z^3 + 49z)K_3(2z)}$$

A. Wiranata, M. Prakash, arXiv:1203.0281 [nucl-th].
 O. N. Moroz, arXiv:1112.0277 [hep-ph].

S. Plumari et al., arXiv:1208.0481 [nucl-th].



- **We know how to fix locally $\eta/s(T)$**
- **We have checked the Chapmann-Enskog:**
 - *CE good already at I° order $\approx 5\%$ ($\approx 3\%$ at II° order)*
 - *RTA even with σ_{tr} severely underestimates η*

Simulating a constant η/s

For the general case of anisotropic cross section and massless particles:

$$\eta(\vec{x}, t)/s = \frac{1}{15} \langle p \rangle \tau_\eta \quad \longrightarrow \quad \sigma_{tot}^{\eta/s} = \frac{1}{15} \frac{\langle p \rangle}{g(m_D/2T)n} \frac{1}{\eta/s} = K \sigma_{tot}^{pQCD}$$

σ is evaluated in such way to keep the $\eta/s = \text{const.}$ during the dynamics. (similar to D. Molnar, arXiv:0806.0026[nucl-th] but our approach is local.)

With this approach we have large cross-section

$$\sigma_{Tot} = K \sigma_{pQCD} \gg \sigma_{pQCD}$$

V.Greco, et al. PLB 670 (2009), PPNP 62 (2009) 562.

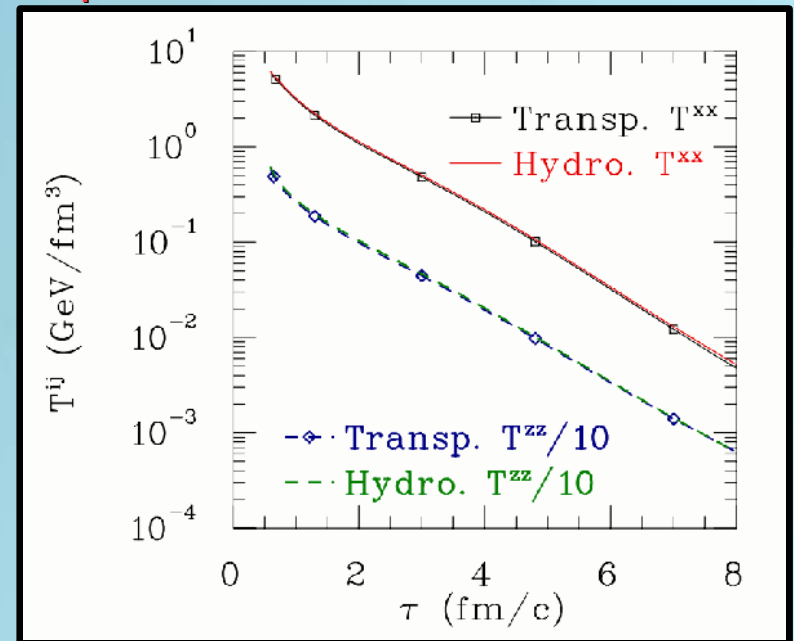
Take into account the non-perturbative effects

BUT NEVER REACH THE pQCD LIMIT WHATEVER IS THE p_T OF THE COLLIDING PARTICLE -

EQUIVALENT TO HYDRO

MORE REALISTIC DESCRIPTION MEANS $K(s)$

At low p_T equivalent to Hydro dynamics



D. Molnar et al., JPG 35, (2008). QM08

Initial condition of our simulation

- ◇ **r-space: standard Glauber model**
- ◇ **p-space: Boltzmann-Juttner $T_{max}=1.7-3.5 T_c$**
- ◇ **[$p_T < 2 \text{ GeV}$]+ minijet [$p_T > 2-3 \text{ GeV}$]
Discarded in viscous hydro**

We fix maximum initial T at RHIC 200 AGeV

$$T_{max0} = 340 \text{ MeV}$$

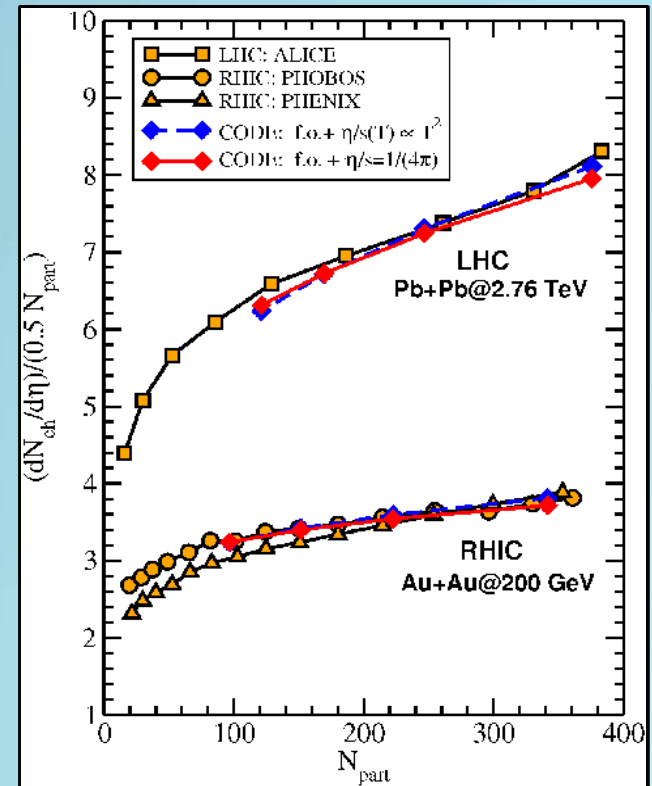
$$T_0 \tau_0 = 1 \rightarrow \tau_0 = 0.6 \text{ fm/c}$$

Typical
hydro
condition

Then we scale it according to

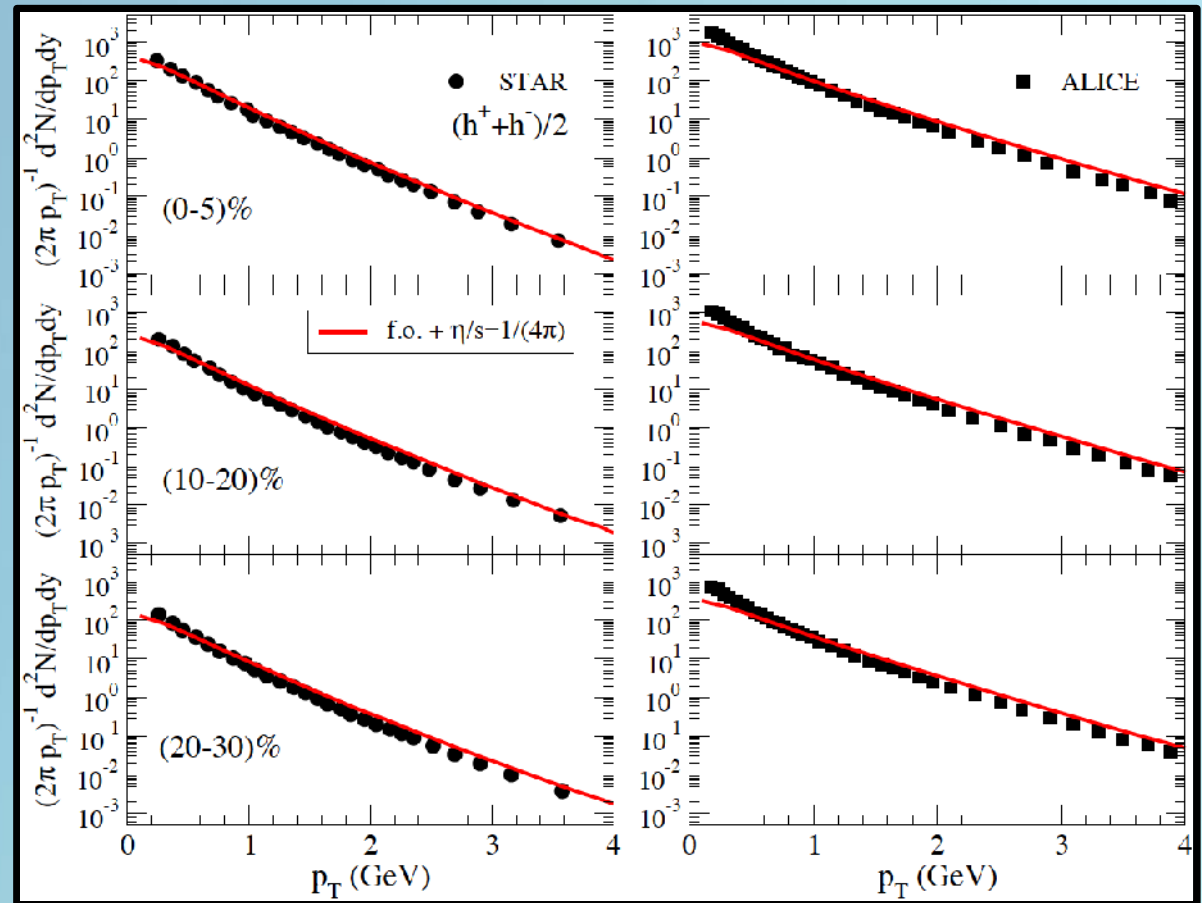
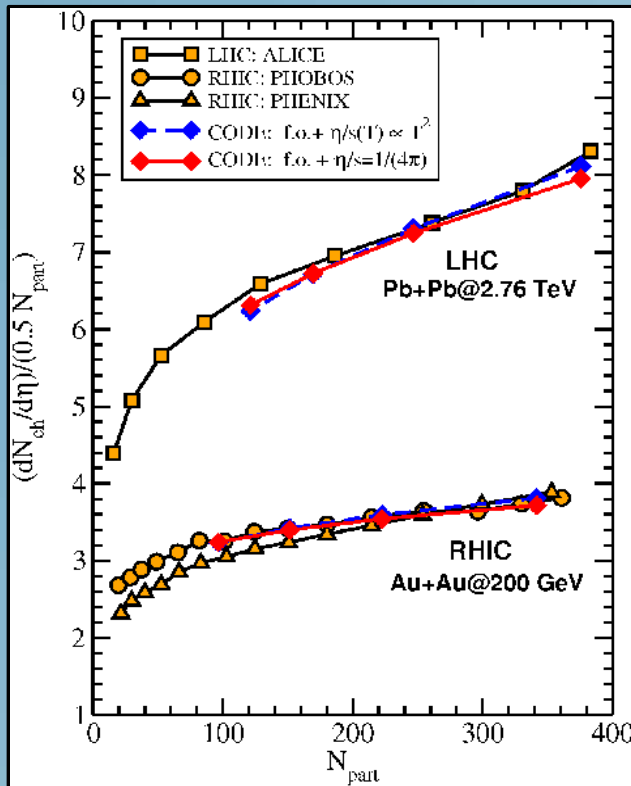
$$\frac{1}{\tau A_T} \frac{dN_{ch}}{d\eta} \propto T^3$$

\sqrt{s}	62 GeV	200 GeV	2.76 TeV
T_0	290 MeV	340 MeV	590 MeV
τ_0	0.7 fm/c	0.6 fm/c	0.3 fm/c



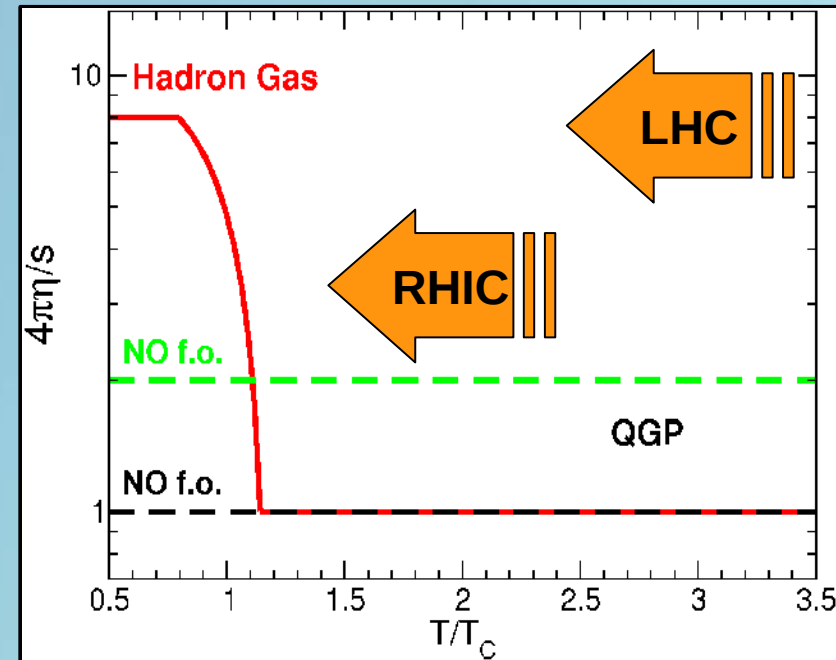
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Discarded in viscous hydro**



kinetic freeze-out scheme

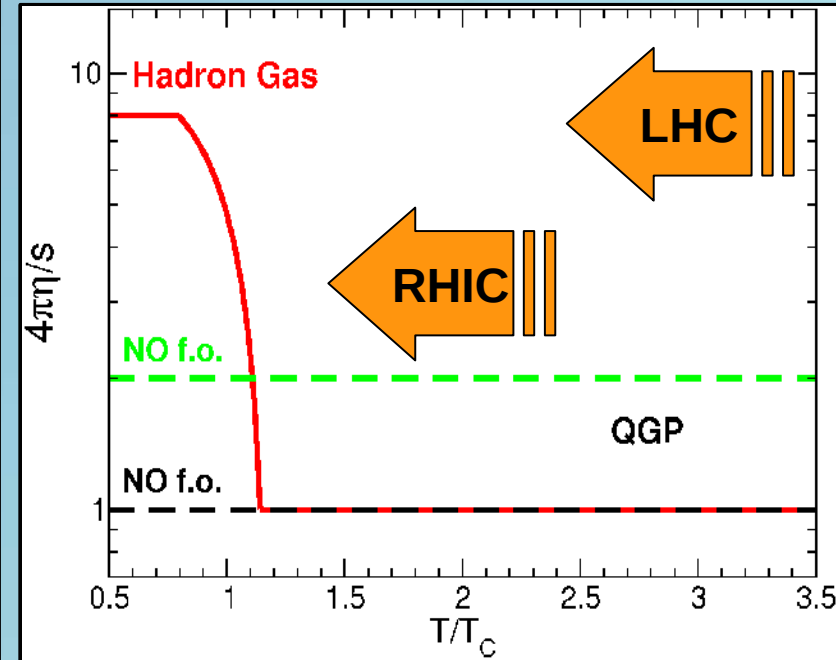
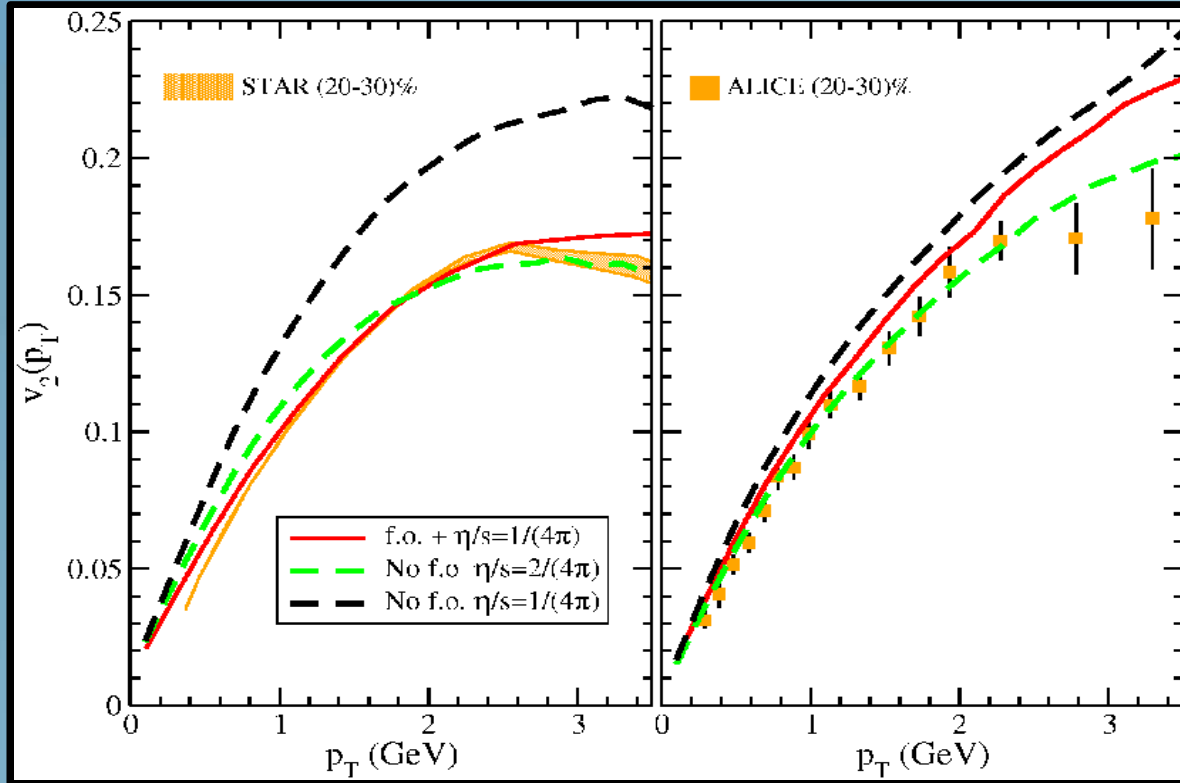
- The f.o. is the increase of η/s in the cross-over region, with a smooth transition between the QGP and the hadronic phase, the collisions are switched off.



For the v_2 similar to cut-off at $\varepsilon_0 = 0.7 \text{ GeV/fm}^3$

kinetic freeze-out scheme

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).



S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

RHIC:

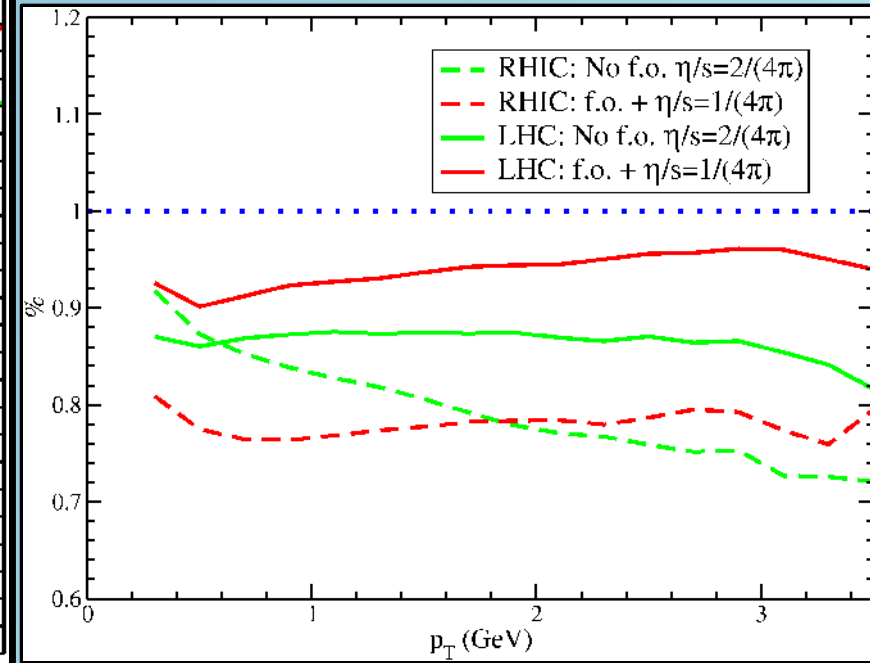
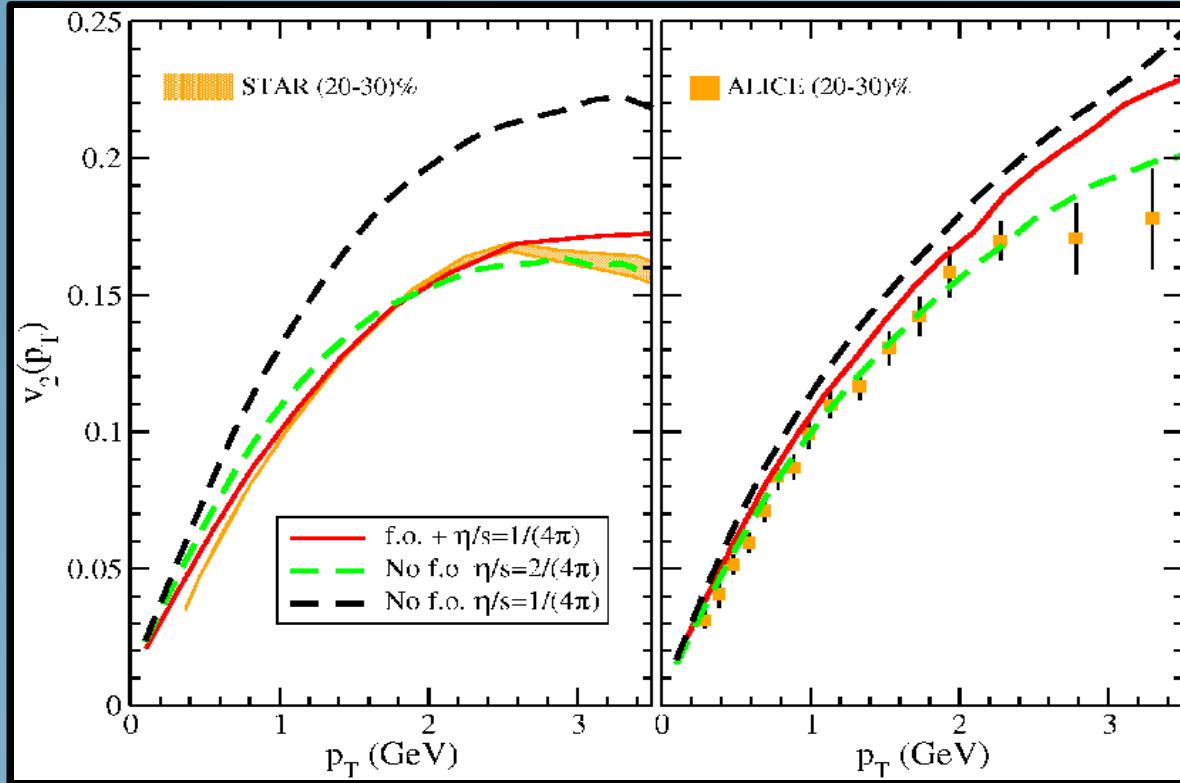
- Like viscous hydro the data are close to $\eta/s=1/(4\pi) + \text{f.o.}$
- Sensitive reduction of the v_2 when the f.o. is included the effect is about of 20%.
- $p_T < 2.5$ GeV good agreement with the experimental data.

LHC:

- $p_T < 2$ GeV like hydro data described with $\eta/s=1/(4\pi) + \text{f.o.}$
- Smaller effect on the reduction of the v_2 when the f.o. is included an effect of about 5%.
- Without the kinetic freezeout the effect of a constant $\eta/s=2(4\pi)^{-1}$ is to reduce the v_2 of 15%.

kinetic freeze-out scheme

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).

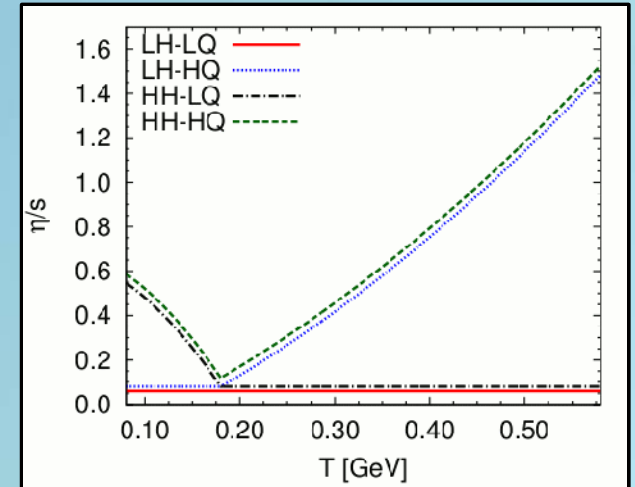
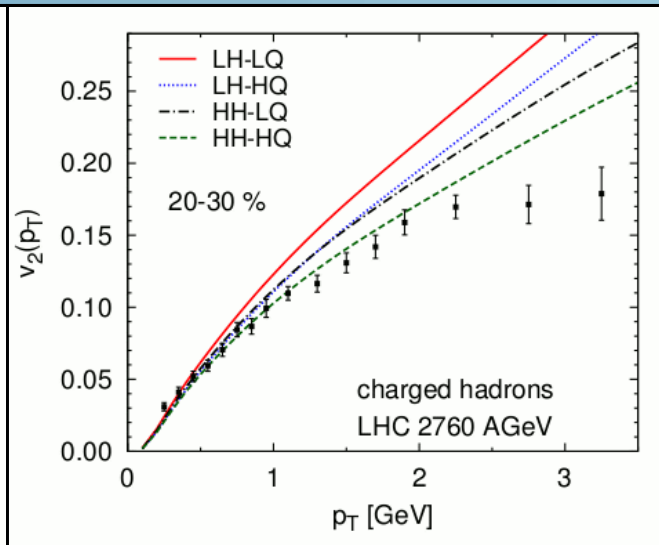
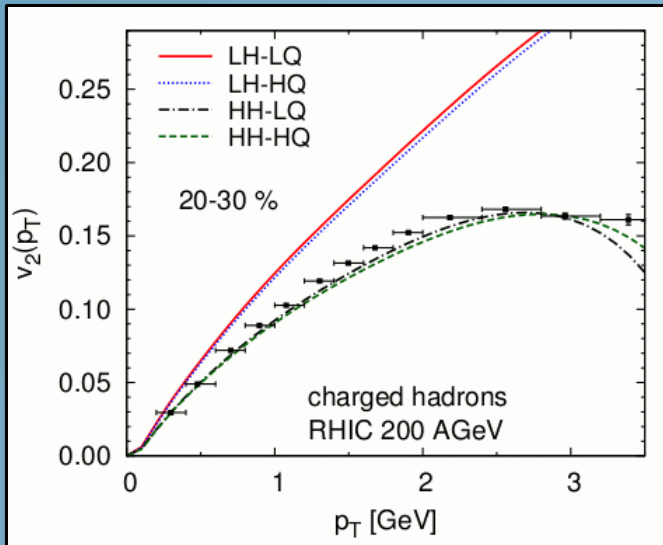


S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

At LHC the contamination of mixed and hadronic phase becomes negligible

Longer life time of QGP $\rightarrow v_2$ completely developed in the QGP phase
(at least up to 3 GeV)

Effect of $\eta/s(T)$ in Hydro: Niemi et al.



Niemi et al., PRL 106 (2011).

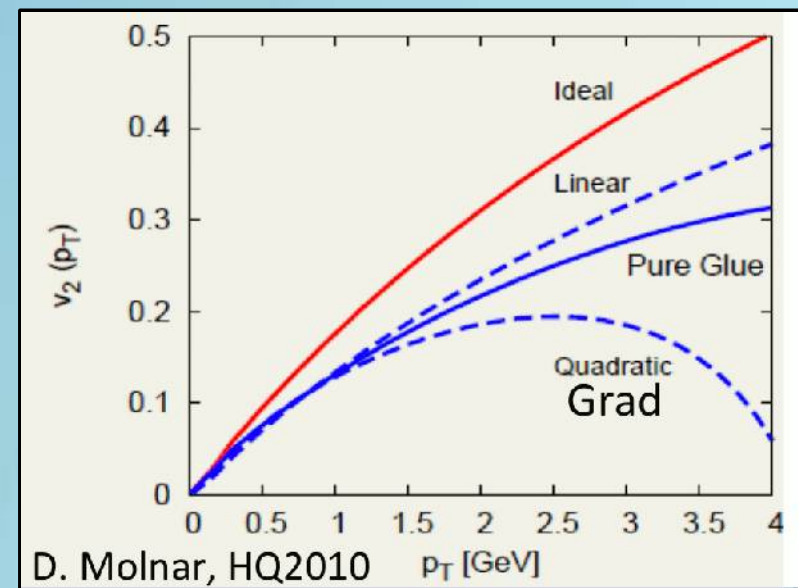
$$T^{\mu\nu} = T_{eq}^{\mu\nu} + \delta T^{\mu\nu} \leftarrow f_{eq} + \delta f$$

Grad ansatz

R. Lacey et al., PRC82

$$\delta f = \frac{\pi^{\mu\nu} p_\mu p_\nu}{(\epsilon + p) T^2} f_{eq} \approx \frac{\eta}{3s} \frac{p_T^2}{\tau T^2} f_{eq}$$

- This implies that the η is in Relaxation Time Approximation
D. Teaney, Phys.Rev. C68 (2003) 034913
- Hydro is valid up to $p_T \sim 3$ GeV



$\eta/s(T)$ around to a phase transition

- Quantum mechanism

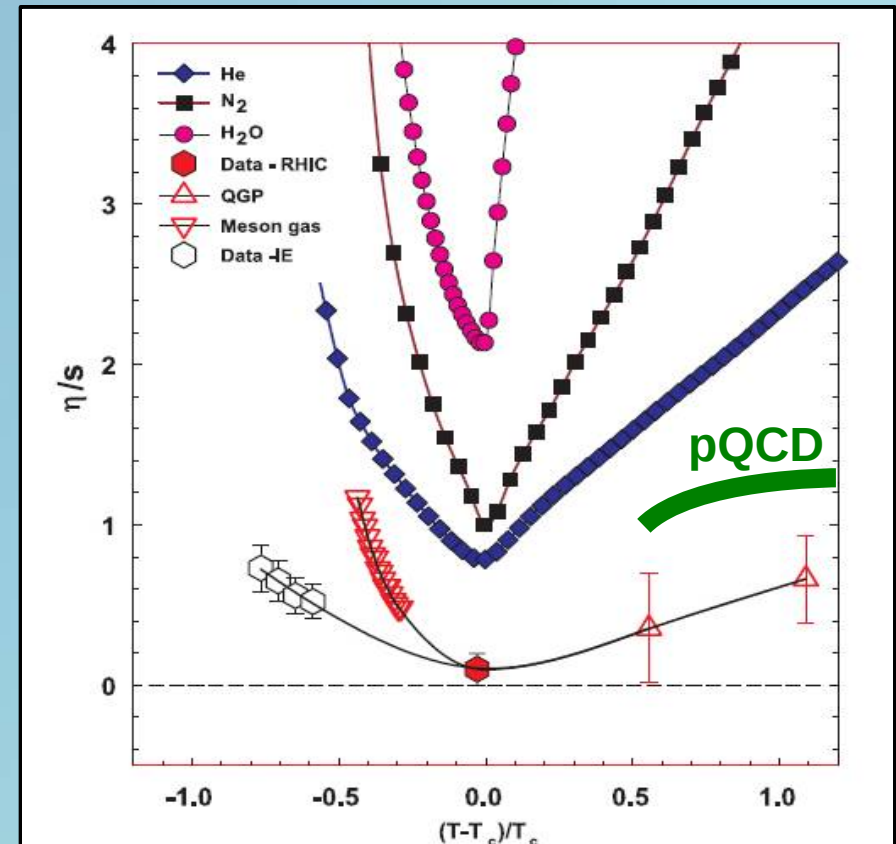
$$\Delta E \cdot \Delta t \geq 1 \rightarrow \eta/s \approx \frac{1}{15} \langle p \rangle \cdot \tau > \frac{1}{15}$$

- AdS/CFT suggest a lower bound $\eta/s = 1/(4\pi) \sim 0.08$

The QGP viscosity is close to this bound!

Do we have signature of a 'U' shape of $\eta/s(T)$ for the QCD matter ?

P. Kovtun et al., Phys.Rev.Lett. 94 (2005) 111601.
 L. P. Csernai et al., Phys.Rev.Lett. 97 (2006) 152303.
 R. A. Lacey et al., Phys.Rev.Lett. 98 (2007) 092301.



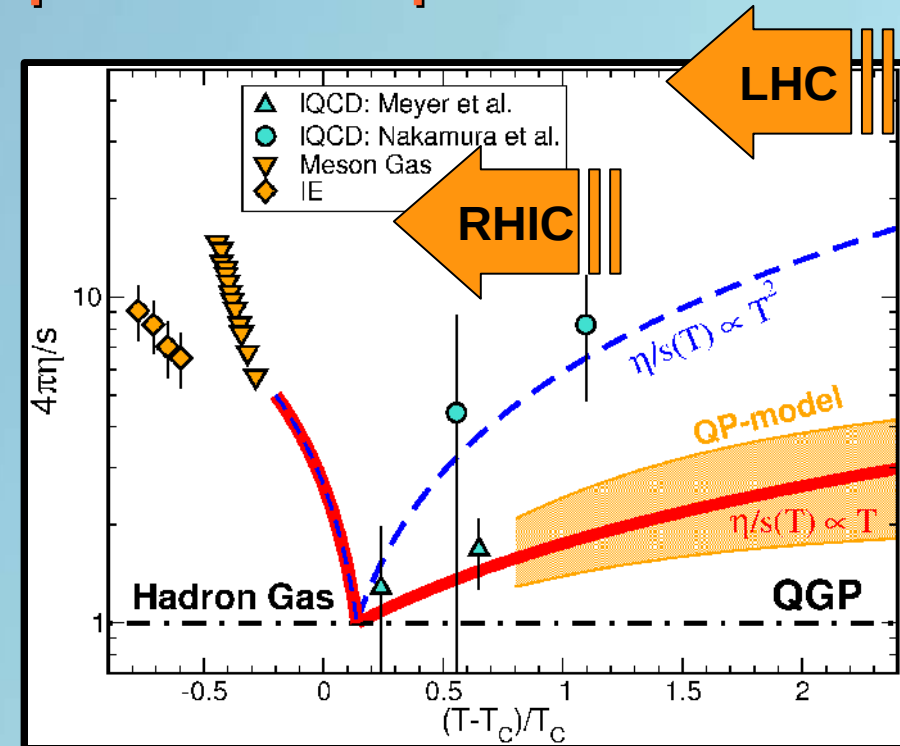
From pQCD: $\eta/s \sim \frac{1}{g^4 \ln(1/g)} \Rightarrow \eta/s \sim 1$

P. Arnold et al., JHEP 0305 (2003) 051.

Temperature dependent $\eta/s(T)$

Phase transition physics suggest a T dependence of η/s also in the QGP phase

- LQCD some results for quenched approx. large error bars
- Quasi-Particle models seem to suggest a $\eta/s \sim T^\alpha$ $\alpha \sim 1 - 1.5$.
- Chiral perturbation theory \rightarrow Meson Gas
- Intermediate Energies – IE ($\mu_B > T$)

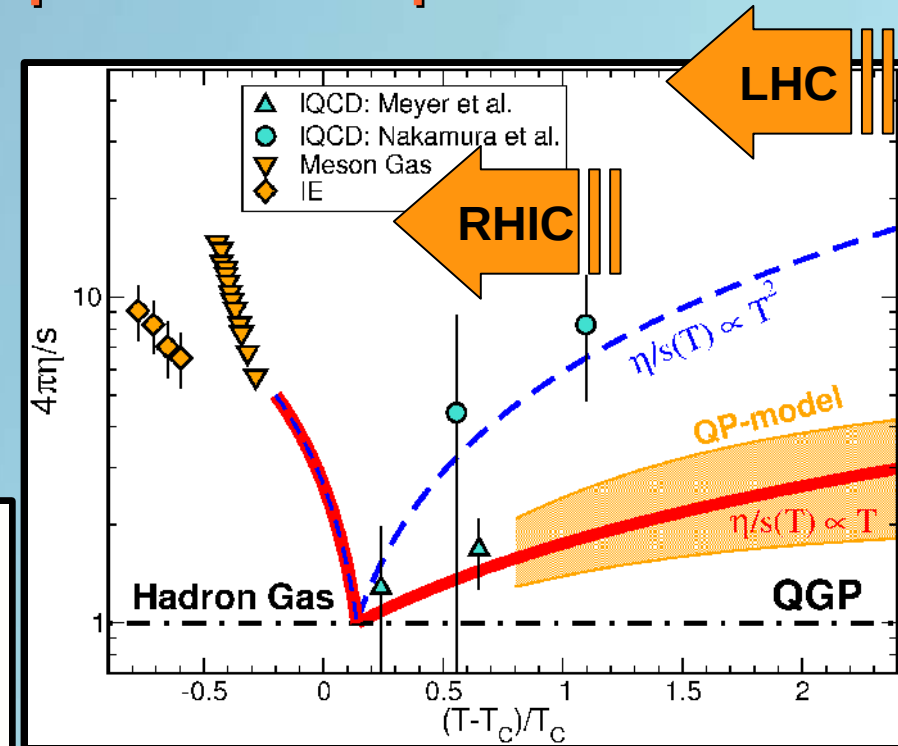
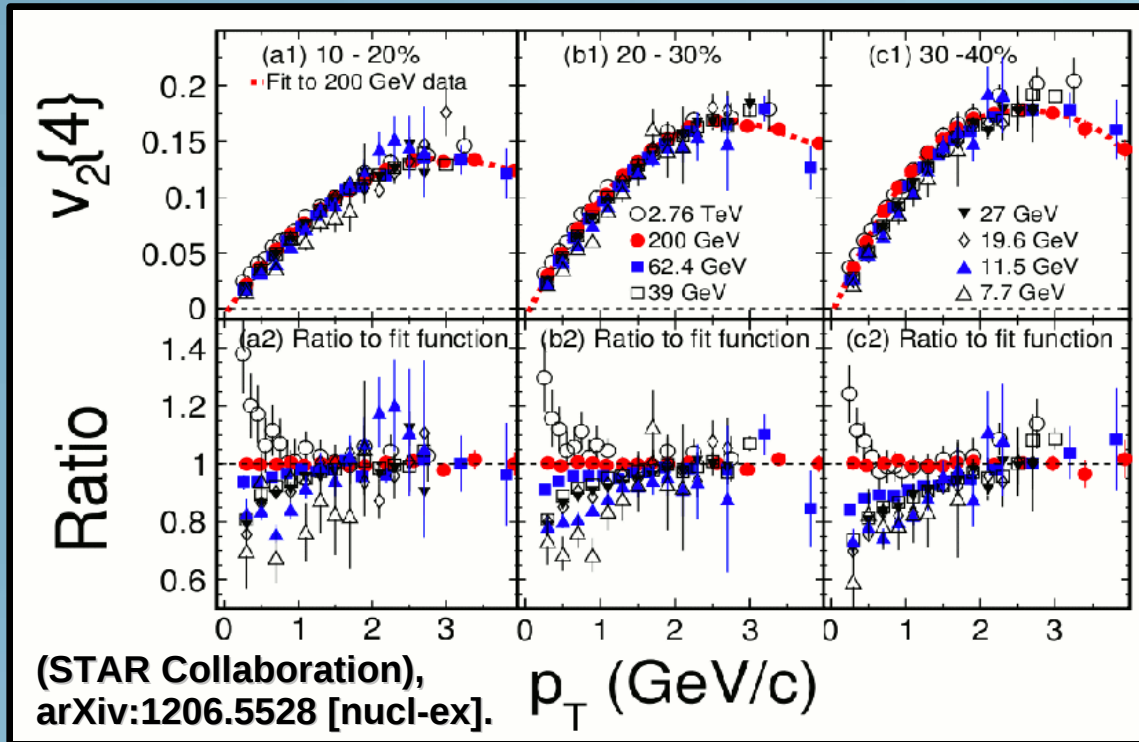


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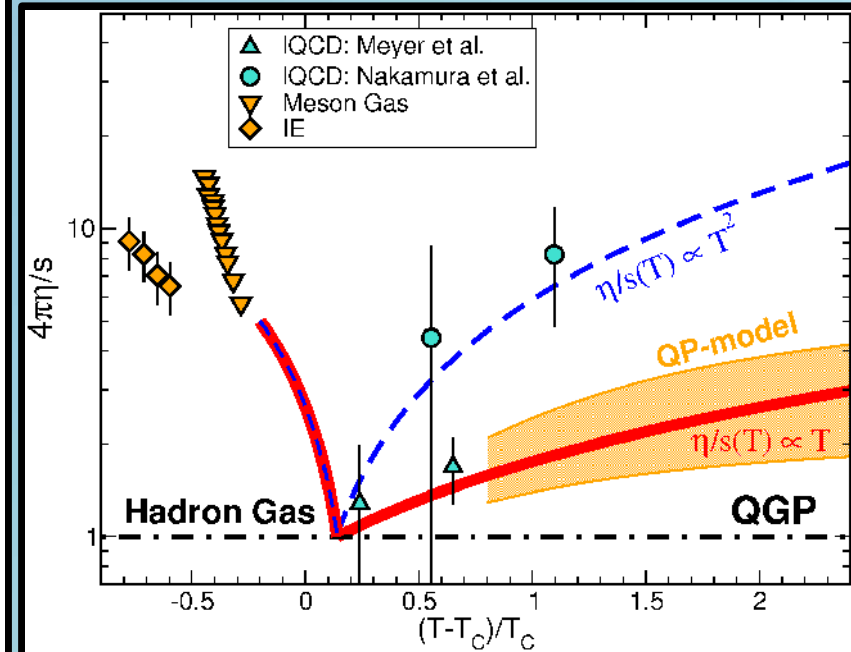
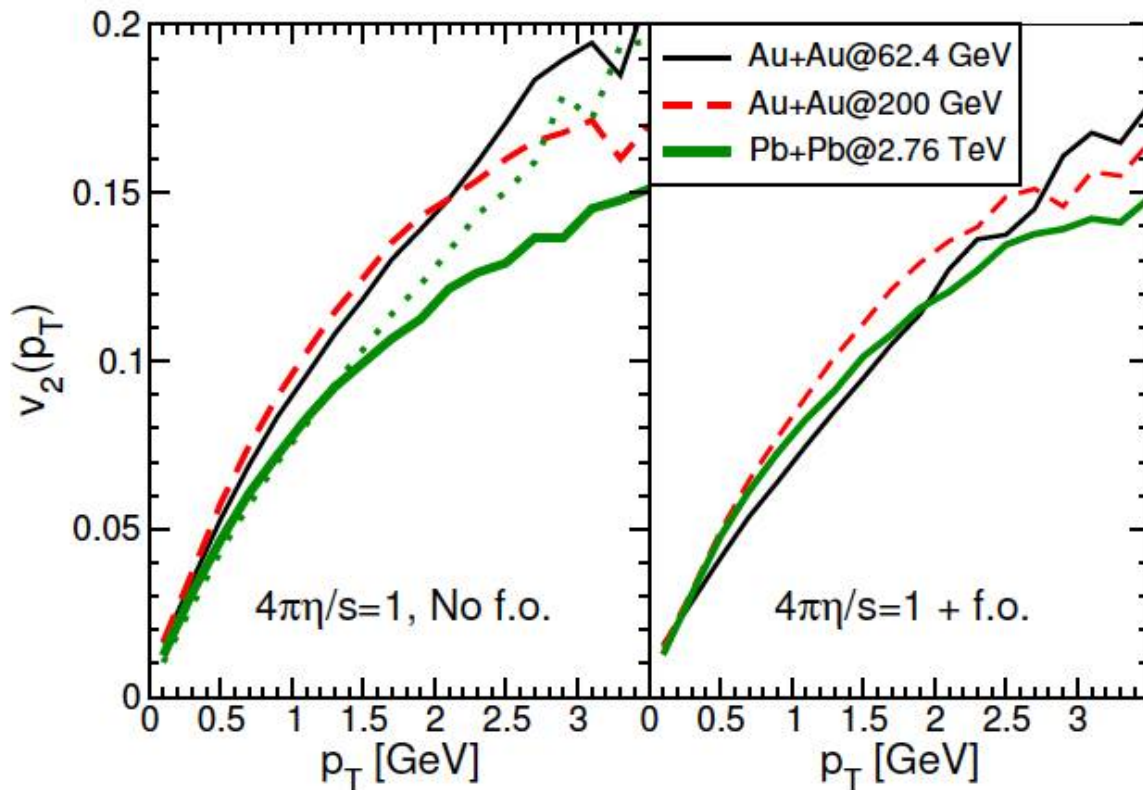
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S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

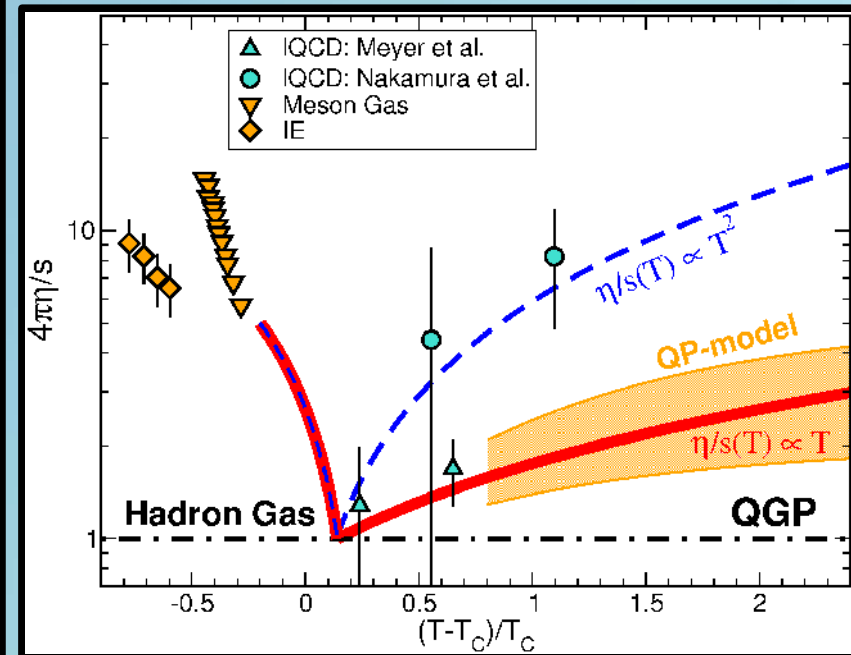
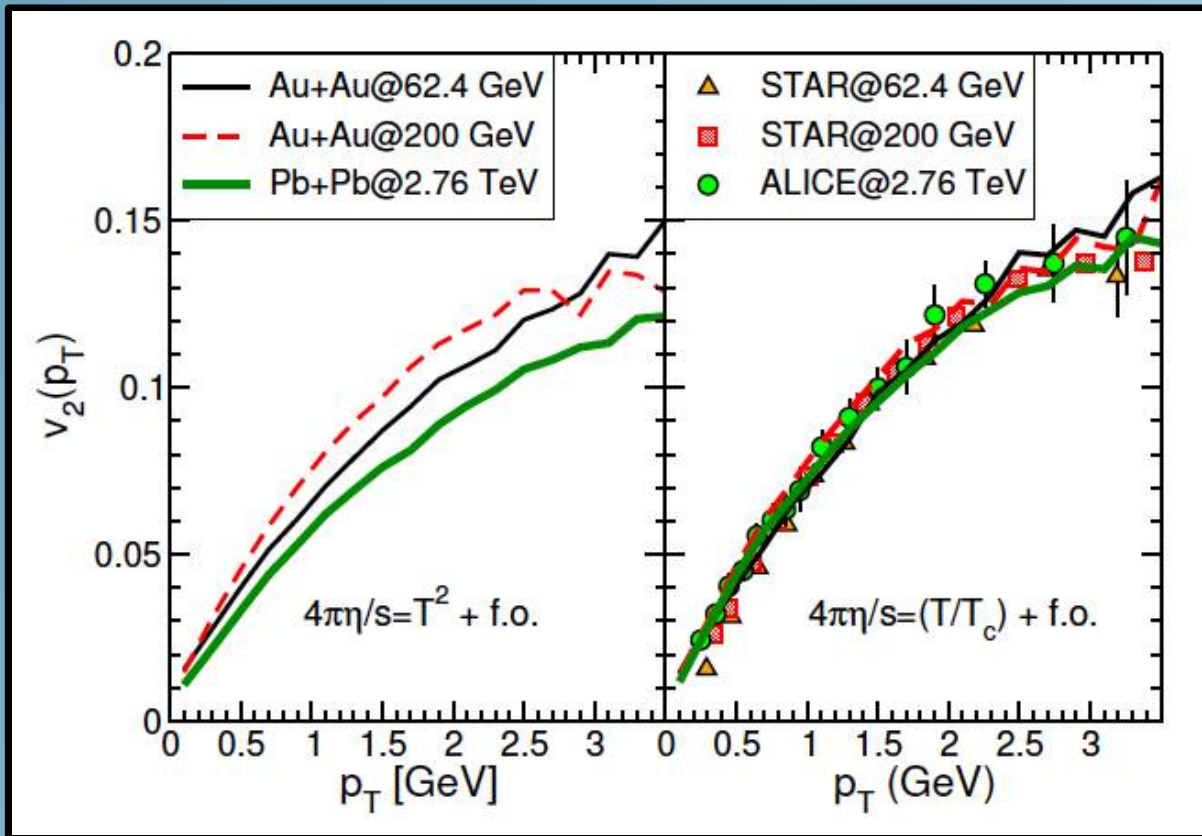
Temperature dependent $\eta/s(T)$



Plumari, Greco, Csernai,
arXiv:1304.6566

- For $4\pi\eta/s=1$ during all the evolution of the fireball we get a discrepancy for the $v_2(p_T)$, in particular we observe a smaller $v_2(p_T)$ at LHC.
- Similar results for $\eta/s \propto T^2 \rightarrow$ a discrepancy about 20%.
- Notice only with RHIC \rightarrow scaling for $4\pi\eta/s=1$ LHC data play a key role

Temperature dependent $\eta/s(T)$



Plumari, Greco, Csernai,
arXiv:1304.6566

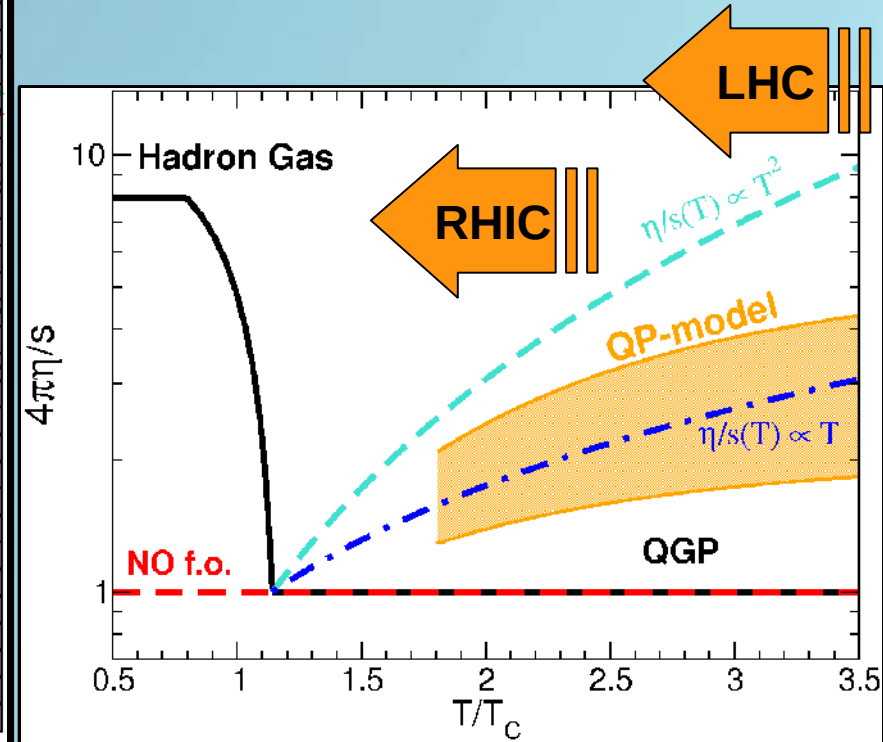
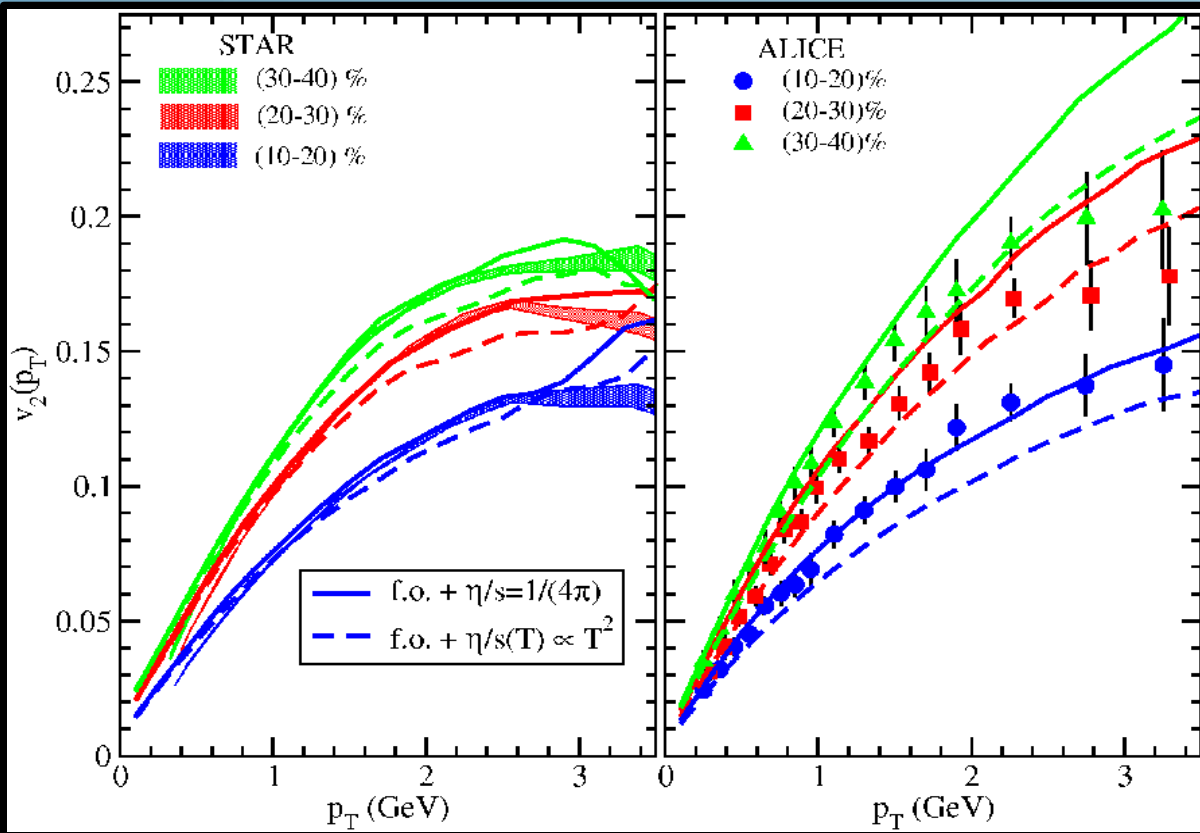
- Invariance of $v_2(p_T)$ in BES suggest that the system goes through a phase transition.
- Hope: v_n , $n > 3$ with an event-by-event analysis will put even stronger constraint
- Implementation of local fluctuation under development
- Similar results: R. A. Lacey et al., arXiv:1305.3341 [nucl-ex].

Conclusions and Outlook

- Enhancement of $\eta/s(T)$ in the cross-over region affect differently the expanding QGP from RHIC to LHC.
- At LHC nearly all the v_2 from the QGP phase.
- The scaling of $v_2(p_T)$ from Beam Energy Scan indicate a 'U' shape of $\eta/s(T)$ this would be a first signature of $\eta/s(T)$ behavior typical of a phase transition.

Temperature dependent $\eta/s(T)$

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).



S. Plumari et al., arXiv:1209.0601 [hep-ph].

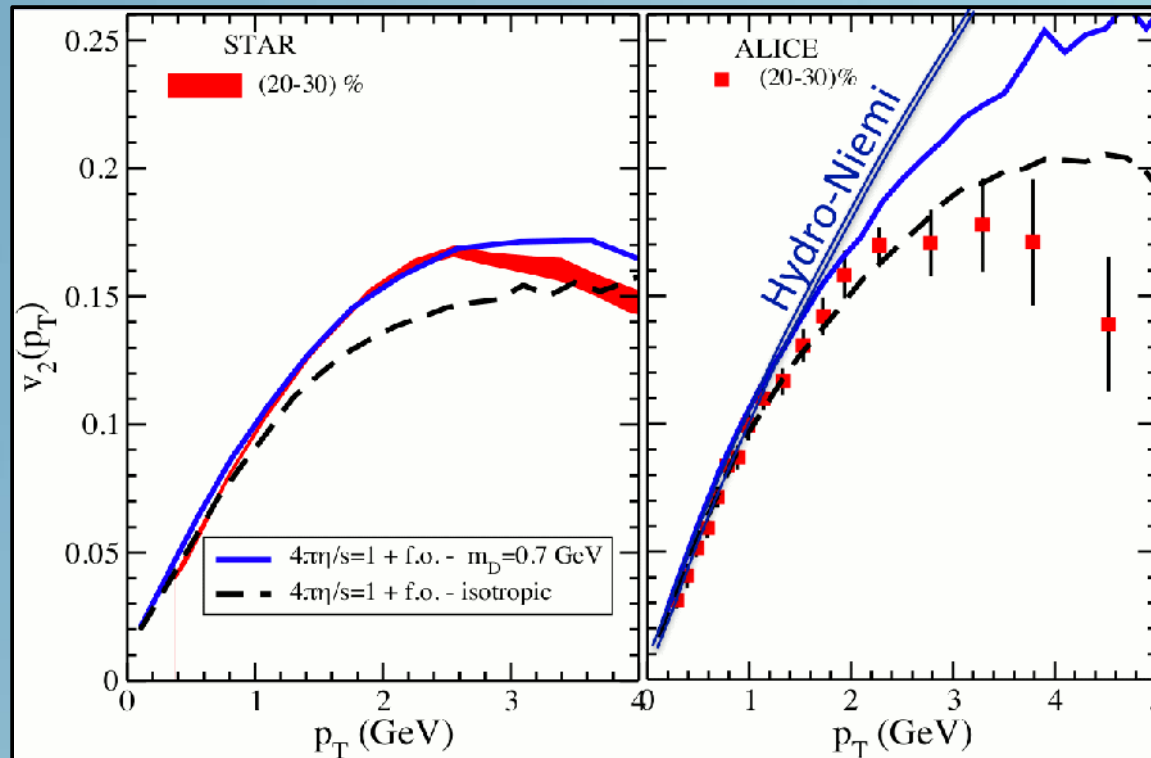
RHIC:

- The v_2 is insensitive to the value of η/s in the QGP phase
- $\eta/s \sim T^2$ cannot account for the v_2 decrease for $p_T > 2.5$ GeV.

LHC:

- The v_2 more sensitive to the QGP phase but still a strong temperature dependence in η/s has a small effect in the $v_2(p_T)$.

Relevance of the microscopic scale



- Microscopic details of the cross section have an effect on v_2 for $p_T > 1.5 \text{ GeV}$
- Fixed the ratio η/s of the fluid the isotropic cross section leads to a smaller v_2
- An appropriate $m_D(T) = g(T) T$ gives a different behaviour between RHIC and LHC.