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# KINETIC THEORY COMPUTATION OF ELLIPTIC FLOW

Based on collaboration with:
V. Greco, S. Plumari and F. Scardina

*Paris, 2013 June 11*

# Plan of the talk

- Heavy ion collisions (HICs) and elliptic flow
- ***Elliptic Flow in HICs from kinetic theory***
- Conclusions and Outlook

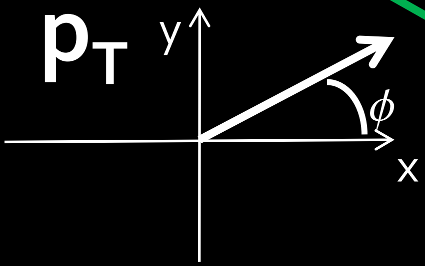
# Elliptic flow in RHICs

**Momentum distribution**

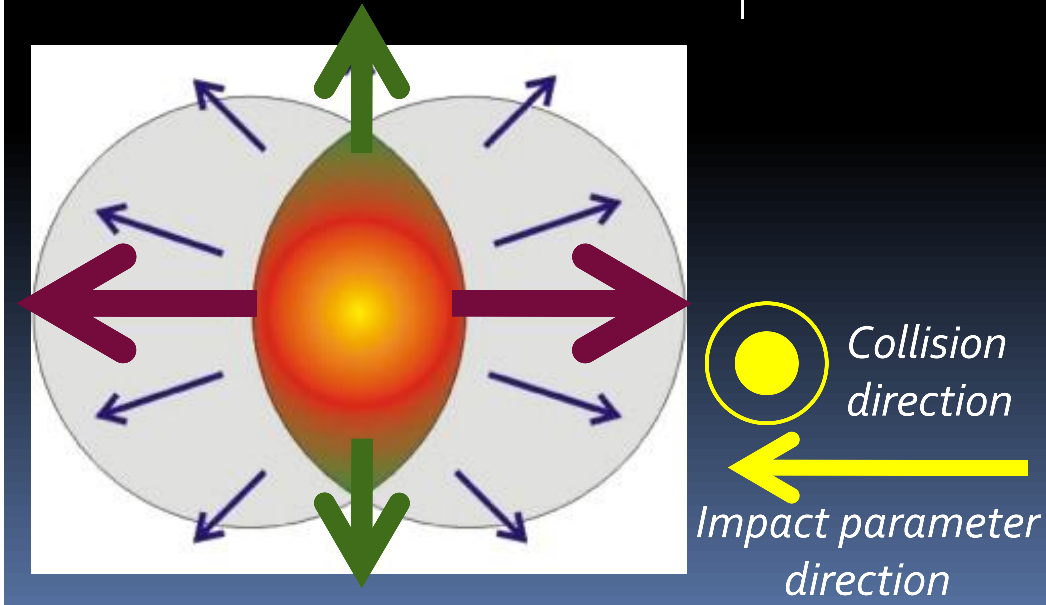
$$\frac{d^3N}{dy\,p_T\,dp_T\,d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy\,p_T\,dp_T}\left[1 + 2v_2(y,p_T)\cos 2\phi\right]$$

***Elliptic flow:***
leading contribution to anisotropy in momentum space
[J. Y. Ollitraut, PRD46 (1992)]

Fourier decomposition in terms of *harmonics* in the *transverse plane*.

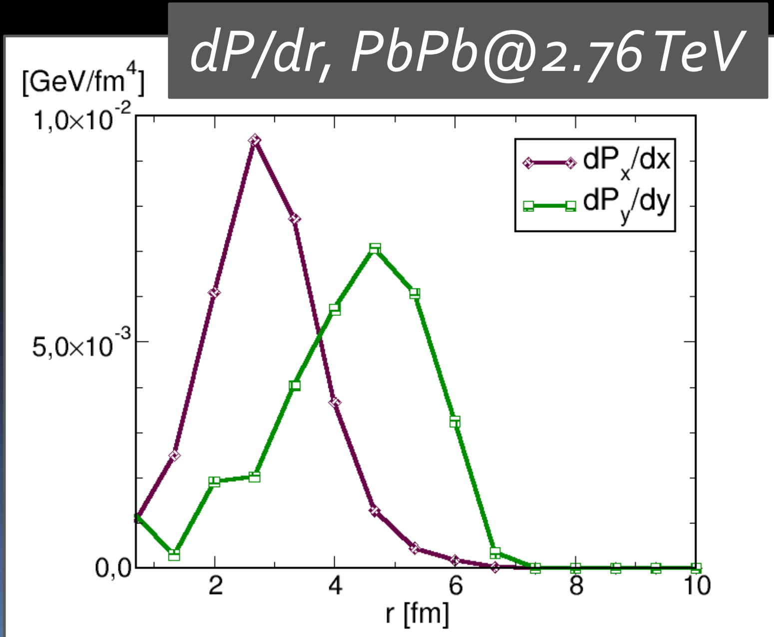


$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$



*Collision direction*

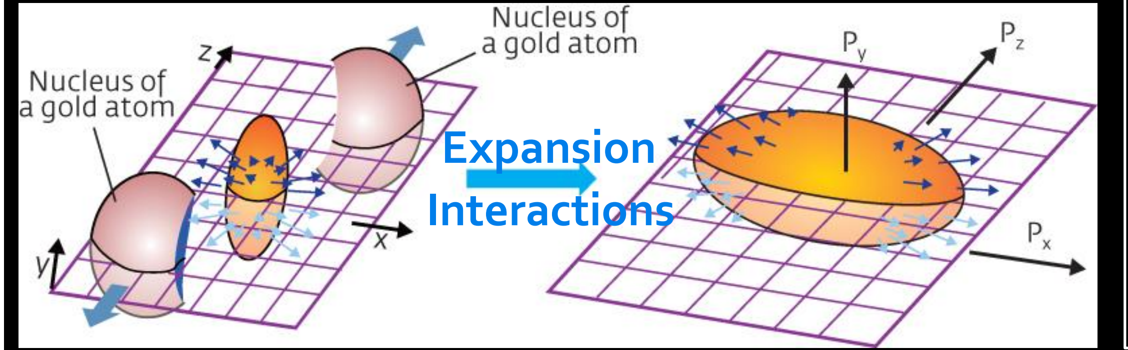
*Impact parameter direction*



*dP/dr, PbPb@2.76 TeV*

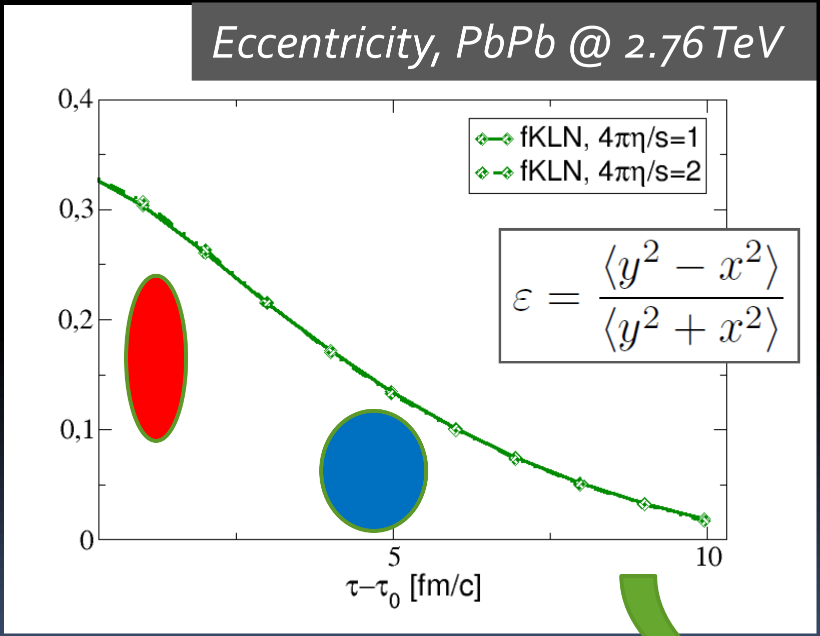
M. R. *et al., work in progress*

# Elliptic flow in RHICs



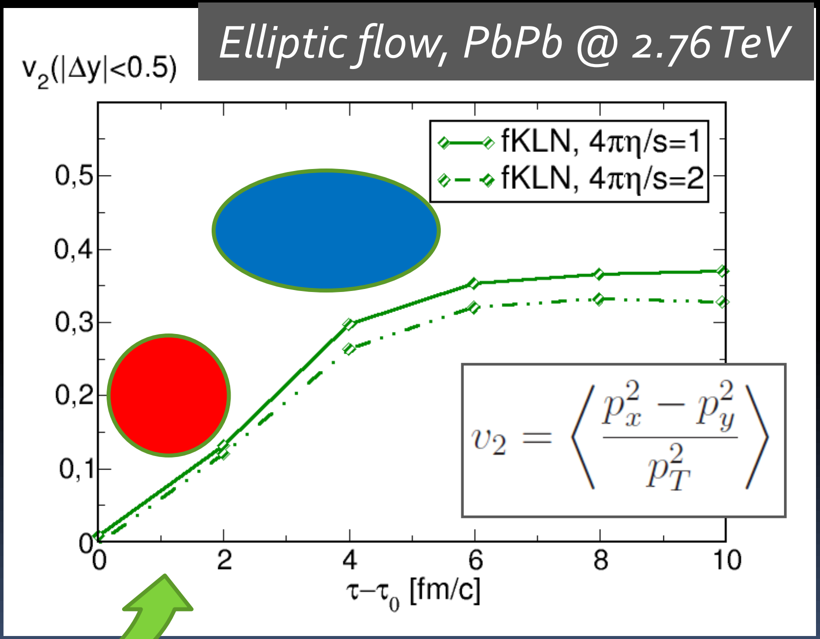
***Collective vs Thermal:***
*v2 measures how efficiently the anisotropy of configuration in the initial state is transmitted to momenta in final states*
[J. Y. Ollitraut, PRD46 (1992)]

*Eccentricity, PbPb @ 2.76 TeV*



$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

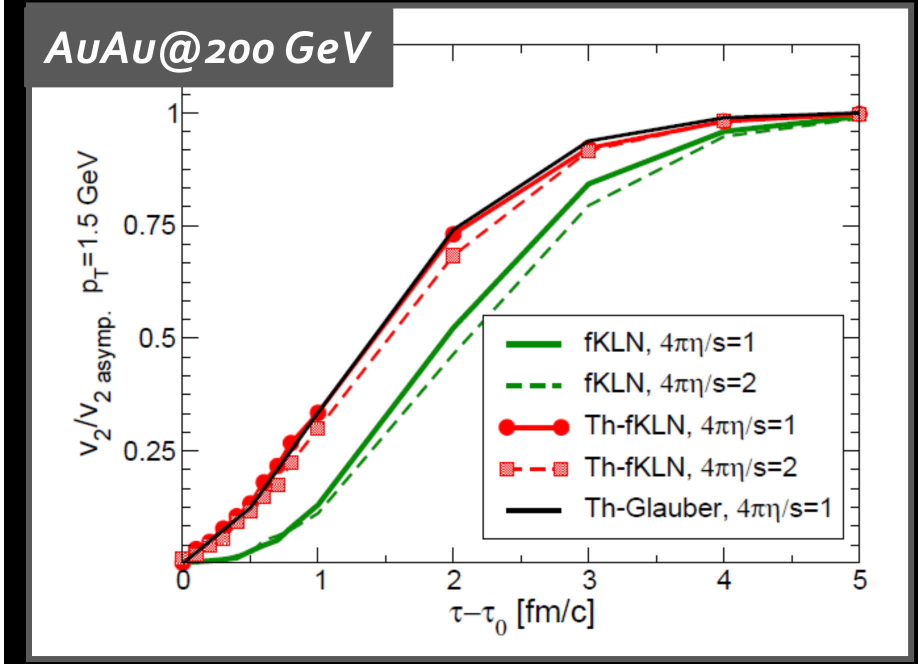
*Elliptic flow, PbPb @ 2.76 TeV*



$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$

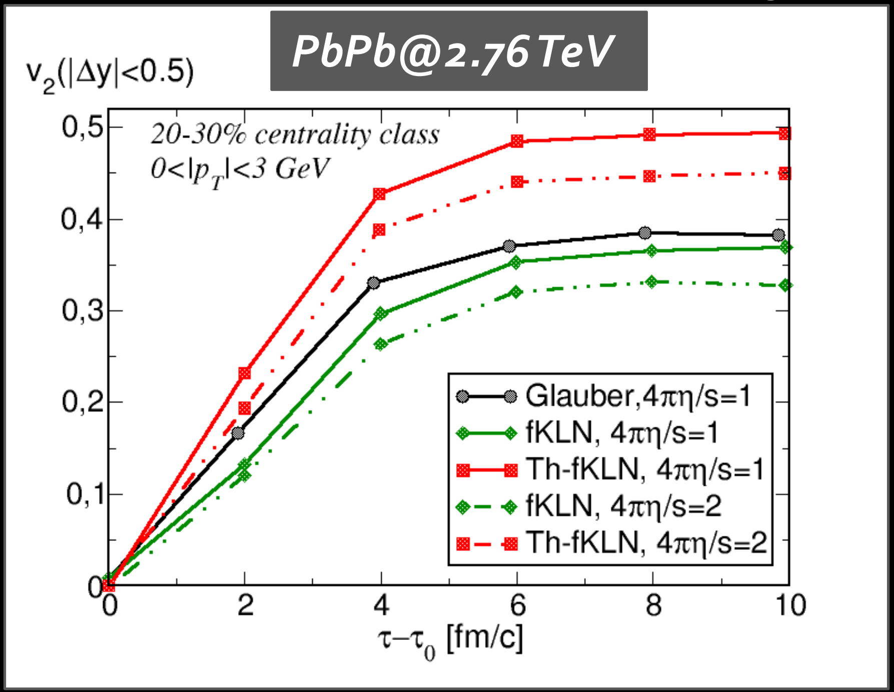
***Transfer of anisotropy***

# Elliptic flow in RHICs



M. R. *et al.*, 1303.3178 [nucl-th]

M. R. *et al., work in progress*



***Elliptic flow is sensitive to the properties of the hot and dense state of partonic matter created after the collision.***

***Reviews:***

Teaney, arXiv:0905.2133 [nucl-th]

Snellings, arXiv:1102.3010 [nucl-ex]

***Scenario in agreement with:***

Csernai *et al.*, PRL **97**, 152303 (2006)
Greco *et al.*, PRC **68**, 034904 (2003)
Peschanski and Saridakis, PRC **80** (2009)
Huovinen and Petreczky, NPA **837** (2010
Zhang *et al.*, PLB 99 (1998)
Heinz and Kolbe, **QGP3**
Zhang *et al.*, PLB 455 (1999)

*More in S. Plumari's talk*

# Boltzmann equation and QGP

In order to ***simulate*** the temporal evolution of the fireball we solve the ***Boltzmann equation*** for the parton distribution function ***f***:

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right) f(\boldsymbol{x},\boldsymbol{p},t) = C[f]$$

L. Boltzmann, 1872

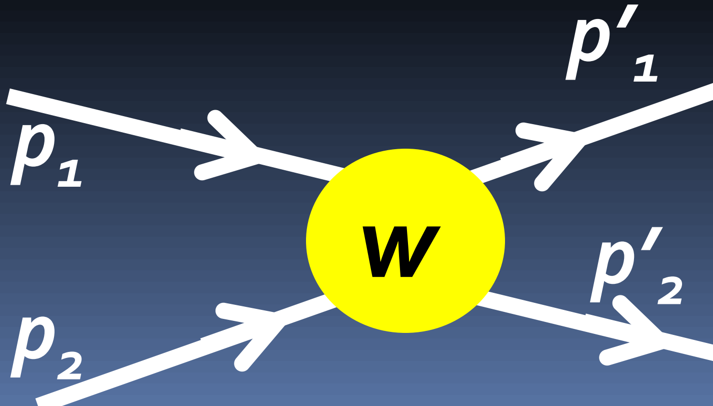
***Drift term*** ***Collision integral***

***Drift term***: change of ***f*** due to particles flowing into and out of the phase space volume centered at (***x***,***p***).

***Collision integral***: change of ***f*** due to collision processes in the phase space volume centered at (***x***,***p***).

For the case of 12->1'2' processes:

$$C[f] = \frac{1}{2}\int d\boldsymbol{p}_2 \int d\boldsymbol{p}'_1 \int d\boldsymbol{p}'_2 \; w(12\to 1'2') \times \left[f(\boldsymbol{x},\boldsymbol{p}'_1,t) f(\boldsymbol{x},\boldsymbol{p}'_2,t) - f(\boldsymbol{x},\boldsymbol{p}_1,t) f(\boldsymbol{x},\boldsymbol{p}_2,t)\right]$$



*More in S. Plumari's talk*

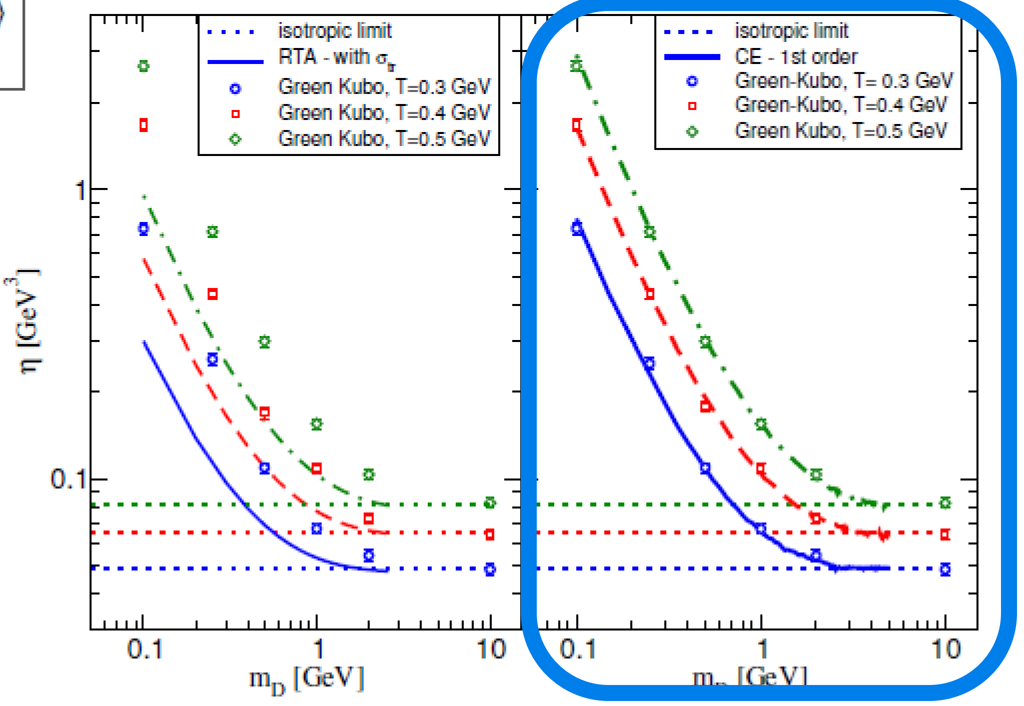
# Boltzmann equation and QGP

$$\eta = \frac{1}{T}\int_0^\infty dt \int_V d^3x \,\langle \pi^{xy}(\mathbf{x},t)\,\pi^{xy}(\mathbf{0},t)\rangle$$

***Viscosity of a gluon plasma***

$$\frac{d\sigma_{gg\to gg}}{dt} = \frac{9\pi^2\alpha_s^2}{2}\,\frac{1}{(t-m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$

Plumari *et al.*, Phys. Rev. C86 (2012).



**CE**

$$f = f_{eq} + \delta f \Rightarrow C[f_{eq} + \delta f]$$

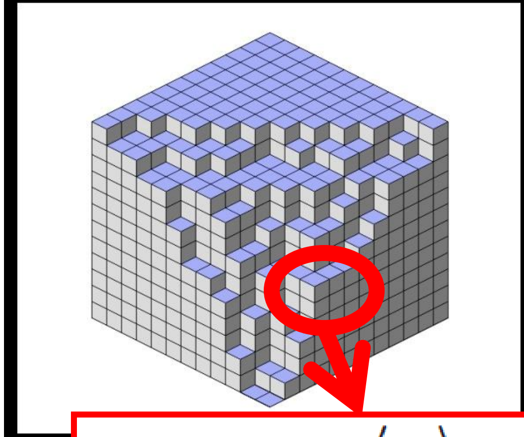
*Chapman-Enskog*

***CE is a better approximation to the Green-Kubo result. This is a useful observation, since CE offers analytical tool to relate $\eta$ to $\sigma$ which we use in our transport code.***

*More in S. Plumari's talk*

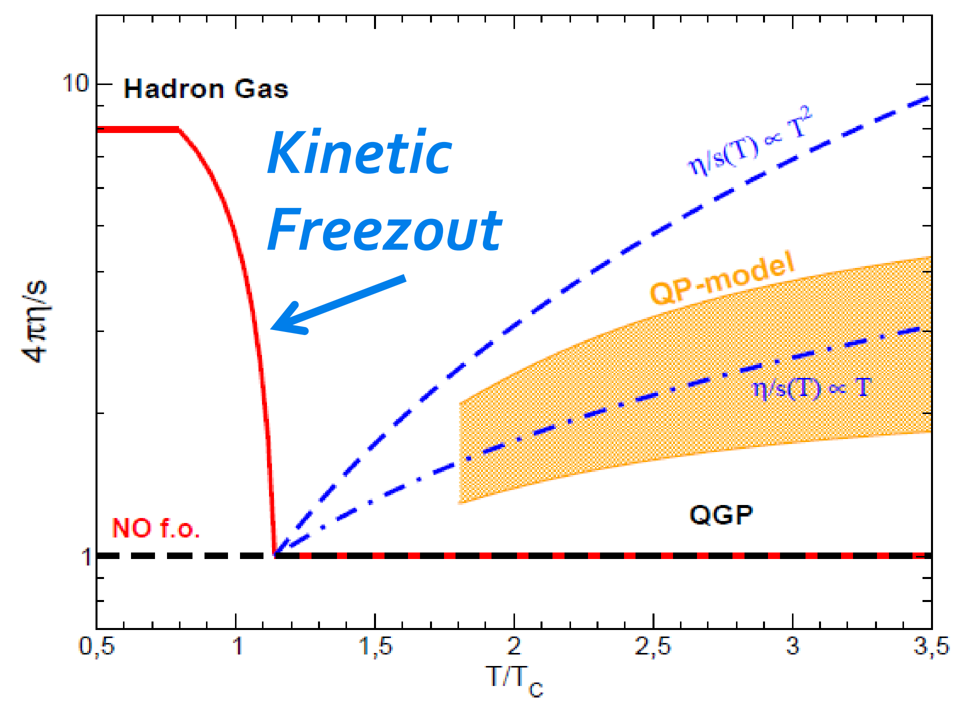
# Kinetic Theory at fixed $\eta/s$

We use Boltzmann equation to simulate a fluid at ***fixed*** $\eta(T)/s$. ***Cross section*** is ***computed*** in ***each configuration space cell*** according to ***Chapman-Enskog equation*** to give the ***wished value of*** $\eta/s$ ***at local T***.



**CE**

$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho}\frac{1}{\sigma}$$

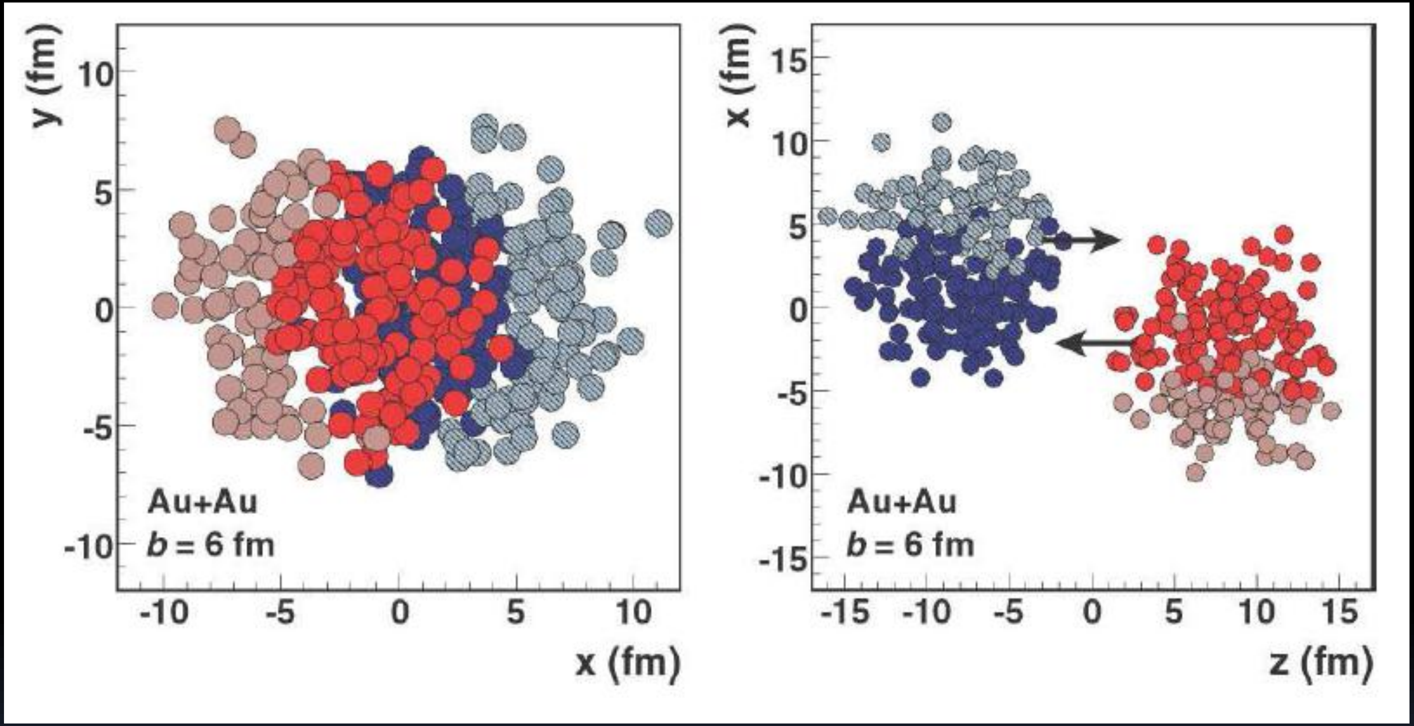


For ***small transverse momenta***, this approach is meaningful since ***this momentum domain*** corresponds to the ***hydro domain***, where ***microscopic details are not important*** and only $\eta/s$ is relevant.

Plumari *et al.*, Phys. Rev. C86 (2012).
Greco *et al.*, Phys. Lett. B670 (2009).
Plumari *et al.*, J.Phys.Conf.Ser. 420 (2013).

# Initial condition: Glauber

**(Almost) Geometrical** description of the fireball:



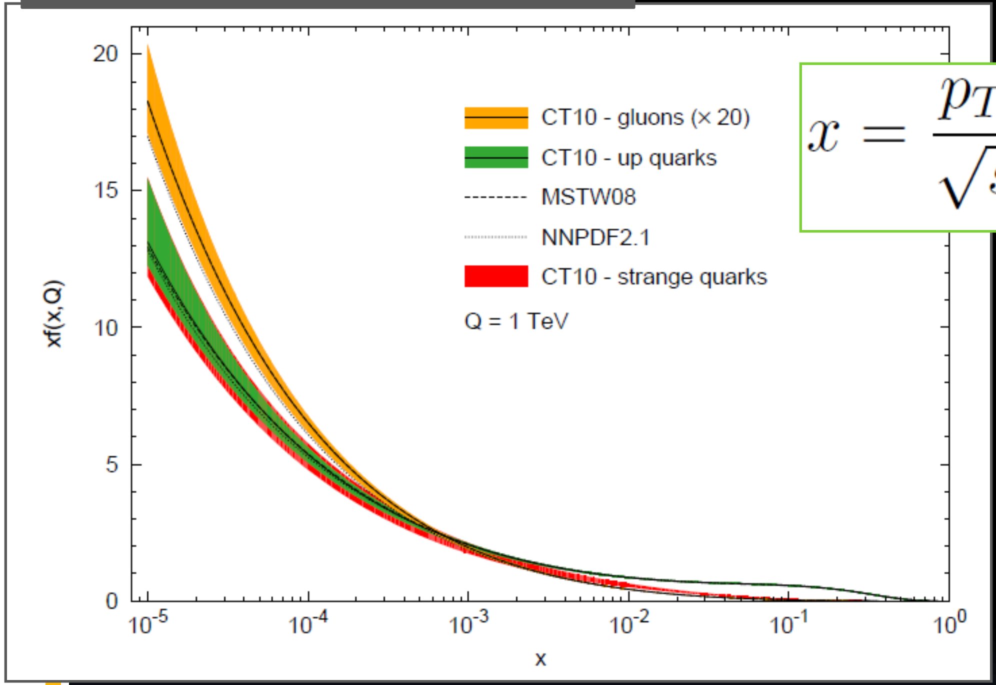
Assuming a nucleon distribution in the parents nuclei (typically a ***Woods-Saxon***), one counts ***how many particles*** from each nucleus are present in the ***overlap region***; among them, the ***participants*** are the nucleons that effectively can have an interaction (in fact, the particles that *are in the overlap region* but *do not interact*, are not considered).

For a review see: Miller *et al.*, Ann.Rev.Nucl.Part.Sci. **57, 205 (2007)**

McLerran and Venugopalan, PRD **49**, 2233 (1994)
McLerran and Venugopalan, PRD **49**, 3352 (1994)

# Saturation in a nutshell

*Parton distribution functions*

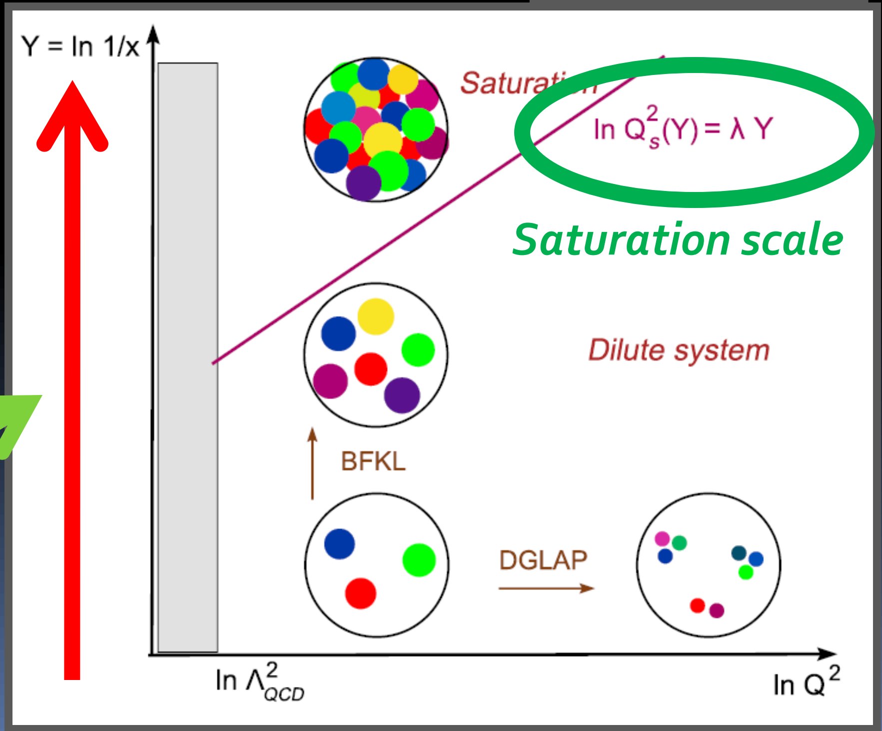


$$x = \frac{p_T}{\sqrt{s}}$$

Brandt and Klasen,arXiv:1305.5677

*Direction of **decreasing** transverse partons area (1/Q²)*

*Phase Diagram*

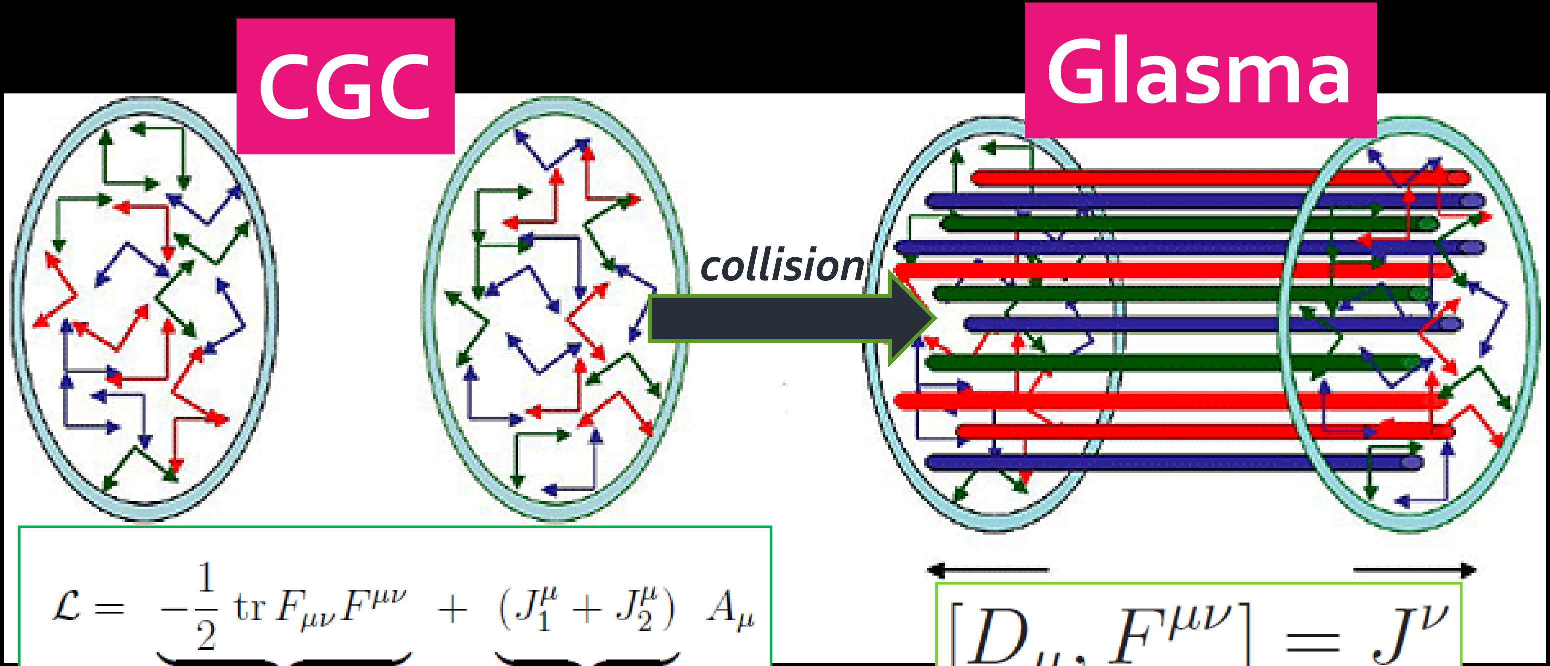


***Saturation scale***

*Direction of **increasing** gluon number*
***The system evolves to a dense state, characterized by a large occupation number: Color Glass Condensate (CGC)***

McLerran and Venugopalan, PRD **49**, 2233 (1994)
McLerran and Venugopalan, PRD **49**, 3352 (1994)

# Initial condition: CYM-Glasma



$$\mathcal{L} = \underbrace{-\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}}_{\text{gluon dynamics}} + \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}}\,A_\mu$$

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

***Reviews/Lectures***

McLerran, 2011
Iancu, 2009
McLerran, 2009
Lappi, 2010
Gelis, 2010
Fukushima, 2011

## *Classical Yang-Mills spectrum*

Mehtar-Tani *et al.*, NPA**846**, (2010)

*Exempli gratia*:
***Mode decomposition***, assuming free dispersion law

$$\frac{dN}{dy d^2\boldsymbol{k}_T} = \frac{1}{(2\pi)^2}\frac{1}{|\boldsymbol{k}_T|}\left[\frac{1}{\tau}\boldsymbol{E}_a(\boldsymbol{k}_T)\cdot\boldsymbol{E}_a(-\boldsymbol{k}_T) + \tau\pi_a(\boldsymbol{k}_T)\pi_a(-\boldsymbol{k}_T)\right]$$

# Initial condition: fKLN-Glasma

## *(f)KLN spectrum*

Nardi *et al.*, Nucl. Phys. A**747**, 609 (2005)
Kharzeev *et al.*, Phys. Lett. B**561**, 93 (2003)
Nardi *et al.*, Phys. Lett. B**507**, 121 (2001)
Drescher and Nara, PRC**75**, 034905 (2007)
Hirano and Nara, PRC**79**, 064904 (2009)
Hirano and Nara, Nucl. Phys. A**743**, 305 (2004)
Albacete and Dumitru, arXiv:1011.5161[hep-ph]
Albacete *et al.*, arXiv:1106.0978 [nucl-th]

$$\frac{dN_g}{d^2x_\perp dy} \propto \int \frac{d^2p_T}{p_T^2}\int_0^{p_T} d^2k_T\,\alpha_s(Q^2) \times\phi_A\left(x_A, \frac{(p_T+k_T)^2}{4}; \boldsymbol{x}_\perp\right) \times\phi_B\left(x_B, \frac{(p_T-k_T)^2}{4}; \boldsymbol{x}_\perp\right)$$

$$\phi_A(x_1, k_T^2; \boldsymbol{x}_\perp) = \frac{\kappa\, Q_s^2}{\alpha_s(Q_s^2)}\left[\frac{\theta(Q_s-k_T)}{Q_s^2+\Lambda^2} + \frac{\theta(k_T-Q_s)}{k_T^2+\Lambda^2}\right]$$

*Saturation effects built in the $\phi$s.*

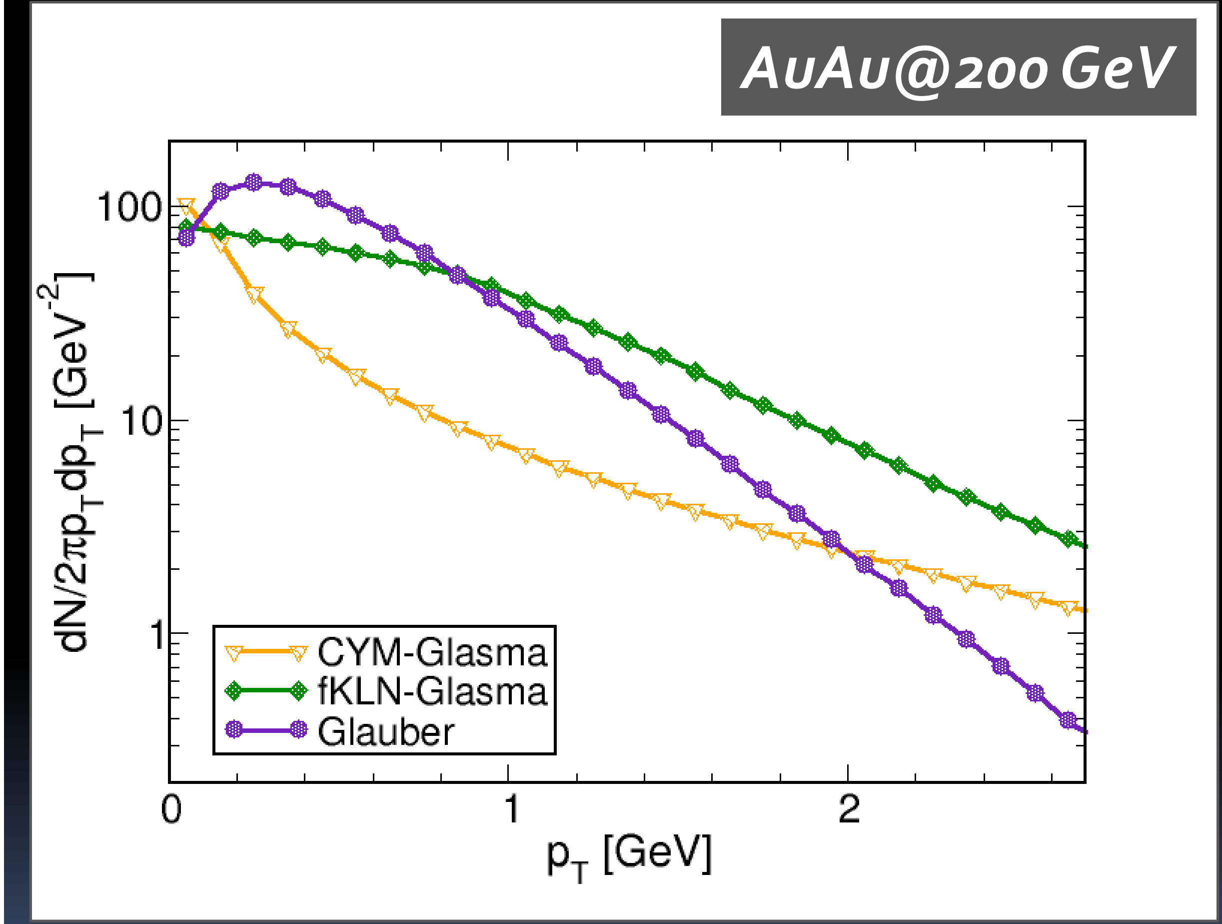
Saturation scale ***Qs*** depends on:
1.) ***position in transverse plane***;
2.) ***gluon rapidity***.

$$Q_{s,A}^2(x, \boldsymbol{x}_\perp) \propto Q_s^2 T_A(\boldsymbol{x}_\perp) x^{-\lambda}$$

***Universal saturation scale***,
in agreement with:
Lappi and Venugopalan, nucl-th/0609021
Drescher and Nara, PRC**75**, 034905 (2007)
Hirano and Nara, PRC**79**, 064904 (2009)

M. R. *et al.*, *work in progress*

# Initial spectra: summary



***CYM data taken from:***

Mehtar-Tani *et al.*, NPA**846**, (2010).

***See also:*** Lappi, PLB**703**, (2011).

# Initial conditions:summary

| IC@RHIC | $\epsilon x$ | Coordinate | Momenta | $\tau$o (fm/c) |
|---|---|---|---|---|
| ***Glauber*** | 0.282 | 0.85Npart+0.15Ncoll | Thermal | 0.6 |
| ***Th-fKLN (Hydro)*** | 0.326 | fKLN | Thermal | 0.6 |
| ***CYM*** | 0.288 | Ncoll | CYM | 0.1-0.2 |
| ***fKLN*** | 0.336 | fKLN | fKLN | 0.1-0.2 |

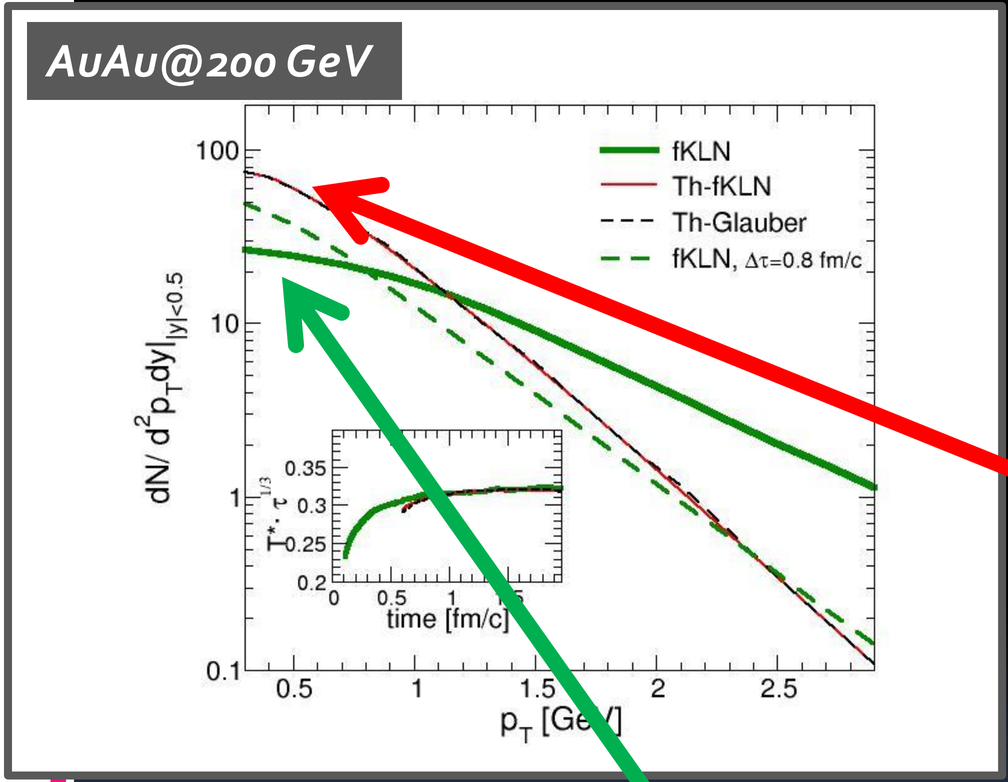
| IC@LHC | $\epsilon x$ | Coordinate | Momenta | $\tau$o (fm/c) |
|---|---|---|---|---|
| ***Glauber*** | 0.282 | 0.85Npart+0.15Ncoll | Thermal | 0.3 |
| ***Th-fKLN (Hydro)*** | 0.326 | fKLN | Thermal | 0.3 |
| ***CYM*** | 0.288 | Ncoll | CYM | 0.1-0.2 |
| ***fKLN*** | 0.336 | fKLN | fKLN | 0.1-0.2 |

Initial eccentricities in agreement with previous estimates:

Drescher and Nara, PRC **75** (2007) 034905
Adil *et al.*, nucl-th/0605012
Gale *et al.*, 1209.6330 [nucl-th]
Schenke *et al.*, 1206.6805 [hep-ph]

M. R. *et al.*, 1303.3178 [nucl-th]

# Initial condition and thermalization



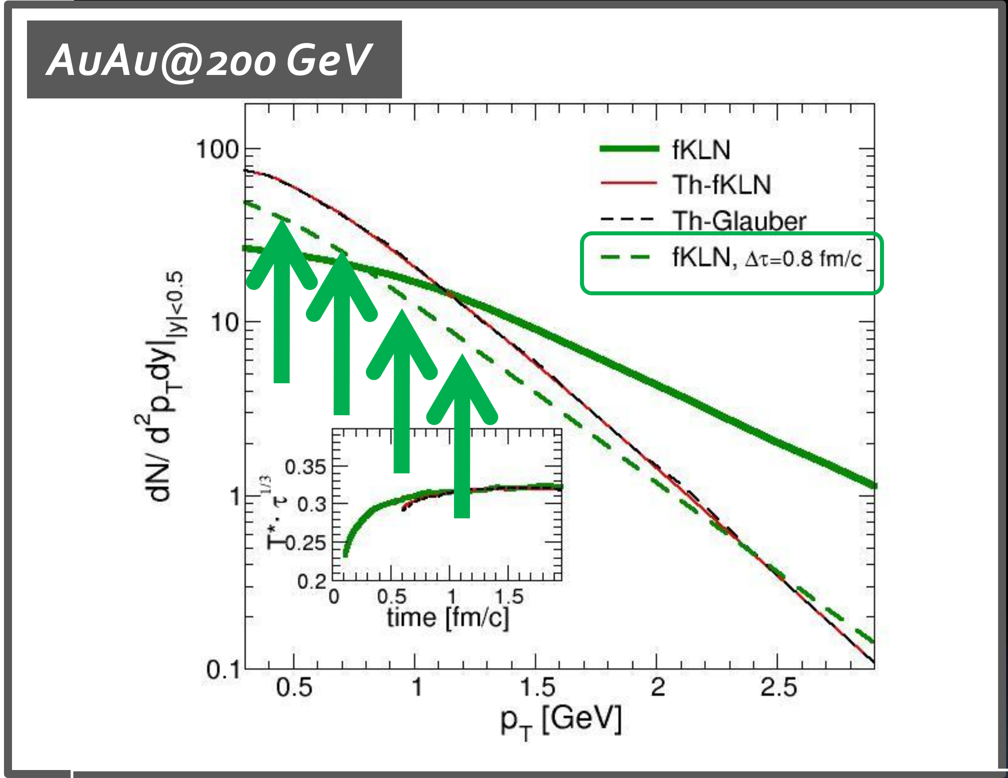
We use Transport to simulate a fluid at ***fixed*** $\eta$/s; the ***cross section*** is ***computed*** in ***each phase space cell*** to give the wished value of $\eta$/s.

***Hydro initial condition:***
*thermalized distribution,*
*to=0.6 fm/c,*
*free streaming*

***fKLN-Glasma initial condition****:*
*saturation at low* $p_T$ *built in,*
*to=0.1 -0.2 fm/c,*
*no free streaming.*

M. R. *et al.*, 1303.3178 [nucl-th]

# Initial condition and thermalization



We use Transport to simulate a fluid at ***fixed*** $\eta$/s; the ***cross section*** is ***computed*** in ***each phase space cell*** to give the wished value of $\eta$/s.

**Thermalization in less than 1 fm/c,**
in agreement with:
Greiner *et al.*, Nucl. Phys. A**806**, 287 (2008).

$$\sigma_{tot} = \frac{\langle p \rangle}{\rho\, g(a)} \frac{1}{\eta/s}$$

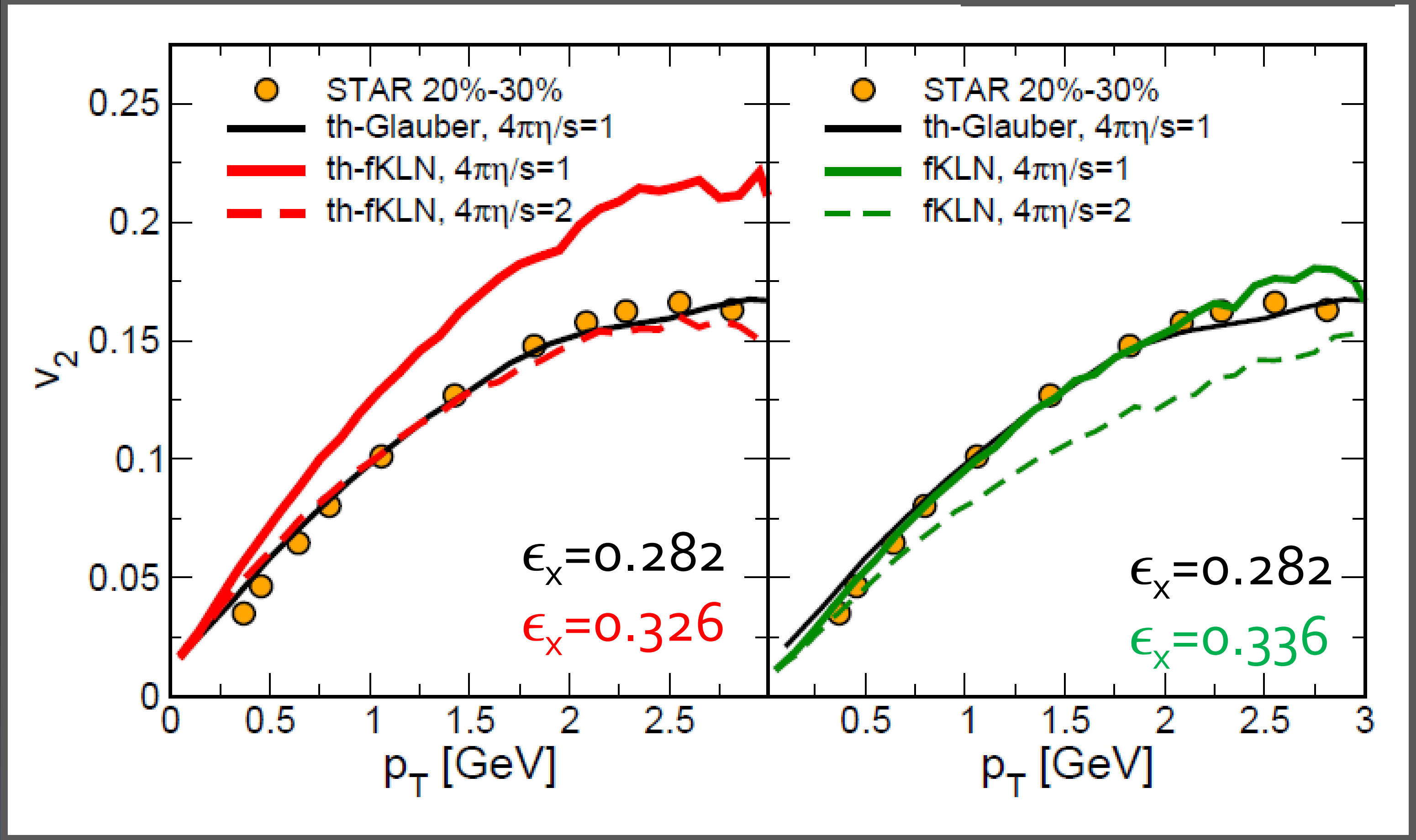
**Not so surprising**:
Because $\eta$/s is fixed, there are large cross sections which naturally lead to fast thermalization.
**However, interesting**:
We have dynamics in the early stages of the simulation, which prepares the momentum distribution to build up the elliptic flow.

M. R. *et al.*, 1303.3178 [nucl-th]

# Elliptic flow: fKLN-Glasma

In agreement with:
Heinz *et al.*, PRC **83**, 054910 (2011)

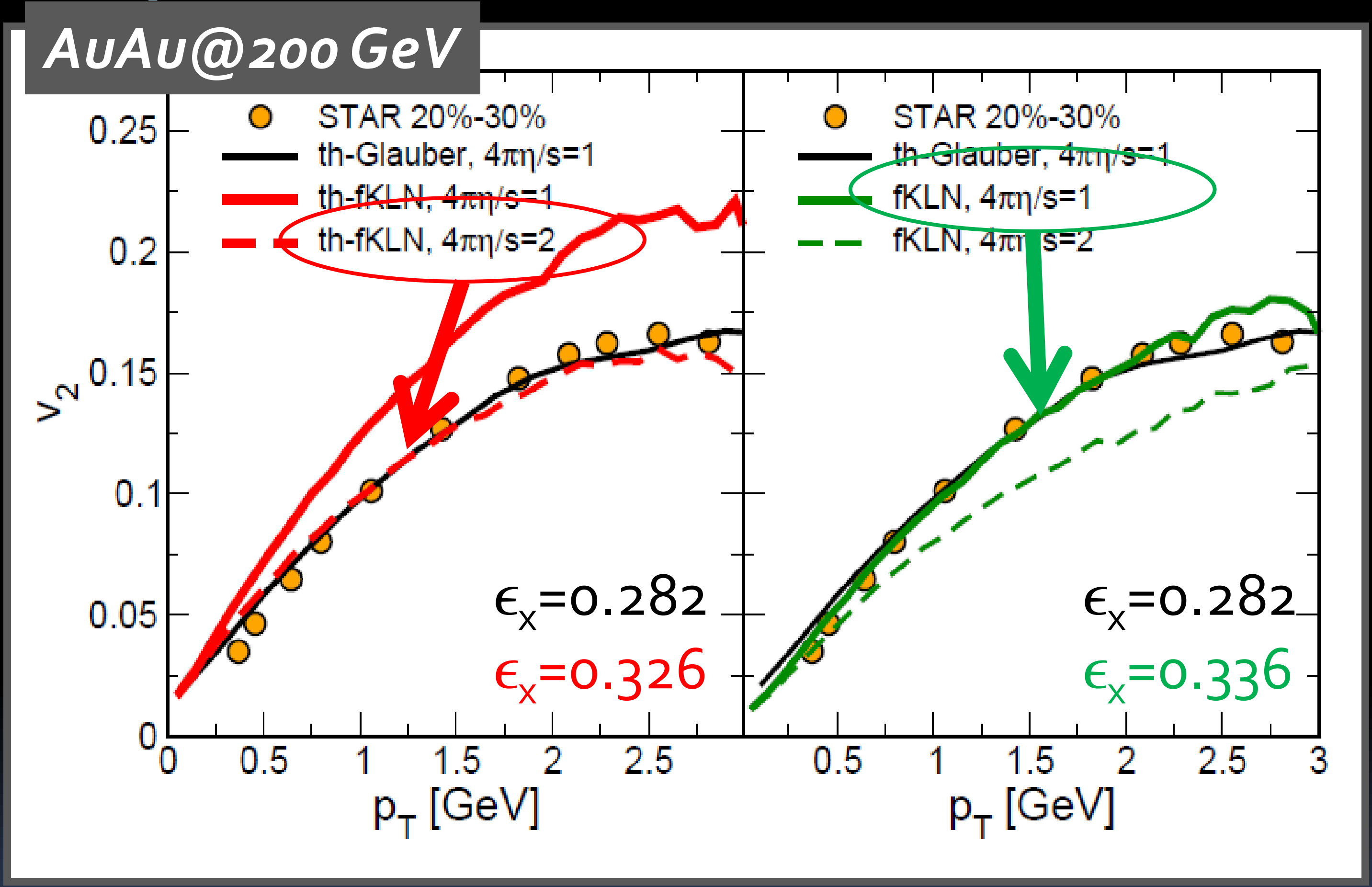
***AuAu@200 GeV***



***Related studies (CYM+Hydro):***
Gale *et al.*, 1209.6330 [nucl-th]
Schenke *et al.*, 1206.6805 [hep-ph]

M. R. *et al.*, 1303.3178 [nucl-th]

# Elliptic flow: fKLN-Glasma

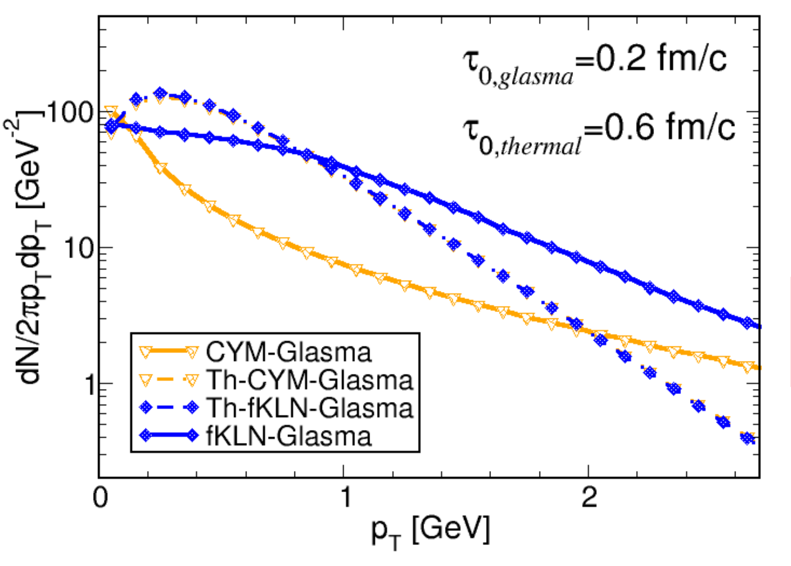


***Implementing the fKLN-Glasma spectrum at initial time leads to an estimate of $\eta/s$ in agreement with the Glauber initial condition.***

***CYM+Hydro: $4\pi\eta/s$ =1.5*** according to Gale *et al.*, 1209.6330 [nucl-th]

M. R. *et al., work in progress*

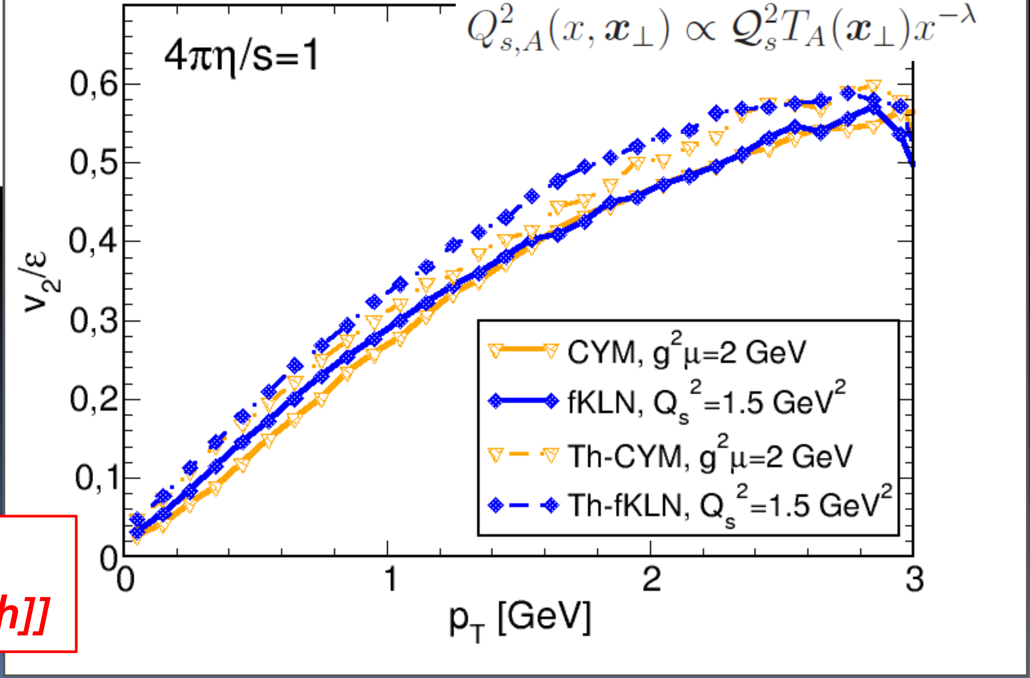
# Elliptic flow: CYM-Glasma



***CYM data taken from:***
Mehtar-Tani *et al.*, NPA**846**, (2010).

***PRELIMINARY***

***AuAu@200 GeV***

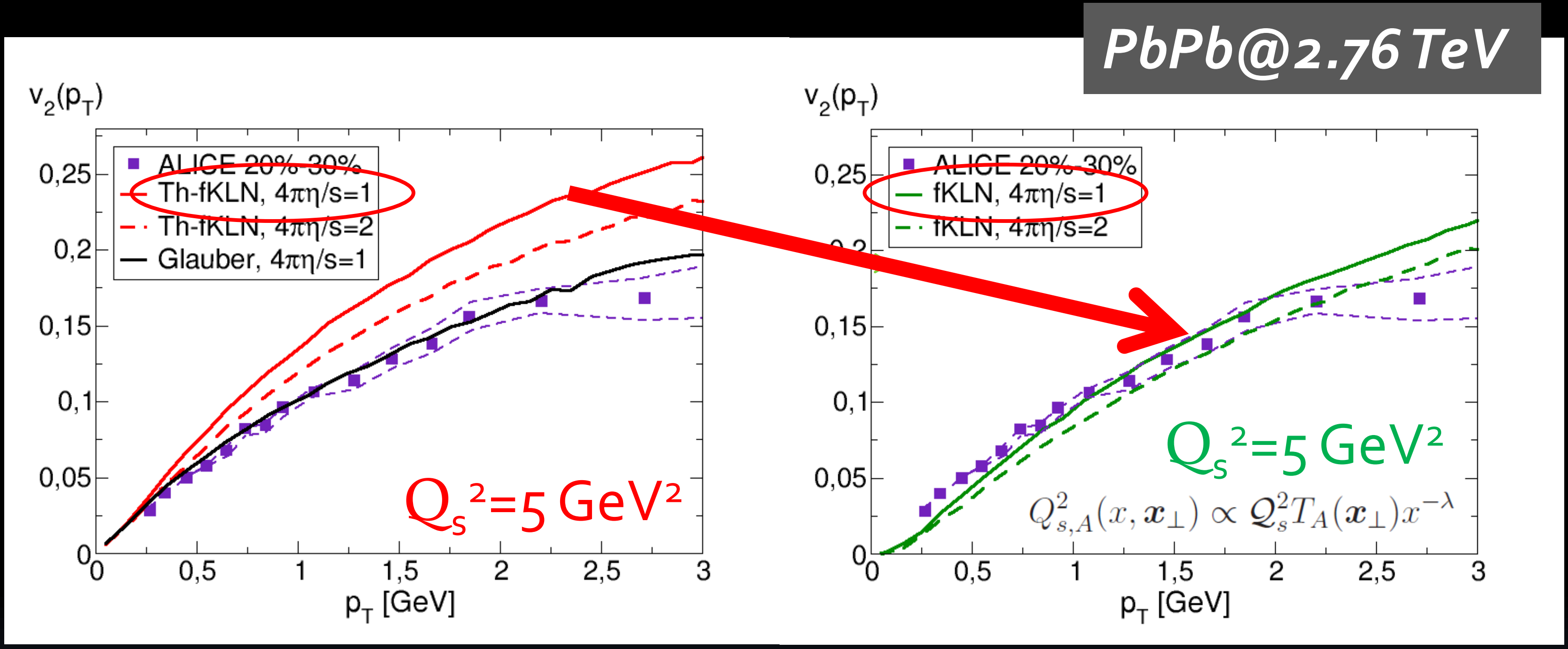


***We measure some effect also changing the average value of the saturation scale in the fKLN-Glasma, and turning to the CYM-Glasma initial condition.***

***CYM x-space: Glauber Ncoll [Schenke et al., 1206.6805 [hep-ph]]***

M. R. *et al.*, *work in progress*

# fKLN-Glasma@LHC

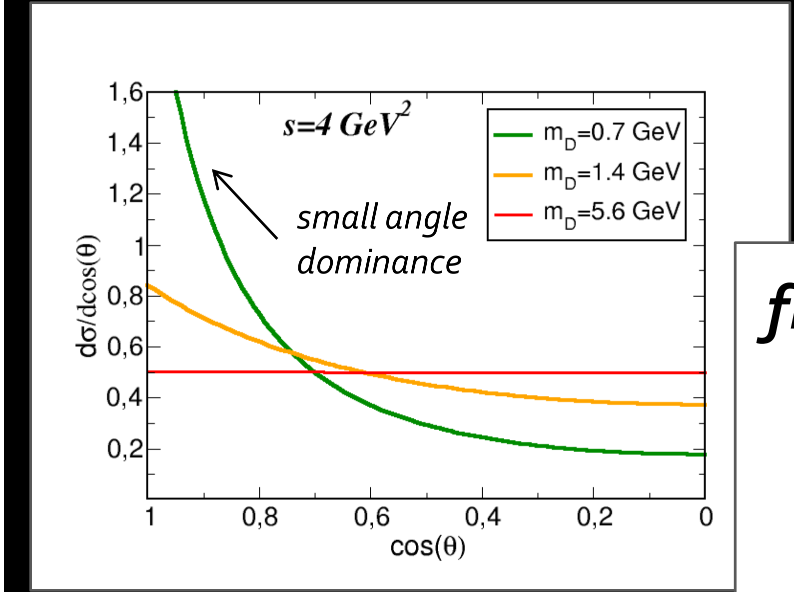


***PbPb@2.76 TeV***

***Implementing the proper initial condition in momentum space, as well as in configuration space, leads to a smaller v2 and to a different estimate of $\eta$/s.***

***CYM+Hydro: 4$\pi\eta$/s =2.5*** according to Gale *et al.*, 1209.6330 [nucl-th]

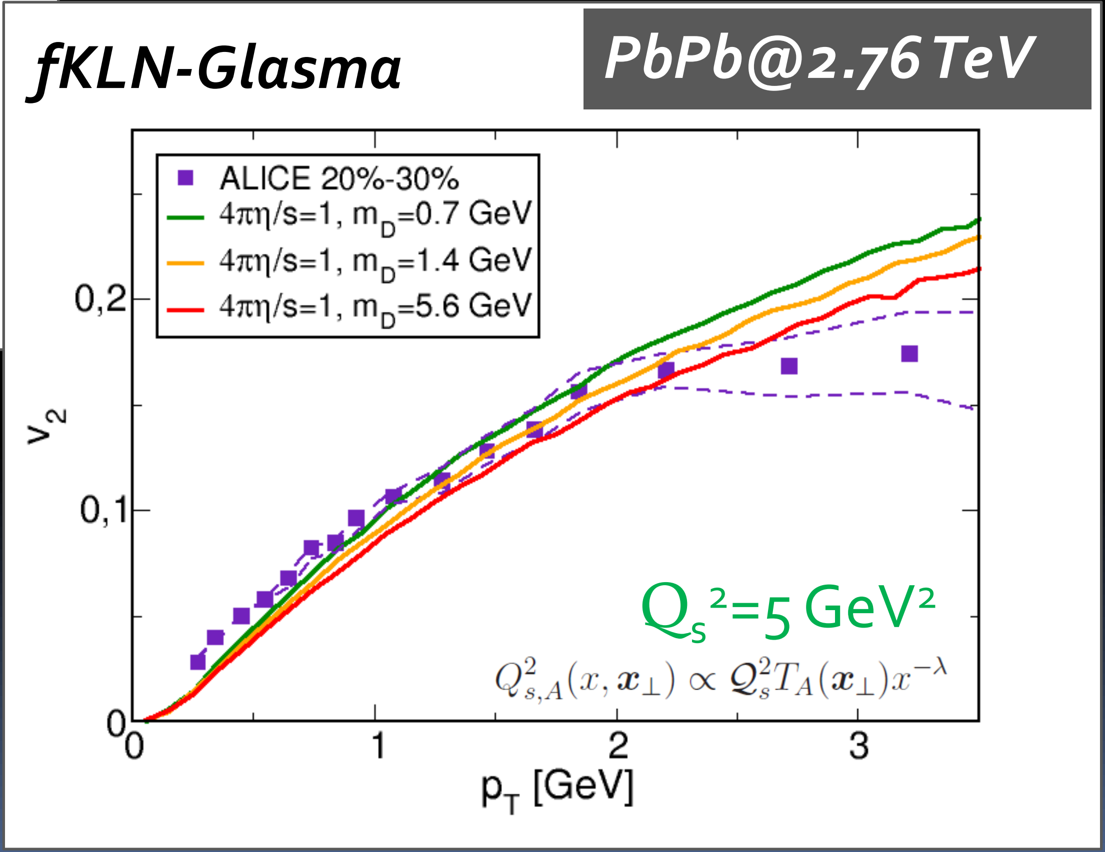
# Are micro-details important?



$$\frac{d\sigma_{gg\to gg}}{dt} = \frac{9\pi^2\alpha_s^2}{2}\frac{1}{(t-m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$

**fKLN-Glasma**

**PbPb@2.76 TeV**



$Q_s^2$=5 GeV$^2$

$Q^2_{s,A}(x,\boldsymbol{x}_\perp) \propto Q_s^2 T_A(\boldsymbol{x}_\perp) x^{-\lambda}$

***Same cross section used in:***

Zhang *et al.*, PLB 455 (1999)
Molnar and Gyulassy, NPA 697 (2002)
Greco et al., PLB 670 (2009)

Increasing $m_D$ makes the cross section isotropic. However: ***Strong change of the cross section does not result in a strong change of the elliptic flow.***

M. R. *et al., work in progress*

# Conclusions and Outlook

- We used ***Kinetic Theory*** to compute the ***elliptic flow*** of a fireball produced in heavy ion collisions, at ***both RHIC and LHC energies***.
- ***Initial distribution in momentum space affects the flow and the building up of momentum anisotropy.***
- ***Microscopic details have a very little relevance for the theoretical computation of the elliptic flow by transport theory.***

***Outlook***

*(.)More merging with initial time CYM spectra.*

*(.)"Cosmology" of HICs (initial state fluctuations).*

(.) *Classical to Quantum: Bose-Einstein condensate?*

(.)Detailed study of *pre-equilibration times*: transport is the *perfect* tool to achieve reliable results.

# KEEP CALM AND THANK YOU FOR YOUR ATTENTION

*I also acknowledge:*

(.) Dr. Hiroaki Abuki
(.) Dr. Santosh Kumar Das
(.) Dr. Marco Frasca
(.) Prof. Kenji Fukushima
(.) Prof. Tetsufumi Hirano
(.) Prof. Akira Ohnishi

*for many discussions about the topics discussed in this talk.*

*There are two breeds of fools: those who do not doubt anything, those who doubt everything.*
(Charles-Joseph de Ligne)

# Appendices

M. R. *et al., work in progress*

# Spectra collection@RHIC