Amplitudes and form factors in 3d superconformal Chern-Simons theory

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Brandhuber, GT, Wen	1205.6705 [hep-th], 1207.6908 [hep-th]
Brandhuber, Gurdogan, Korres, Mooney, GT	1305.2421 [hep-th]
Brandhuber, GT, Wen, Wiegandt	in progress

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Plan

- N=6 superconformal Chern-Simons matter theory (known as ABJM) (Aharony, Bergman, Jafferis, Maldacena)
  - original motivation: find the dual field theory to M-theory on $AdS_4 \times S^7$ (Schwarz)
- ABJM amplitudes have surprising similarities to N=4 super Yang-Mills amplitudes
- One-loop amplitudes
  - explain certain intriguing regularities in one-loop amplitudes
- Two-loop Sudakov form factor
  - very interesting properties of integral functions, peculiar to 3d

ABJM in a nutshell

(Aharony, Bergman, Jafferis, Maldacena)

Johansson's talk

- Gauge fields $(A, \hat{A})$
  - Chern-Simons levels k and -k respectively

$$S_{\mathrm{ABJM}} \ni S_{\mathrm{CS}}[A] + \hat{S}_{\mathrm{CS}}[\hat{A}]$$

$$= \frac{k}{4\pi}\int \left[\mathrm{Tr}(A\wedge dA - \frac{2}{3}iA\wedge A\wedge A) - (\hat{A}\wedge d\hat{A} - \frac{2}{3}i\hat{A}\wedge\hat{A}\wedge\hat{A})\right]$$

- gluons appear only as internal states! $\partial_{[\mu}A_{\nu]} = 0$
- peculiar role of gluon zero-momentum mode

- Matter fields:

  - 4 complex bosons & 4 complex fermions $(\phi^A, \psi_A^\alpha)^I_{\bar{I}}$, $(\bar{\phi}_A, \bar{\psi}^A_\alpha)^{\bar{I}}_I$
    - particles / anti-particles transform in the bi-fundamental $(N, \bar{N})$, $(\bar{N}, N)$ of $\mathrm{U}(N) \times \mathrm{U}(N)$ $I\,, \bar{I} = 1, \ldots, N$
    - $A = 1, \ldots, 4$ SU(4) R-symmetry index, $\alpha = 1, 2$ spin index
    - all particles transform in the (anti)-fundamental of R-symmetry group (unlike N=4 SYM)

  - N=6 supersymmetry in 3d (for appropriately tuned 6-scalar and 2-fermion/2-scalar couplings)

  - superconformal OSp(6|4)

- New example of AdS/CFT duality in 3d

  - at large N and $k << N$
    - dual to M-theory on $AdS_4 \times S^7/Z_k$, weakly curved for $N >> k^5$
    - dual to type IIA string theory on $AdS_4 \times CP^3$ for $N << k^5 << N^5$:

  - at large k , there is a weakly-coupled Lagrangian description
    - 't Hooft limit: large N and k with $\boxed{\lambda \equiv N/k}$ fixed
    - weak coupling for $\lambda << 1$
    - $1/N$ expansion at fixed λ

  - this talk: amplitudes and form factors at small λ

Amplitudes

Spinor helicity formalism

- crucial to expose the simplicity of amplitudes (as in 4d)
  - Lorentz group isomorphic to $\mathrm{SL}(2,\mathbf{R})$: $p^\mu \to p_{\alpha\beta} := p^\mu \sigma_{\mu,\alpha\beta}$
  - For null vectors: $p_{\alpha\beta} = \lambda_\alpha \lambda_\beta$ with $\alpha,\beta = 1, 2$
    - automatically enforces $p^2 = \det(\mathrm{p}) = 0$
    - similar to $p_{\alpha\dot\beta} = \lambda_\alpha \tilde\lambda_{\dot\beta}$ in 4d
  - little group is $\lambda \longrightarrow -\lambda$ hence no helicity (unlike in 4d!)
  - Lorentz invariant products: $\langle i\, j \rangle := \lambda_{i\alpha}\lambda_{j\beta}\, \epsilon^{\alpha\beta}$
    - only one kind of invariant product (no [...] brackets !)

Simplest amplitude

- Four-point (super)amplitude at tree level
  (Agarwal, Beisert, McLoughlin)

$$\mathcal{M}(\bar{1},2,\bar{3},4) \;=\; \frac{\delta^{(3)}\left(\sum_{i=1}^{4}\lambda_i\lambda_i\right)\,\delta^{(6)}\left(\sum_{i=1}^{4}\lambda_i\eta_i\right)}{\langle 12\rangle\langle 23\rangle}$$

  - all amplitudes with a fixed number of legs packaged into a single superamplitude
    - η^A fermionic variables, A =1, 2, 3 is an SU(3) index ($\subset$ SU(4))
    - N=6 supersymmetric delta functions: $\delta^{(3)}\left(\sum_i \lambda_i\lambda_i\right)\,\delta^{(6)}\left(\sum_i \lambda_i\eta_i\right)$
  - Because of gauge invariance, particles and antiparticles must alternate, hence only amplitudes with an even number of legs are nonvanishing

- The only amplitude reminiscent of 4d MHV amplitudes

- in 3d

$$\mathcal{M}(\bar{1},2,\bar{3},4) \;=\; \frac{\delta^{(3)}\left(\sum_{i=1}^{4}\lambda_i\lambda_i\right)\,\delta^{(6)}\left(\sum_{i=1}^{4}\lambda_i\eta_i\right)}{\langle 12\rangle\langle 23\rangle}$$

- in 4d:

$$\mathcal{M}_{\mathrm{MHV}}(1,\ldots,4) \;=\; \frac{\delta^{(4)}\left(\sum_{i=1}^{4}\lambda_i\tilde{\lambda}_i\right)\,\delta^{(8)}\left(\sum_{i=1}^{4}\lambda_i\eta_i\right)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

Parke-Taylor (super)amplitude

Facts & similarities with N=4 SYM

- At four points in ABJM:
  - One-loop amplitude vanishes (Agarwal, Beisert, McLoughlin)
  - Two-loop amplitude matches the one-loop amplitude in N=4 SYM
    (Chen & Huang; Bianchi, Leoni, Mauri, Penati, Santambrogio)
  - Two-loop Wilson loop matches the one-loop Wilson loop in N=4 SYM
    (Henn, Plefka, Wiegandt; Wiegandt)
- Conjectured scattering amplitude-Wilson loop duality
  (at four points)
- Conjectured correlation function-Wilson loop duality
  (at four points) (Bianchi, Leoni, Mauri, Penati, Ratti, Santambrogio)

- Dual (super)conformal symmetry
  - for the Wilson loop (Henn, Plefka, Wiegandt)
  - for the amplitudes (Gang, Huang, Koh, Lee, Lipstein; Bargheer, Beisert, Loebbert, McLoughlin)

- Yangian symmetry (Bargheer, Loebbert, Meneghelli)
  - by commuting dual conformal with conformal generators

- Amplitudes represented as a Grassmannian integral (Lee)

- Spectrum of (planar) anomalous dimensions in terms of integrable spin chain (Minahan & Zarembo; Bak & Rey)

Differences with N=4 SYM

- n-point amplitudes have Grassmann degree $3n / 2$
  - no MHV amplitudes, no helicity
  - no amplitudes with odd number of particles

- n-point amplitudes at one loop are non-vanishing for $n \geq 6$
  (Bargheer, Beisert, Loebbert, McLoughlin; Bianchi, Leoni, Mauri, Penati, Santambrogio; Brandhuber, GT, Wen)

- Wilson loop with odd number of edges is non-vanishing, but there is no corresponding amplitude!

One-loop amplitudes

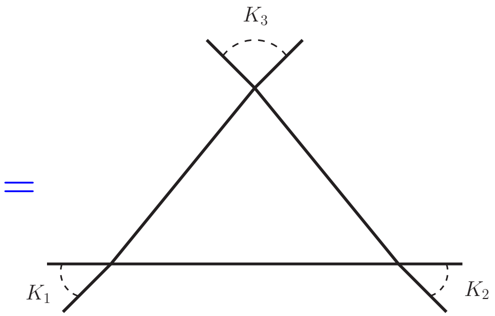
One-loop ABJM amplitudes

- Only scalar triangles because of dual conformal symmetry
  - one-mass and two-mass triangles vanish in $d=3$ hence

$$\mathcal{M}_n^{(1)} = \sum_{K_1,K_2,K_3} \mathcal{C}_{K_1,K_2,K_3}\, \mathcal{I}^{3m}(K_1,K_2,K_3)$$

  - three-mass triangles are finite $(K_i^2 \neq 0)$

$$\mathcal{I}^{3m}(K_1,K_2,K_3) =$$



$$= \frac{-i\,\pi^3}{\sqrt{-(K_1^2+i\varepsilon)}\sqrt{-(K_2^2+i\varepsilon)}\sqrt{-(K_3^2+i\varepsilon)}}$$

- All one-loop amplitudes are finite!
  - we provide later a recursion relation for their coefficients

Six-point amplitude

- **Tree-level:**

$$\mathcal{M}^{(0)}(\bar{1},2,\bar{3},4,\bar{5},6) \;=\; Y^{(1)}_{12;4} \;+\; Y^{(2)}_{12;4}$$

  - Y-functions:

$$Y^{(1)}_{12;4} = \frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{P_{24}^2} \frac{\delta^{(3)}(\epsilon_{ijk}\langle j\,k\rangle\eta_i - i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\langle\bar{j}\,\bar{k}\rangle\eta_{\bar{i}})}{(\langle 2|P_{34}|5\rangle + i\langle 3\,4\rangle\langle 6\,1\rangle)(\langle 1|P_{23}|4\rangle + i\langle 2\,3\rangle\langle 5\,6\rangle)} \qquad i,j = 2,3,4$$

$$Y^{(2)}_{12;4} = \frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{P_{24}^2} \frac{\delta^{(3)}(\epsilon_{ijk}\langle j\,k\rangle\eta_i + i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\langle\bar{j}\,\bar{k}\rangle\eta_{\bar{i}})}{(\langle 2|P_{34}|5\rangle - i\langle 3\,4\rangle\langle 6\,1\rangle)(\langle 1|P_{23}|4\rangle - i\langle 2\,3\rangle\langle 5\,6\rangle)} \qquad \bar{i},\bar{j} = 5,6,1$$

- **one-loop:**

$$\mathcal{M}^{(1)}(\bar{1},2,\bar{3},4,\bar{5},6) \;=\; \pi^3\mathcal{S}(Y^{(1)}_{12;4} \;-\; Y^{(2)}_{12;4})$$

  - $S \;=\; \text{sgn}(\langle 1\,2\rangle)\text{sgn}(\langle 3\,4\rangle)\text{sgn}(\langle 5\,6\rangle) + \text{sgn}(\langle 2\,3\rangle)\text{sgn}(\langle 4\,5\rangle)\text{sgn}(\langle 6\,1\rangle)$
  - $\text{sgn}\;(\langle m\,n\rangle) \;:=\; -i\,\dfrac{\langle m\,n\rangle}{\sqrt{-(\langle m\,n\rangle^2 + i\varepsilon)}}$

- Determined with maximal cuts (Bargheer, Beisert, Loebbert, McLoughlin) and supergraphs (Bianchi, Leoni, Mauri, Penati, Santambrogio)

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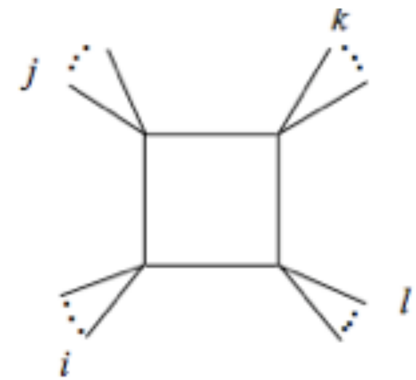
- **Goal:** explain (and possibly extend) the remarkable similarity between the tree and one-loop results observed at 6 points

- **Strategy:** look for similarities with N=4 SYM in 4d

- **More specifically:** look for links between tree-level and one-loop expressios...

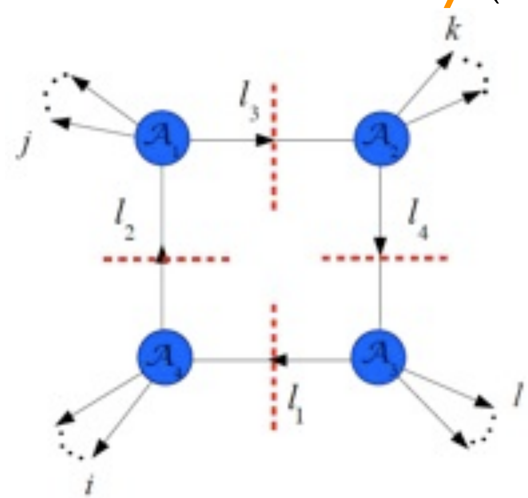
- 4d N=4 SYM:
  - one-loop amplitudes in N=4: rational coefficient x box function
    (Bern, Dixon, Dunbar, Kosower)

$$\mathcal{A}^{1-\mathrm{loop}} = \sum_{i,j,k,l} \mathcal{C}(i,j,k,l)$$



  - Box coefficient from generalised unitarity (Britto, Cachazo, Feng)

$$\mathcal{C}(i,j,k,l) =$$

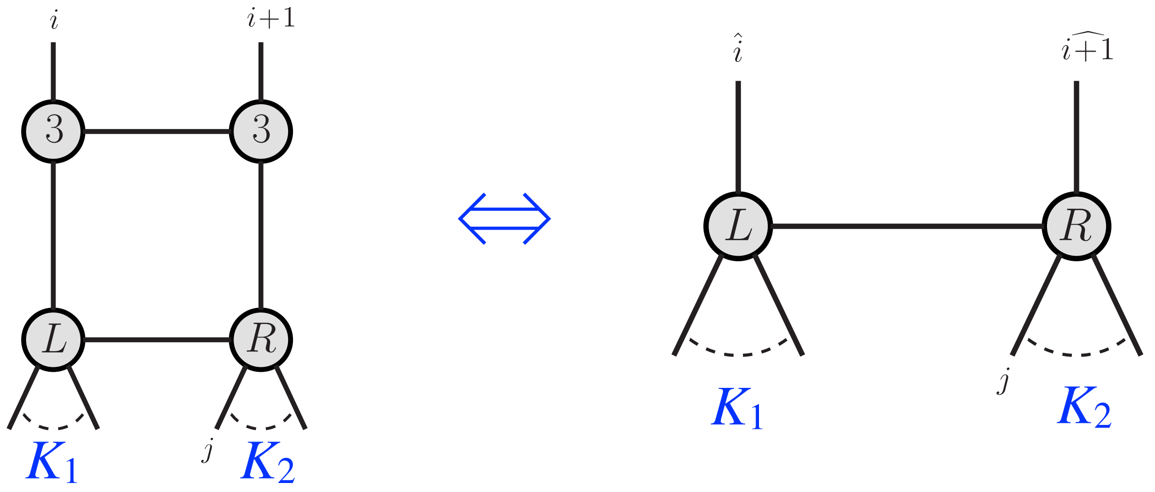


- **Tree/one-loop link:**
  - RSV equations: n equations relating sums of two-mass hard (and one-mass) box supercoefficients to the N=4 tree amplitude

    (Roiban, Spradlin, Volovich)

$$\sum_{j=i+2}^{i+n-2} \mathcal{C}^{2\mathrm{mh}}(i,j) \;=\; 2\,\mathcal{M}^{(0)}\,, \quad i=1,\ldots,n$$

  - key picture:



- LHS: quadruple cut evaluates 2mh coefficient. Note: 2 three-point vertices
  RHS: BCFW diagram contributing to the tree amplitude

- First hint: solutions for cut momenta $\hat{l}_a, \hat{l}_b$ same as BCFW shifts $\hat{i}, \widehat{i+1}$

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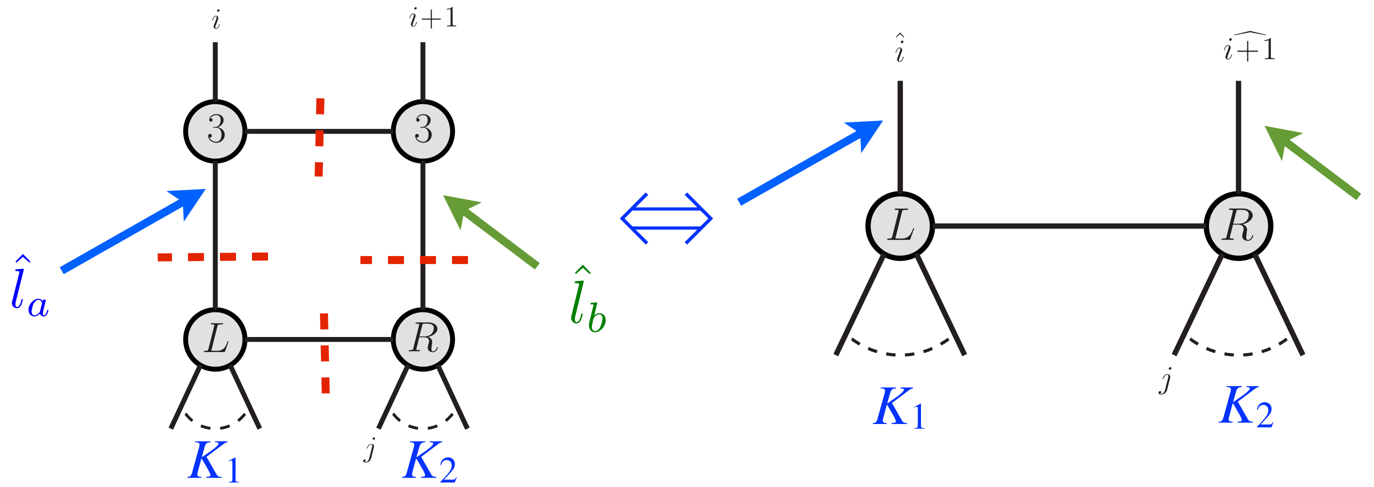

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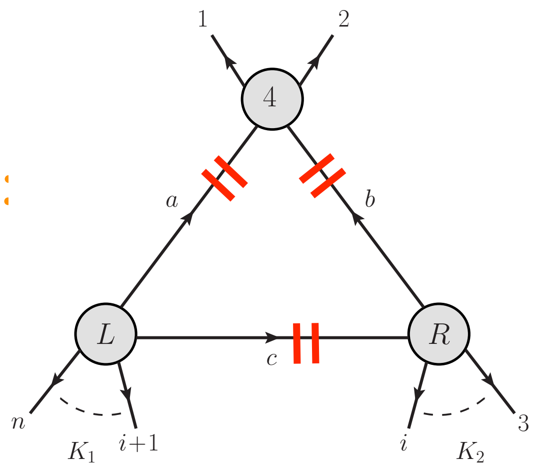
- RSV relations proved by direct calculation or IR consistency conditions (Arkani-Hamed, Cachazo, Kaplan) or using dual conformal equations (Brandhuber, Heslop, GT)
- question: do we have a similar connection in 3d?
- pessimistic answer: RSV relations are related to infrared divergences. 3d at one loop is finite, hence answer is NO.
- optimistic answer: the RSV equations are related to anomalous dual conformal symmetry, which ABJM does have

  (Bargheer, Beisert, Loebbert, McLoughlin)
- our answer: try!

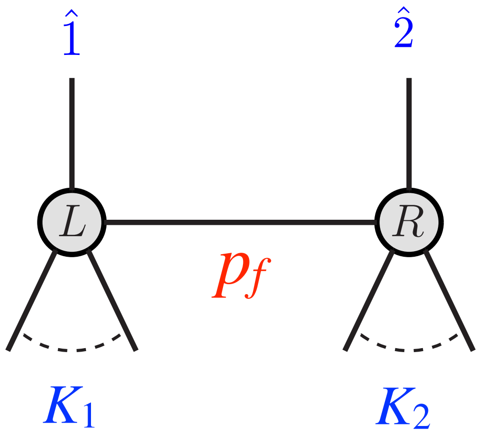
One-loop amplitudes & BCFW diagrams

(Brandhuber, GT, Wen)

- 3d key picture:



$\Longleftrightarrow$



  - LHS: triple cut evaluates coefficient. Note: one 4-point amplitude
    RHS: BCFW diagram contributing to the tree amplitude
  - First hint: solutions for cut momenta same as BCFW shifts
- Opposite sign for the two residues
  - curious minus signs in the one-loop amplitudes vs tree level explained

- In brief:

contribution of the two residues at z_1 and z_2

  - Recursion diagram: $\mathcal{R}_{12;i} = Y^{(1)}_{12;i} + Y^{(2)}_{12;i}$
  - Supercoefficient: $\mathcal{C}_{12;i} = -\langle 12\rangle \sqrt{K_1^2 K_2^2}\left(Y^{(1)}_{12;i} - Y^{(2)}_{12;i}\right)$
  - Supercoefficient x integral function:

$$\mathcal{C}_{12;i}\,\mathcal{I}_{12,K_1,K_2} = -i\frac{\pi^3}{4}\frac{\langle 12\rangle}{\sqrt{-(P_{12}^2+i\varepsilon)}}\frac{\langle \xi\mu\rangle}{\sqrt{-(K_1^2+i\varepsilon)}}\frac{\langle \xi'\mu'\rangle}{\sqrt{-(K_2^2+i\varepsilon)}}\left(Y^{(1)}_{12;i} - Y^{(2)}_{12;i}\right)$$

    - $K_{1\,ab} := \xi_{(a}\mu_{b)}$, $K_{2\,ab} := \xi'_{(a}\mu'_{b)}$
    - prefactor involves sign functions

  - Result obtained by adding all cut diagrams
  - Can derive complete amplitudes up to 10 points

Examples

- <u>Six-point amplitude</u>
  - Tree level: $\mathcal{M}^{(0)}(\bar{1},2,\bar{3},4,\bar{5},6) \;=\; Y^{(1)}_{12;4} \;+\; Y^{(2)}_{12;4}$
    - Y-functions from recursive diagrams:

$$Y^{(1)}_{12;4} \;=\; \frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{P^2_{24}}\,\frac{\delta^{(3)}(\epsilon_{ijk}\langle j\,k\rangle\eta_i - i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\langle\bar{j}\,\bar{k}\rangle\eta_{\bar{i}})}{(\langle 2|P_{34}|5\rangle + i\langle 3\,4\rangle\langle 6\,1\rangle)(\langle 1|P_{23}|4\rangle + i\langle 2\,3\rangle\langle 5\,6\rangle)} \qquad i,j = 2,3,4$$

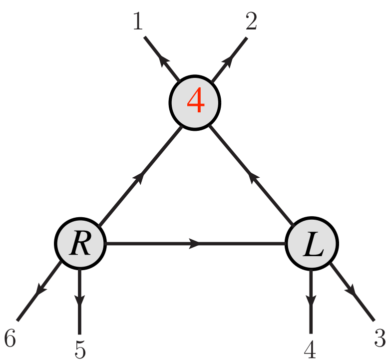
$$Y^{(2)}_{12;4} \;=\; \frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{P^2_{24}}\,\frac{\delta^{(3)}(\epsilon_{ijk}\langle j\,k\rangle\eta_i + i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\langle\bar{j}\,\bar{k}\rangle\eta_{\bar{i}})}{(\langle 2|P_{34}|5\rangle - i\langle 3\,4\rangle\langle 6\,1\rangle)(\langle 1|P_{23}|4\rangle - i\langle 2\,3\rangle\langle 5\,6\rangle)} \qquad \bar{i},\bar{j} = 5,6,1$$

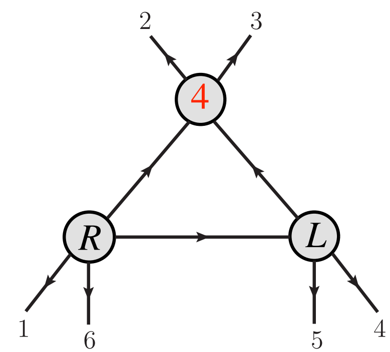
  - Six-point amplitude at one loop:

$$\mathcal{M}^{(1)}(\bar{1},2,\bar{3},4,\bar{5},6) \;=\; \pi^3\mathcal{S}(Y^{(1)}_{12;4} \;-\; Y^{(2)}_{12;4})$$
$$\;=\; i\pi^3\mathcal{S}\,\mathcal{M}^{(0)}(\bar{6},1,\bar{2},3,\bar{4},5)$$

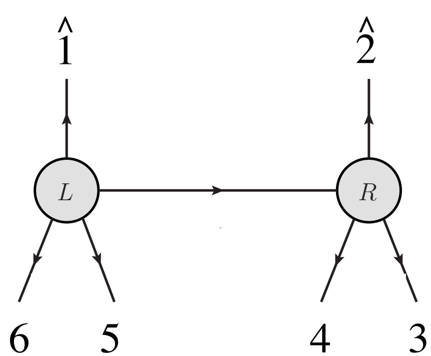
$$\mathcal{S} \;=\; \mathrm{sgn}(\langle 1\,2\rangle)\mathrm{sgn}(\langle 3\,4\rangle)\mathrm{sgn}(\langle 5\,6\rangle) + \mathrm{sgn}(\langle 2\,3\rangle)\mathrm{sgn}(\langle 4\,5\rangle)\mathrm{sgn}(\langle 6\,1\rangle)$$

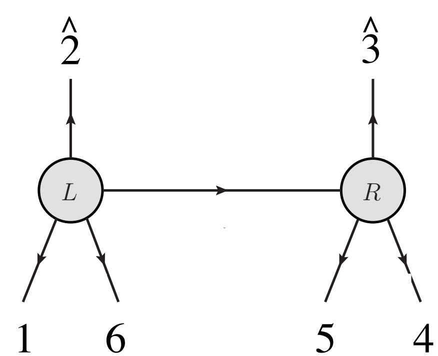
- Derivation from earlier result:
  - Anomalous cut diagrams:





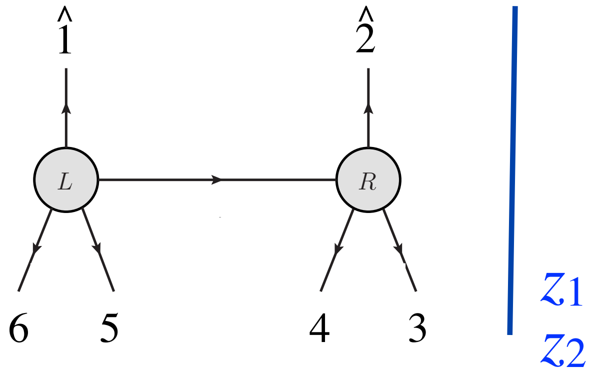
  - Associated recursive diagrams:





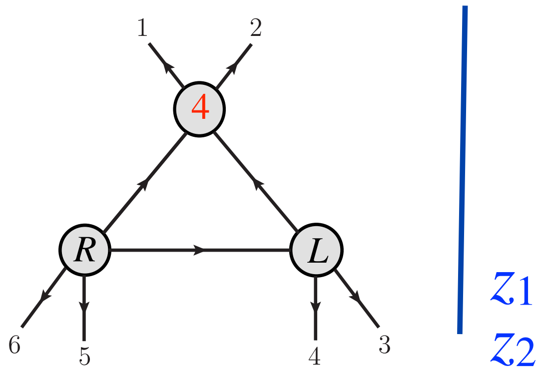
    - Note: two BCFW diagrams with different shifts (same amplitude!)

- BCFW recursive diagram associated to the anomalous cut:



$$= \begin{cases} Y^{(1)}_{12;4} \\ Y^{(2)}_{12;4} \end{cases}$$

- Anomalous triple-cut diagram:



$$\sim \quad \mathrm{sgn}\left(\langle 1\,2\rangle\right)\mathrm{sgn}\left(\langle 3\,4\rangle\right)\mathrm{sgn}\left(\langle 5\,6\rangle\right) \begin{cases} Y^{(1)}_{12;4} \\ (-)\,Y^{(2)}_{12;4} \end{cases}$$

- Final result from adding other diagram

- Side remark:
  - In general, each recursion diagram has two contributions from the two poles z_1 , z_2 :

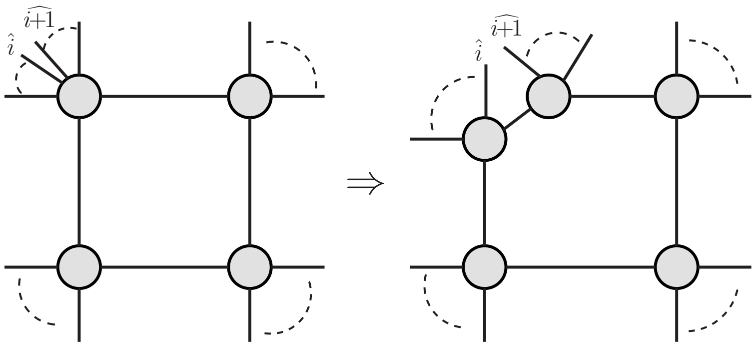
$$\mathcal{R}_{12;i} = Y^{(1)}_{12;i} + Y^{(2)}_{12;i}$$

  - The two residues are separately dual conformal invariant

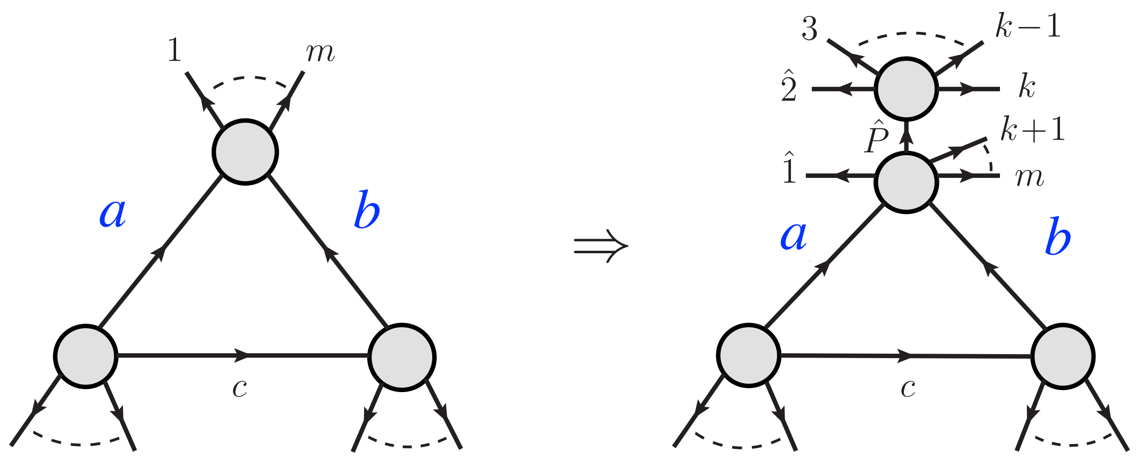
Recursion relation for one-loop coefficients

(Brandhuber, GT, Wen)

- Just the main idea:
  - Used already in QCD (Bern, Bjerrum-Bohr, Dunbar, Ita)
  - Typical problems:
    - spurious poles
    - large-z behaviour not understood
  - Example of a problematic case (in 4d gauge theory):



- Problematic situation can always be avoided in ABJM
  - can choose shifts such that legs a and b belong to the same amplitude



  - reason: no amplitudes with odd number of legs
  - shift i and $i+1$ with i odd (e.g. 1 and 2)
- All one-loop amplitudes in ABJM under control!

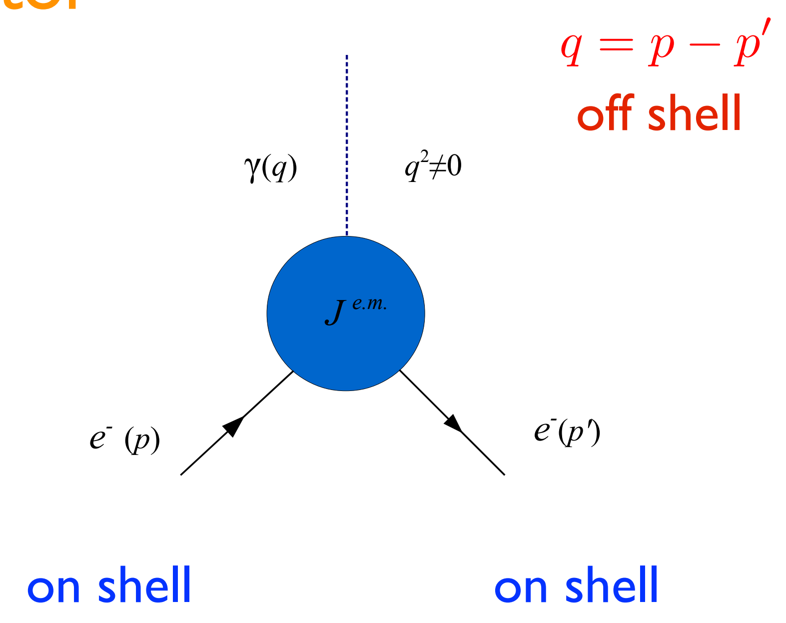
Form Factors

- Partially off-shell quantities

$$F = \int d^4x\, e^{-iqx} \langle state|\mathcal{O}(x)|0\rangle = \delta^{(4)}(q - p_{state}) \langle state|\mathcal{O}(0)|0\rangle$$

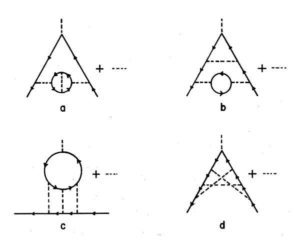
- Electromagnetic form factor

$$\langle e^-(p')|\, J_\mu^{e.m.}(0)\, |e^-(p)\rangle =$$



$$J_\mu^{e.m.} = \bar{\psi}\gamma_\mu\psi$$

- Three-loop correction to electron g–2

72 diagrams like  $= \ (1.181241456...)\,(\alpha_{\mathrm{e.m.}}/\pi)^3$

(Cvitanovic & Kinoshita '74)

(Laporta & Remiddi '96)

- wild oscillations between the values of each diagram/integral
- final result is $O(1)$
- another example of "unexplained" simplicity...

- A number of interesting recent results:
  - surprising similarities between two-loop, three-point form factors of 1/2 BPS operators in N=4 SYM and:
    1. Higgs + 3 jet amplitudes in QCD
       - maximally transcendental parts are identical!
    2. a slice $(u+v+w=1)$ of the six-point MHV amplitude remainder in N=4 SYM (Brandhuber, GT, Yang)

$$
\begin{aligned}
\mathcal{R}_3^{(2)} \;=\;& -2\left[J_4\left(-\frac{uv}{w}\right)+J_4\left(-\frac{vw}{u}\right)+J_4\left(-\frac{wu}{v}\right)\right]-8\sum_{i=1}^{3}\left[\mathrm{Li}_4\left(1-u_i^{-1}\right)+\frac{\log^4 u_i}{4!}\right]\\
& -2\left[\sum_{i=1}^{3}\mathrm{Li}_2(1-u_i^{-1})\right]^2+\frac{1}{2}\left[\sum_{i=1}^{3}\log^2 u_i\right]^2-\frac{\log^4(uvw)}{4!}-\frac{23}{2}\zeta_4
\end{aligned}
$$

$$
J_4(z) := \mathrm{Li}_4(z)-\log(-z)\mathrm{Li}_3(z)+\frac{\log^2(-z)}{2!}\mathrm{Li}_2(z)-\frac{\log^3(-z)}{3!}\mathrm{Li}_1(z)-\frac{\log^4(-z)}{48}\,.
$$

Form factors in ABJM

(Brandhuber, Korres, Gurdogan, Mooney, GT; Young)

- Simplest form factors: scalar 1/2 BPS operators
  - e.g. $O(x) = \mathrm{Tr}(\phi^A \bar{\phi}_4)(x)$
  - Sudakov form factor:

$$\langle \phi^A(p_1)\bar{\phi}_4(p_2)|\, O(0)\, |0\rangle$$

O is a colour singlet

  - equal to 1 at tree level, one-loop correction vanishes

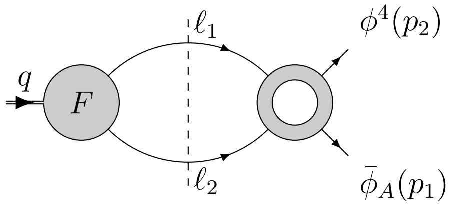
- Sudakov form factor controls IR divergences of amplitudes and UV divergences of Wilson loops with cusps (Korchemsky & Radyushkin)
  - zero at one-loop (consistent with finiteness of one-loop amplitudes)
  - at two loops expect $\sim \frac{\gamma_{\mathrm{cusp}}}{\epsilon^2} + \mathrm{finite}$ from known 4- and 6-pt amplitudes

- Goal: evaluate $F(q^2) = \langle \phi^A(p_1)\bar{\phi}_4(p_2)|\,\mathrm{Tr}(\phi^A\bar{\phi}_4)(0)\,|0\rangle$ at two loops, $q = p_1 + p_2$
- Known technical challenge: non-planar amplitudes enter the cuts of planar form factors
- Strategy: use a combination of
  - two-particle cuts
  - three-particle cuts fix all remaining ambiguities
  - note: we work at the integrand level!

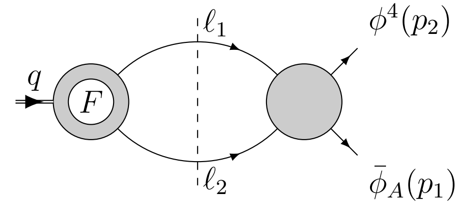
Two-loop form factors in ABJM

(Brandhuber, Korres, Gurdogan, Mooney, GT)

- Two-particle cuts:



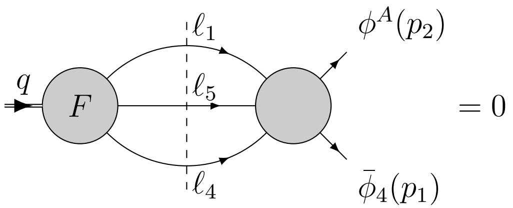
and



  - LHS: glue tree-level Sudakov form factor to a four-point one-loop complete amplitude

  - RHS: glue one-loop Sudakov form factor with four-point tree-level amplitude

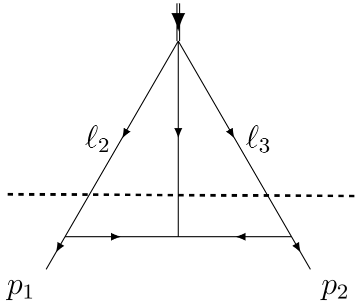
- Triple cuts:



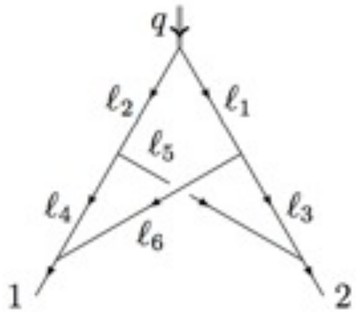
  - no odd-particle amplitudes in ABJM

  - very powerful constraint!

  - triple cuts uniquely fix potential remaining integral (which is free of double two-particle cuts)



- Final result: $$F^{(2)}(q^2) = \left(\frac{N}{k}\right)^2 \mathbf{XT}(q^2)$$

$\mathbf{XT}(q^2) =$ 

$\times\, q^2 \left[-\operatorname{Tr}(p_1 p_2 l_3 l_1) + q^2 l_3^2 \right]$

$$= -\frac{1}{(4\pi)^3}\left(-\frac{q^2 e^{\gamma_E}}{4\pi\mu^2}\right)^{-2\epsilon}\left[\frac{\pi}{\epsilon^2} + \frac{2\pi\log 2}{\epsilon} - 4\pi\log^2 2 - \frac{2\pi^3}{3} + \mathcal{O}(\epsilon)\right]$$

- $$F^{(2)}(q^2) = \frac{1}{64\pi^2}\left(\frac{N}{k}\right)^2\left(-\frac{q^2}{\mu'^2}\right)^{-2\epsilon}\left[-\frac{1}{\epsilon^2} + 6\log^2 2 + \frac{2\pi^2}{3} + \mathcal{O}(\epsilon)\right]$$

$$\mu'^2 := 8\pi e^{-\gamma_E}\mu^2$$

- agreement with the IR divergences of the known two-loop amplitudes, result has maximal degree of transcendentality

- **Final result:** $$F^{(2)}(q^2) = \left(\frac{N}{k}\right)^2 \mathbf{XT}(q^2)$$

note particular numerator

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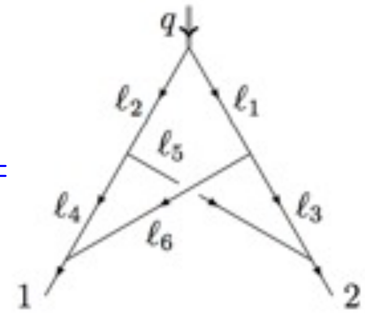

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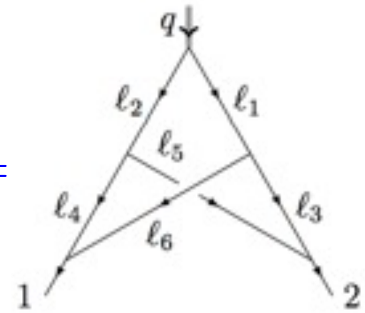

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π, log 2

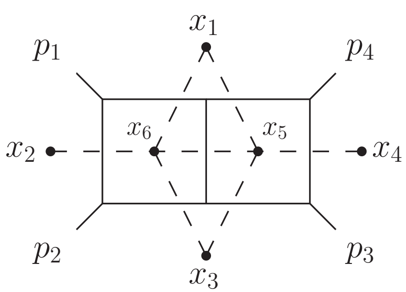
- agreement with the IR divergences of the known two-loop amplitudes, result has maximal degree of transcendentality

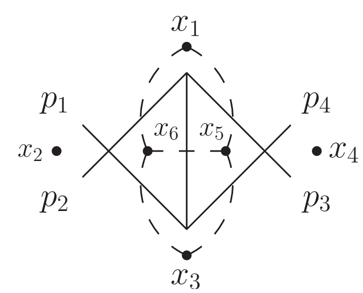
Comment I

- special numerator removes unwanted/unphysical infrared divergences associated to three internal momenta becoming soft. These are present even for massive external kinematics

- Already observed in amplitudes, where numerators are crucial to maintain dual conformal invariance (Bianchi, Leoni, Mauri, Penati, Santambrogio)

I_{4s}





I_{1s}

$$I_{4s} = \int \frac{d^3x_5}{(2\pi)^D} \frac{d^3x_6}{(2\pi)^D} \frac{x_{13}^2 x_{25}^2 x_{46}^2}{x_{15}^2 x_{35}^2 x_{45}^2 x_{56}^2 x_{16}^2 x_{26}^2 x_{36}^2}$$

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- Only I_{1s} - I_{4s} is dual conformal. I_{1s} and I_{4s} separately IR divergent!

- Dual conformal symmetry absent in form factors, however the cancellation of unwanted IR divergences is still present and powerful

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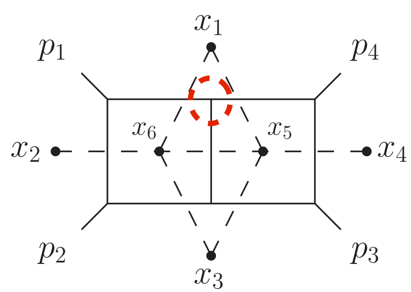

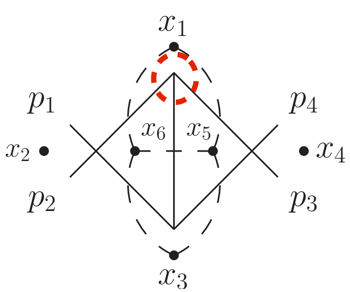

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- Comment 2
  - Numerators make the integrals maximally transcendental!
  - an experimental observation so far
  - amplitudes and Wilson loops have uniform degree of transcendentality as in N=4 SYM

Summary

- Hidden structures/regularities in ABJM amplitudes
- One-loop amplitudes and recursion relations
    - connection between special triple cuts and BCFW diagrams
    - recursion relations for supercoefficients
- Two-loop Sudakov form factor
    - very interesting properties of integral functions, transcendental result
- Plenty of questions to ask!