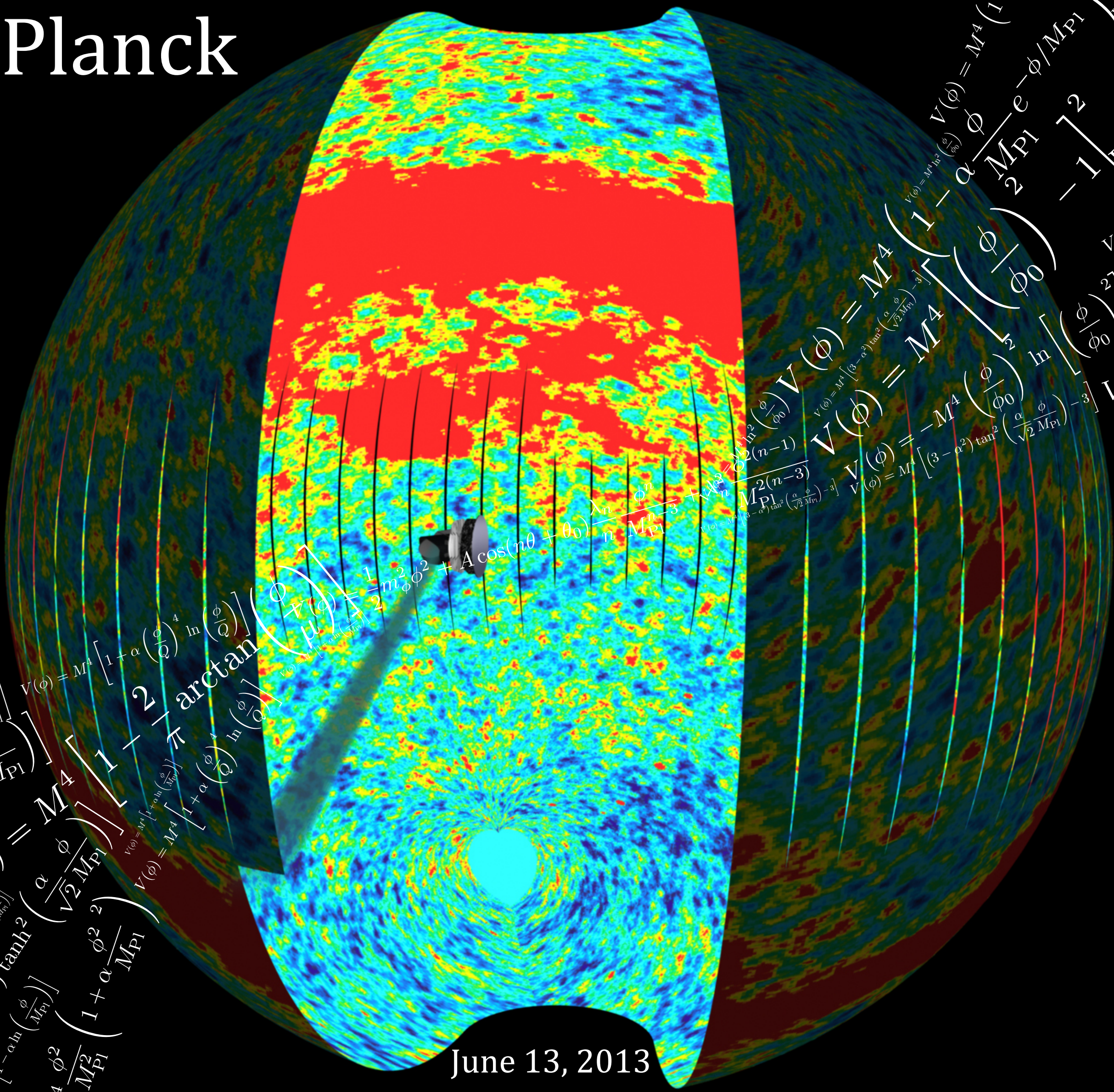
Inflationary Models after Planck



June 13, 2013

IAP, France

Vincent Vennin

12th Workshop on Non-Perturbative QCD

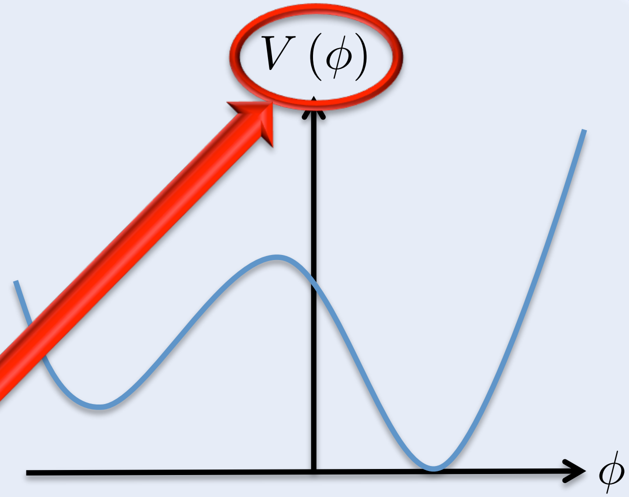
Cosmological Inflation

- Is a high energy phase of accelerated expansion in the early Universe $\ddot{a} > 0$
- Solves the Hot Big Bang horizon and flatness problem $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$
- Can be implemented with a single scalar field

$$S = -\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right]$$

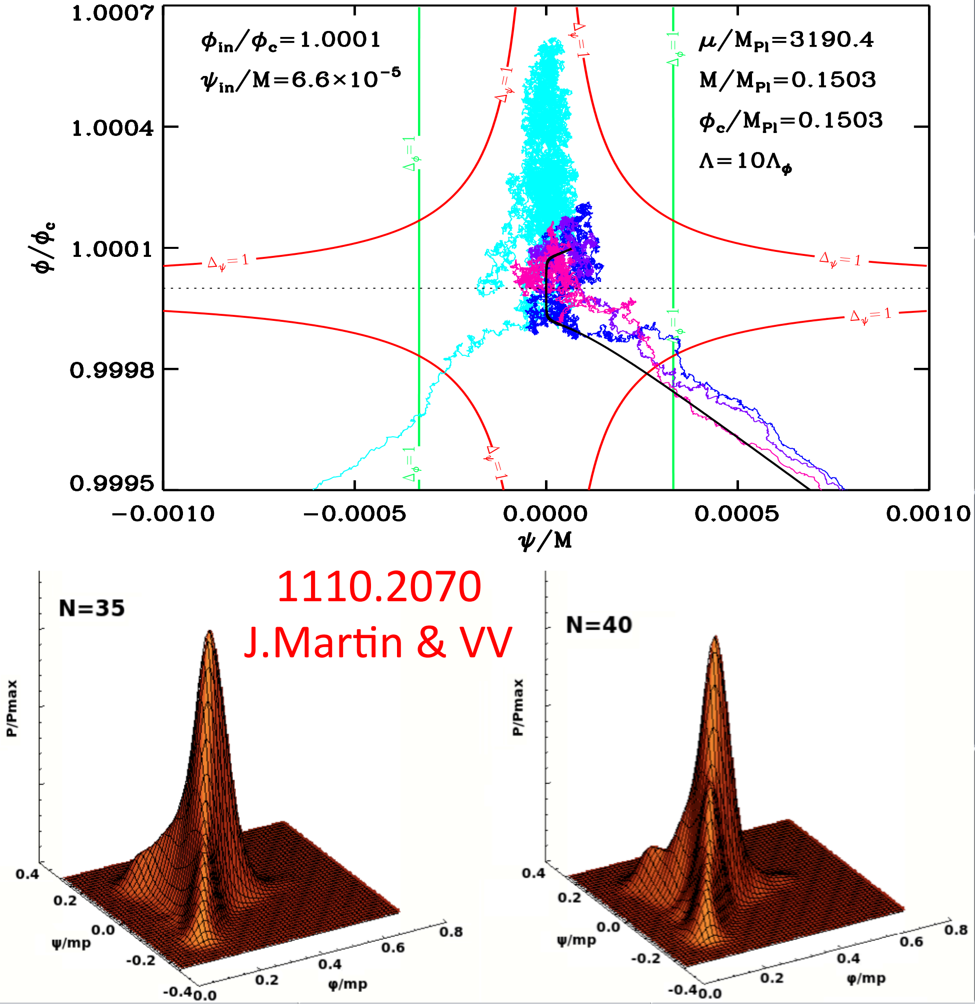
$$\Rightarrow \begin{cases} \rho = \frac{1}{2}\left(\dot{\phi}\right)^2 + V(\phi) \\ p = \frac{1}{2}\left(\dot{\phi}\right)^2 - V(\phi) \end{cases}$$

$$\ddot{a}/a = -\frac{1}{6M_\mathrm{P}^2}(\rho + 3p) \quad \Longrightarrow \quad V(\phi) \gg \dot{\phi}^2$$

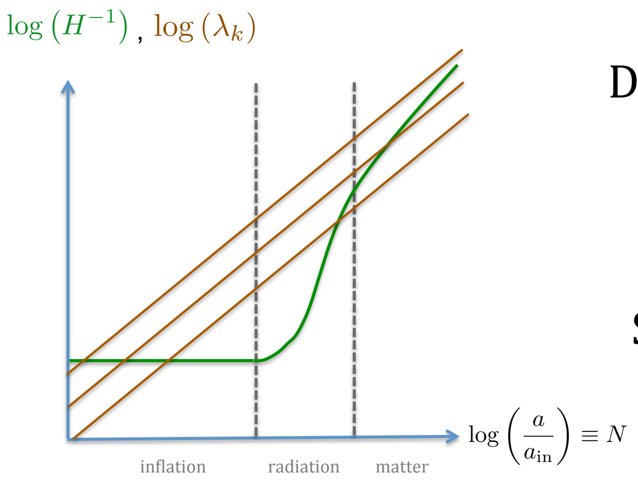


- Combined with QM, accounts for an almost scale invariant power spectrum

Observables / Physical Models	Regular Single Field	Single Field with Features	Single Field Non-canonical K-term	Multiple Field	…
scalar power spectrum $n_S \sim 1$ $\alpha_S \sim 0$					…
entropic & adiabatic perturbations $\mathcal{I} \ll \mathcal{R}$					…
gravity waves $r < 1$					…
non gaussianities $f_{\rm nl}^{\rm local} \lesssim 1$					…



Slow-Roll Approximation



During inflation, H is almost constant

$$\epsilon_0 = \frac{H_{\mathrm{in}}}{H} \simeq \text{constant}$$

Slow-Roll hierarchy: $\epsilon_{n+1} = \frac{1}{\epsilon_n}\frac{\mathrm{d}\epsilon_n}{\mathrm{d}N}$

Friedman equation: $3M_{\mathrm{Pl}}^2 H^2 = V + \dot{\phi}^2/2$

$$\Rightarrow \epsilon_1 = 3\frac{\dot{\phi}^2/\left(2V\right)}{1+\dot{\phi}^2/\left(2V\right)} \ll 1$$

Klein Gordon equation: $\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0$

$$\Rightarrow \dot{\phi}^2/V \ll 1 \quad \text{« slow roll »}$$

$$\epsilon_1 \simeq \frac{1}{2M_{\mathrm{Pl}}^2}\left(\frac{V_\phi}{V}\right)^2 \qquad \epsilon_2 \simeq \frac{2}{M_{\mathrm{Pl}}^2}\left[\left(\frac{V_\phi}{V}\right)^2 - \frac{V_{\phi\phi}}{V}\right] \qquad \epsilon_3 \simeq \text{etc...}$$

Scalar Power Spectrum

Cosmological Fluctuations:

- are combined gauge invariant perturbations of the metric and of the inflaton field v
- are the seeds of temperature anisotropies in the CMB $v \propto \frac{\delta T}{T}$
- Follow a parametric amplifying equation of motion

$$v_{\boldsymbol{k}}'' + \left[k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}\right] v_{\boldsymbol{k}} = 0$$

Power Spectrum:

$$P_v(k) = \frac{k^3}{2\pi^2}\left\langle \hat{v}_k^2 \right\rangle$$

$$= \frac{a^2 H^2}{8\pi^2 M_{\rm Pl}^2 \epsilon_{1\diamond}}\left[1 - \left(2\epsilon_{1\diamond} + \epsilon_{2\diamond} + \cdots\right)\ln\frac{k}{k_\diamond} + \left(2\epsilon_{1\diamond}^2 + \epsilon_{1\diamond}\epsilon_{2\diamond} + \frac{\epsilon_{2\diamond}^2}{2} - \frac{\epsilon_{2\diamond}\epsilon_{3\diamond}}{2} + \cdots\right)\ln^2\frac{k}{k_\diamond} + \cdots\right]$$

Spectral index $n_{\rm S} = \left.\frac{\mathrm{d}\ln P}{\mathrm{d}\ln k}\right|_{k_*}$

$$n_{\rm S}^{\rm Planck} \sim 0.96$$

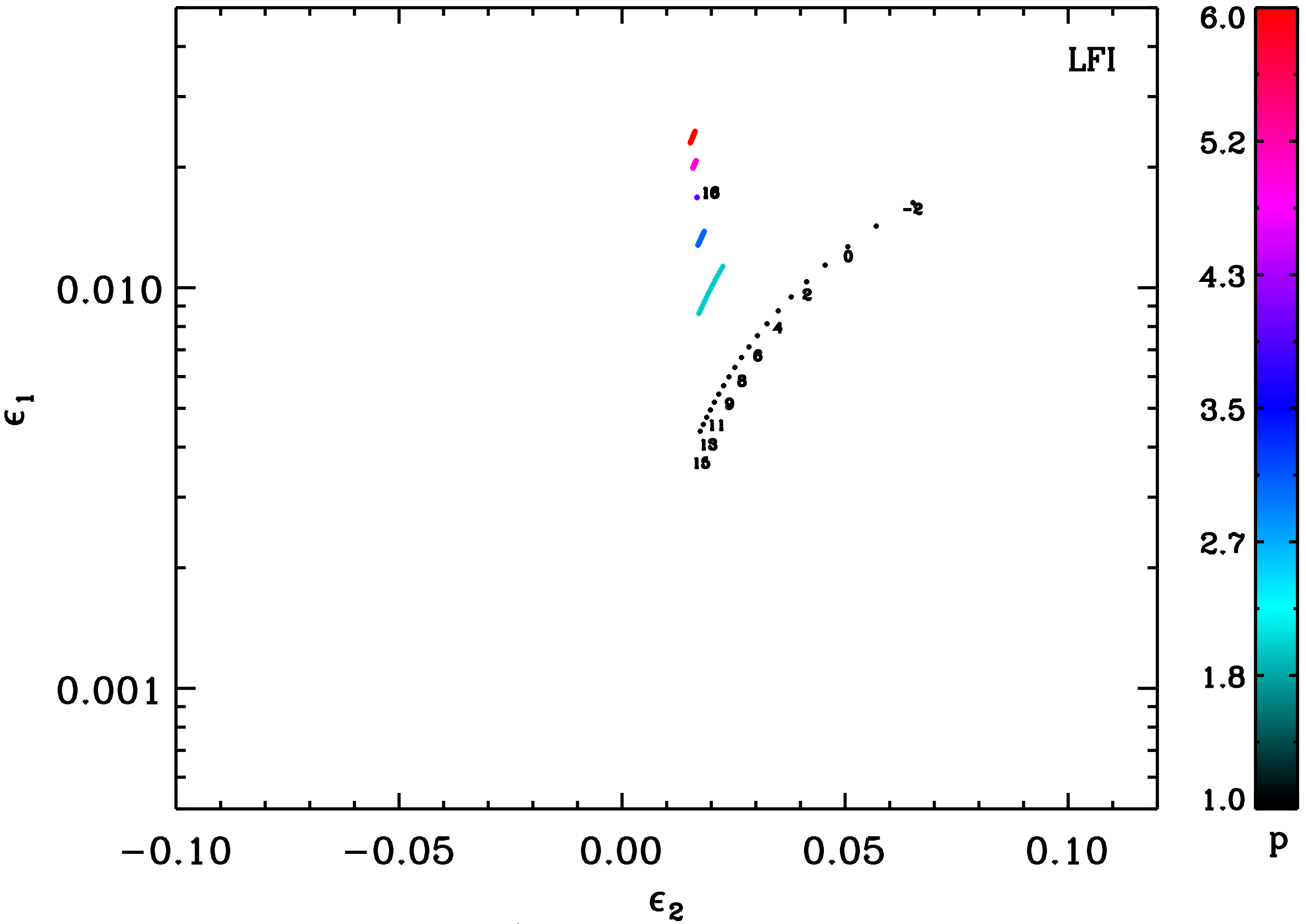
Gravity waves:

$$r = \frac{P_h(k_\diamond)}{P_v(k_\diamond)} = 16\epsilon_{1\diamond} + \cdots$$

Confronting Models with Data

An example: « large field inflation »

$$V\left(\phi\right) = M^4 \left(\frac{\phi}{M_{\rm Pl}}\right)^p$$



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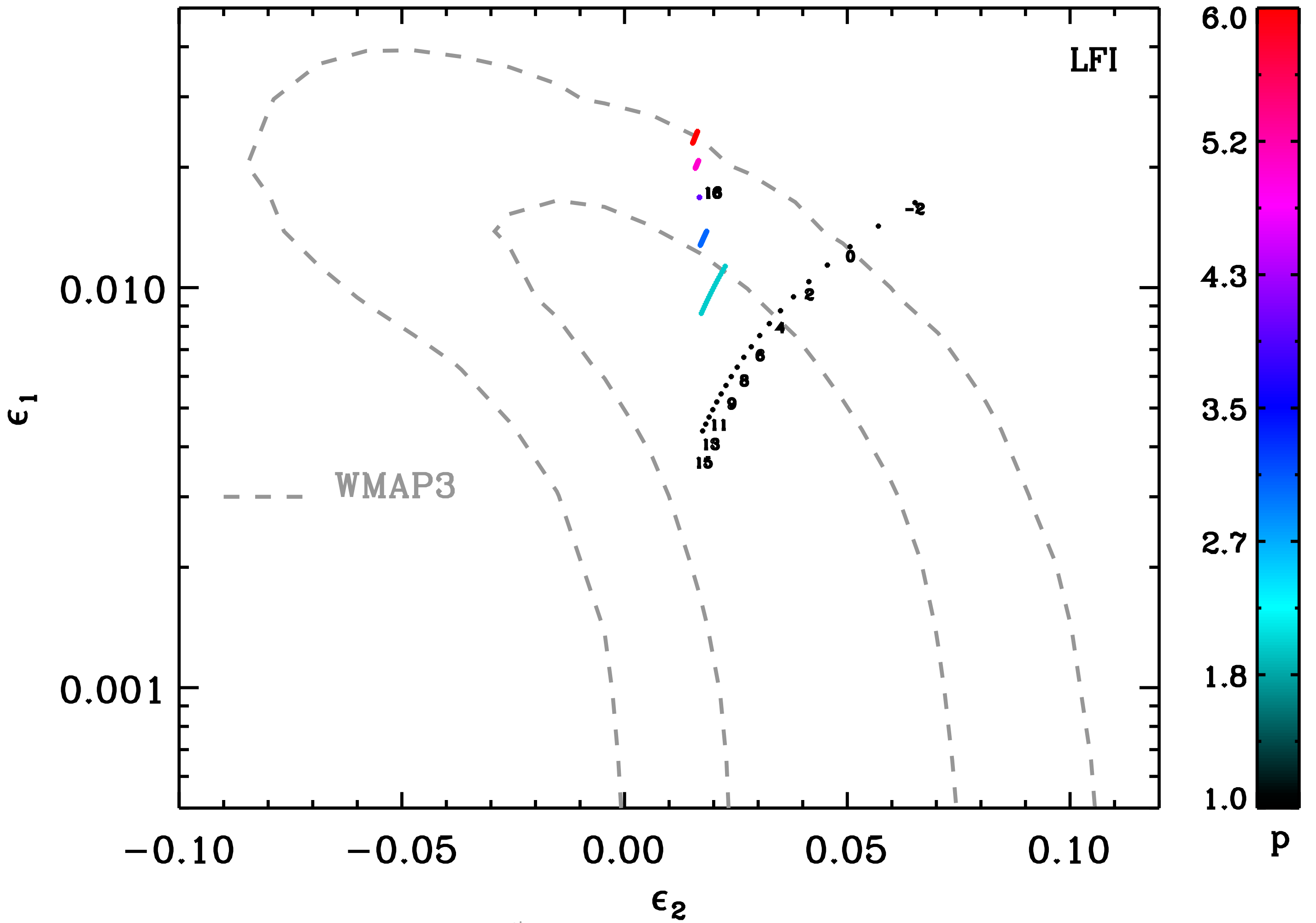

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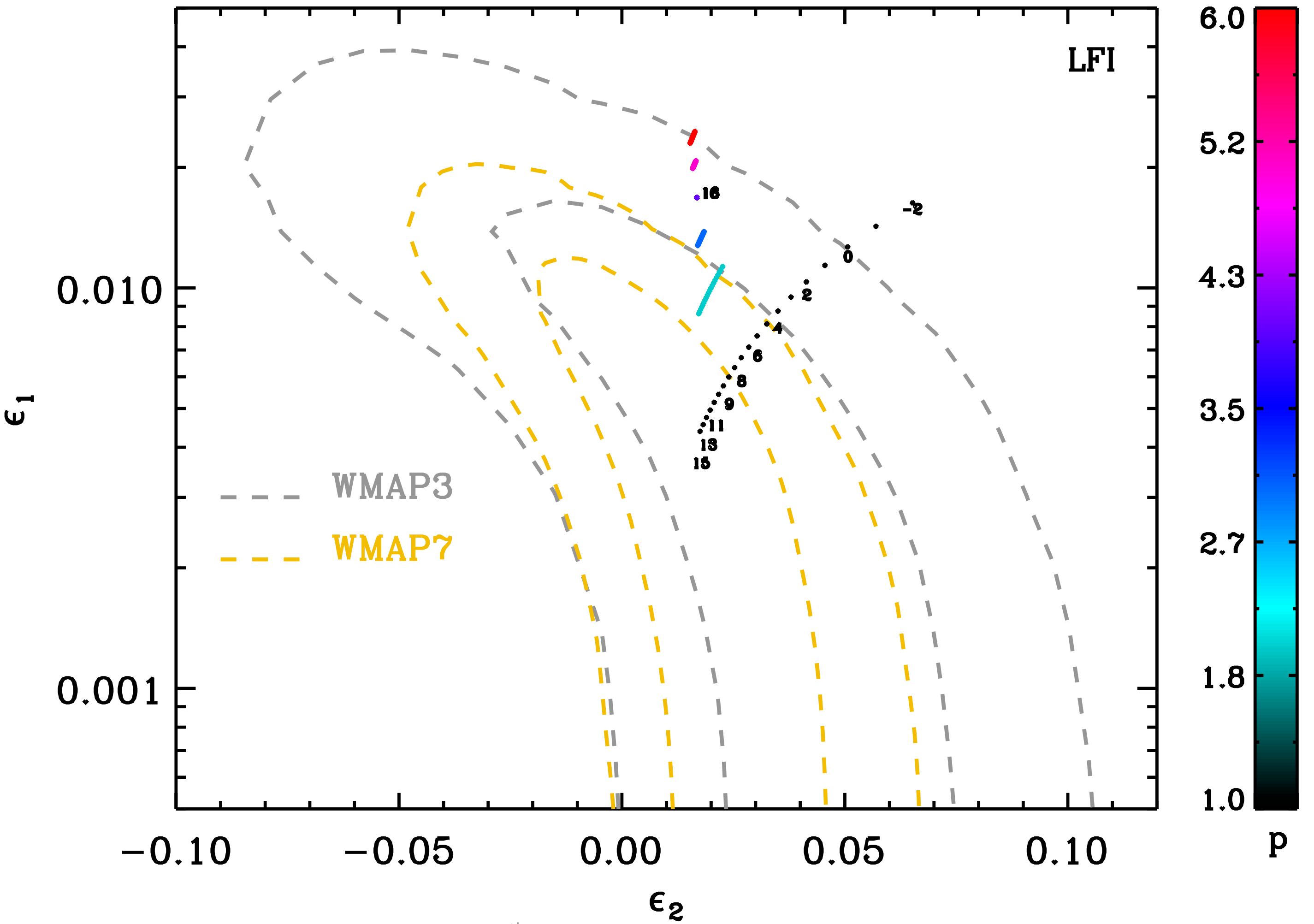

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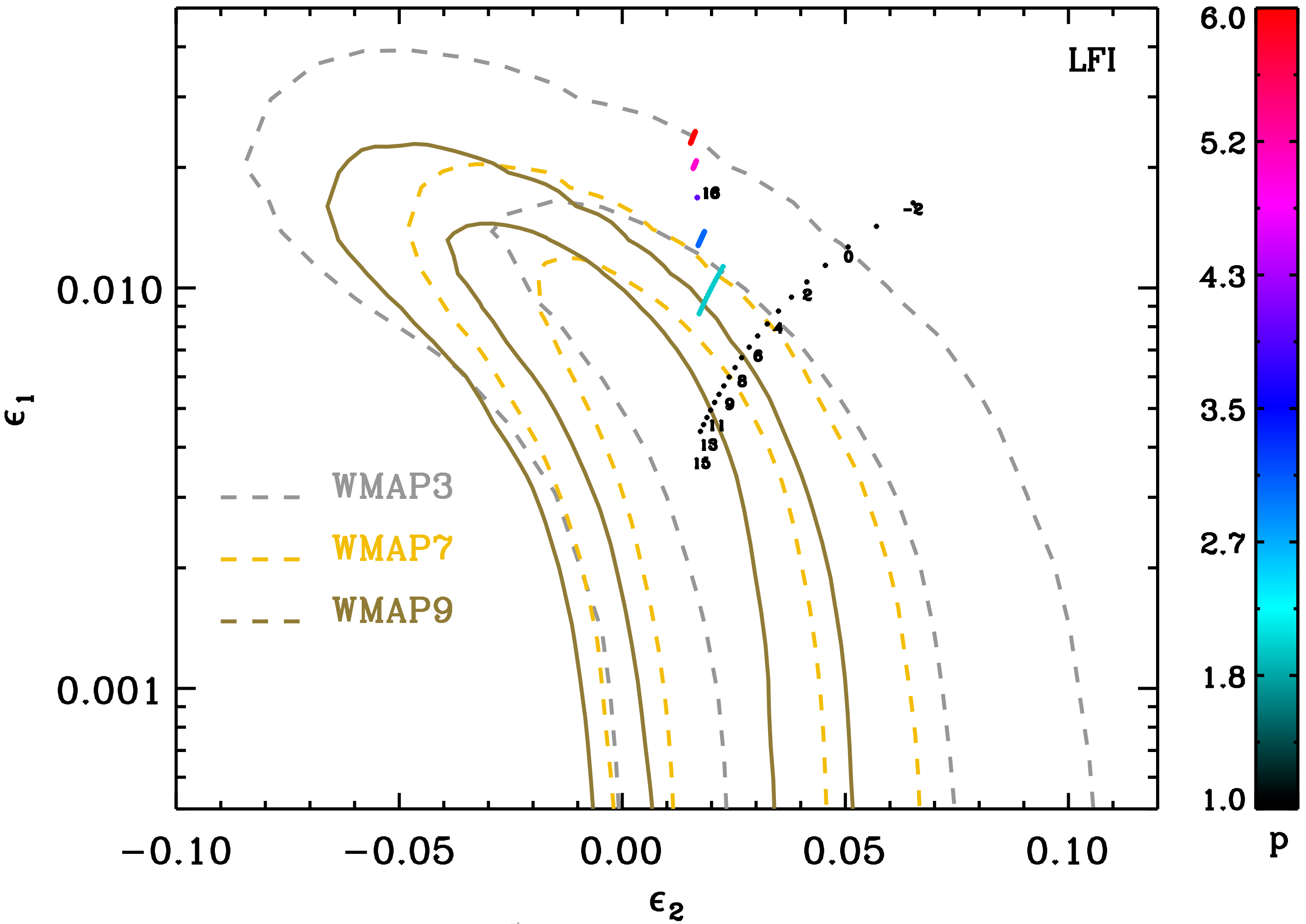

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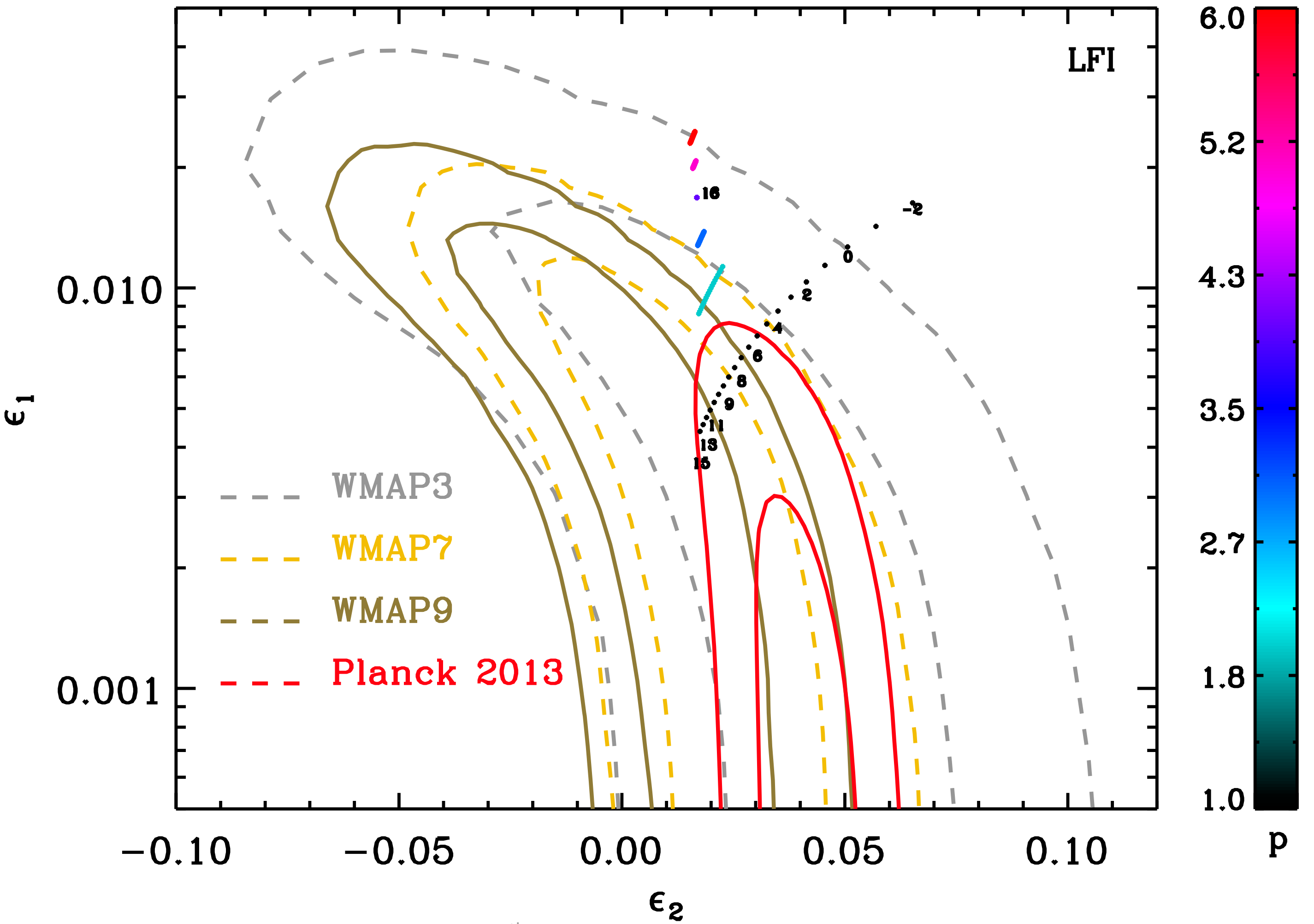
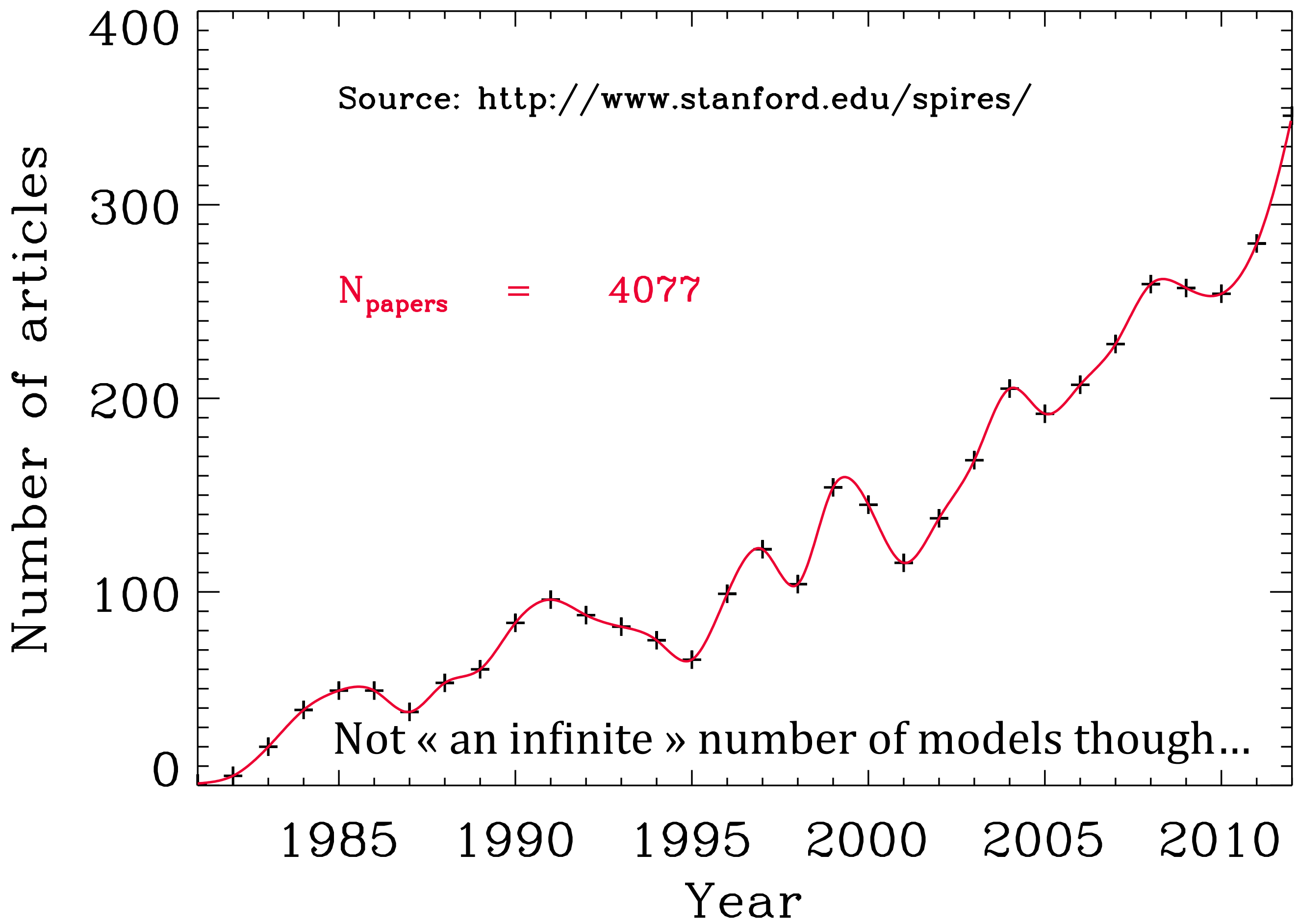


Repeat the analysis for all single field, canonical k, model?

Which models? How many of them?



Proliferation of inflationary models[1]

5-dimensional assisted inflation
anisotropic brane inflation
anomaly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic hybrid inflation
chaotic inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation
dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false vacuum inflation
false vacuum chaotic inflation
fast-roll inflation
first order inflation
gauged inflation
generalised inflation
generalized assisted inflation
generalized slow-roll inflation
gravity driven inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
induced gravity open inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation
K inflation
kinetic inflation
lambda inflation
large field inflation
late D-term inflation
late-time mild inflation
low-scale inflation
low-scale supergravity inflation
M-theory inflation
mass inflation
massive chaotic inflation
moduli inflation
multi-scalar inflation
multiple inflation
multiple-field slow-roll inflation
multiple-stage inflation
natural inflation
natural Chaotic inflation
natural double inflation
natural supergravity inflation
new inflation
next-to-minimal supersymmetric hybrid inflation
non-commutative inflation
non-slow-roll inflation
nonminimal chaotic inflation
old inflation
open hybrid inflation
open inflation
oscillating inflation
polynomial chaotic inflation
polynomial hybrid inflation
power-law inflation
pre-Big-Bang inflation
primary inflation
primordial inflation
quasi-open inflation
quintessential inflation
R-invariant topological inflation
rapid asymmetric inflation
running inflation
scalar-tensor gravity inflation
scalar-tensor stochastic inflation
Seiberg-Witten inflation
single-bubble open inflation
spinodal inflation
stable starobinsky-type inflation
steady-state eternal inflation
steep inflation
stochastic inflation
string-forming open inflation
successful D-term inflation
supergravity inflation
supernatural inflation
superstring inflation
supersymmetric hybrid inflation
supersymmetric inflation
supersymmetric topological inflatior
supersymmetric new inflation
synergistic warm inflation
TeV-scale hybrid inflation

A partial list of ever-increasing number of inflationary models!

[1] From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

[1303.3787]

Encyclopædia Inflationaris

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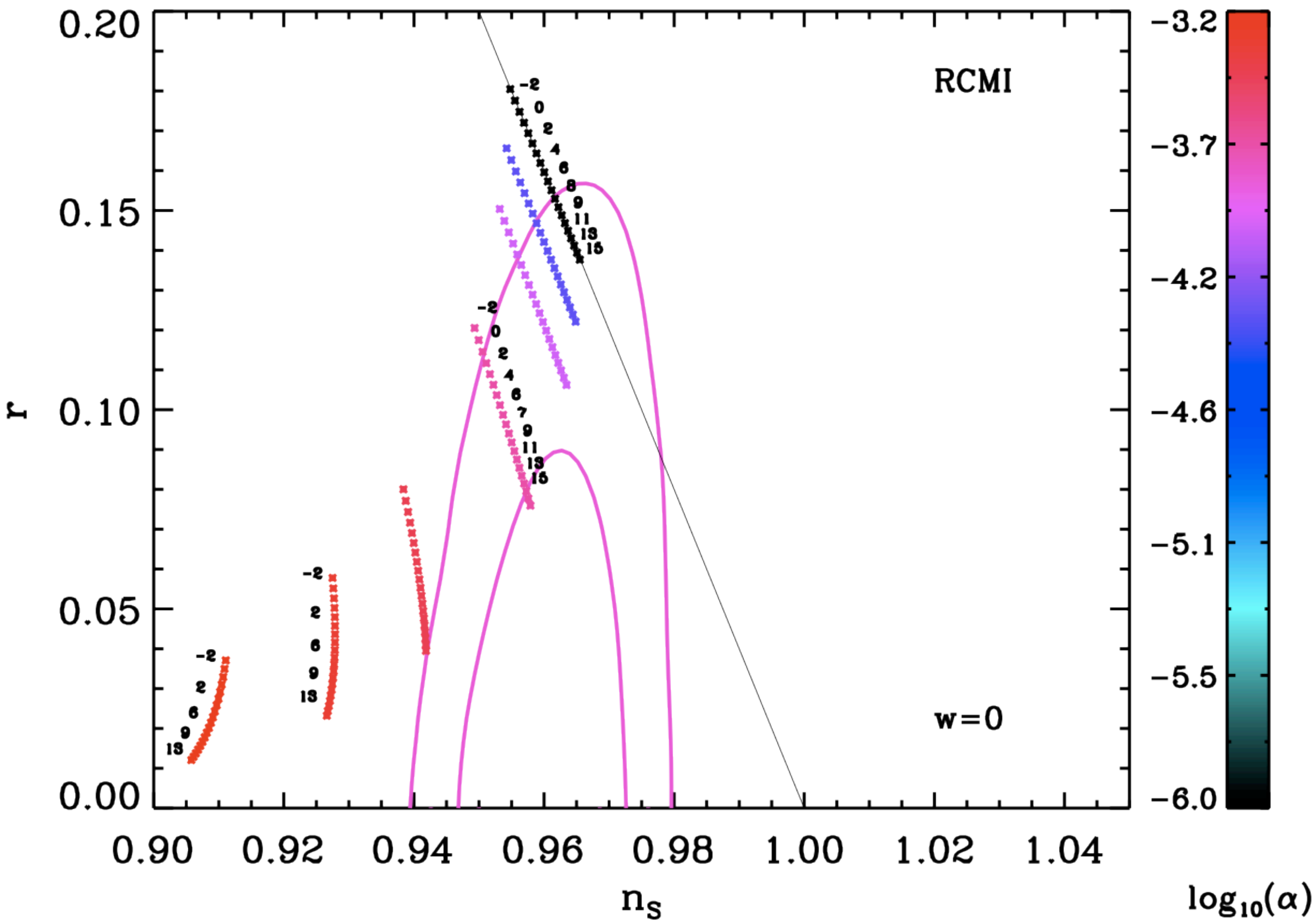
Keywords: Cosmic Inflation, Slow-Roll, Reheating, Cosmic Microwave Background, Aspic

≈ 70 models

≈ 700 slow roll formulas

≈ 320 pages

A few examples



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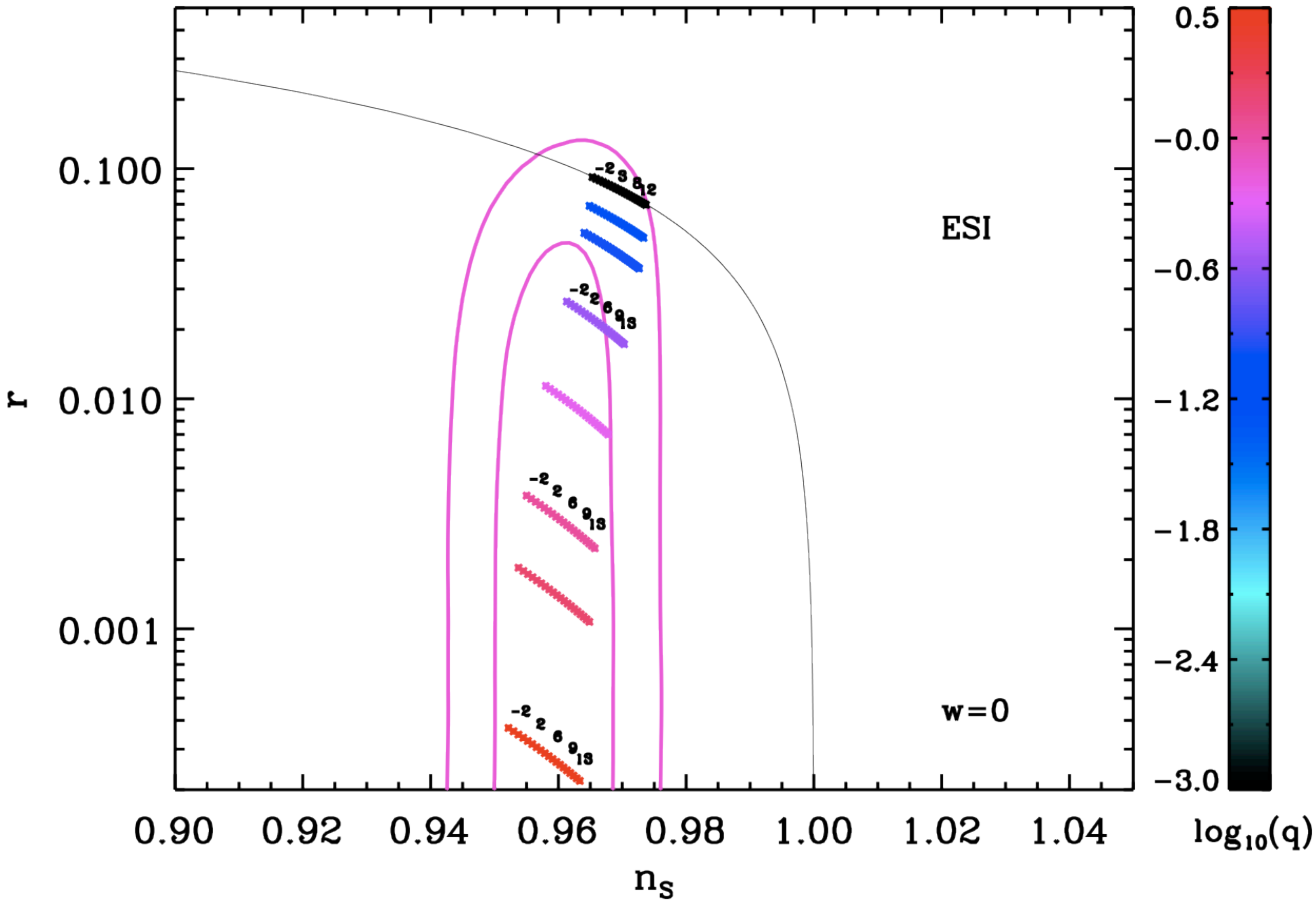

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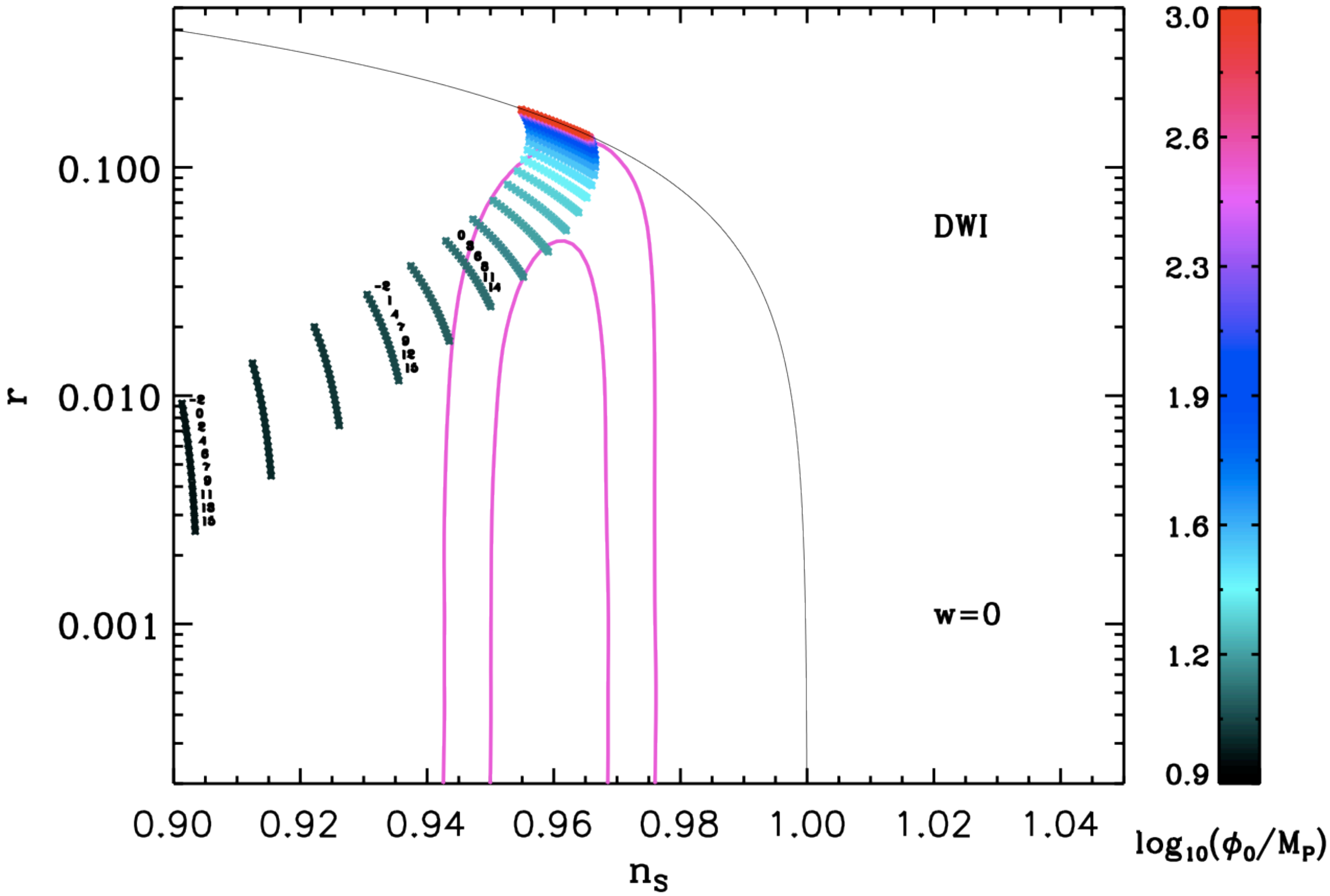

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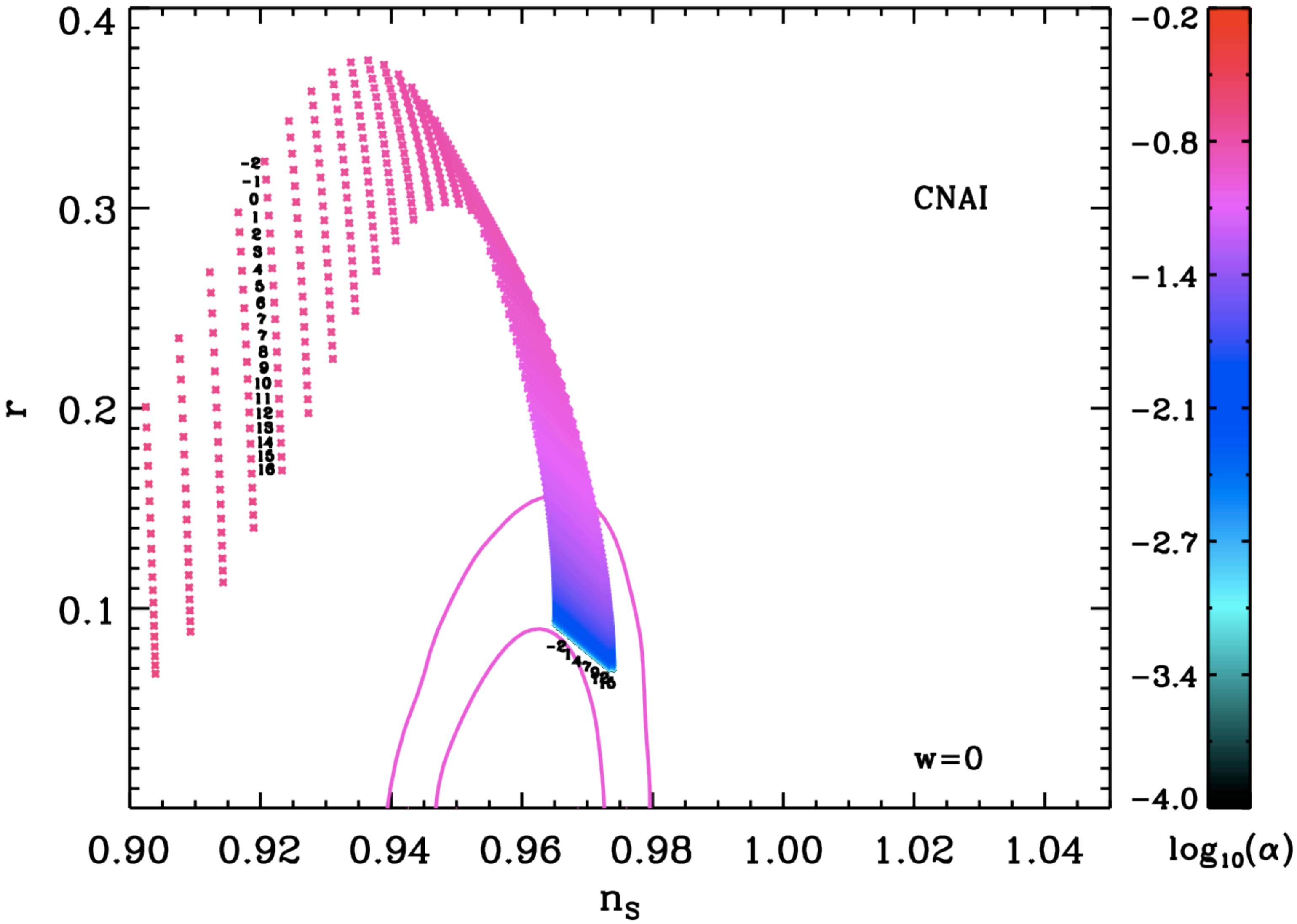

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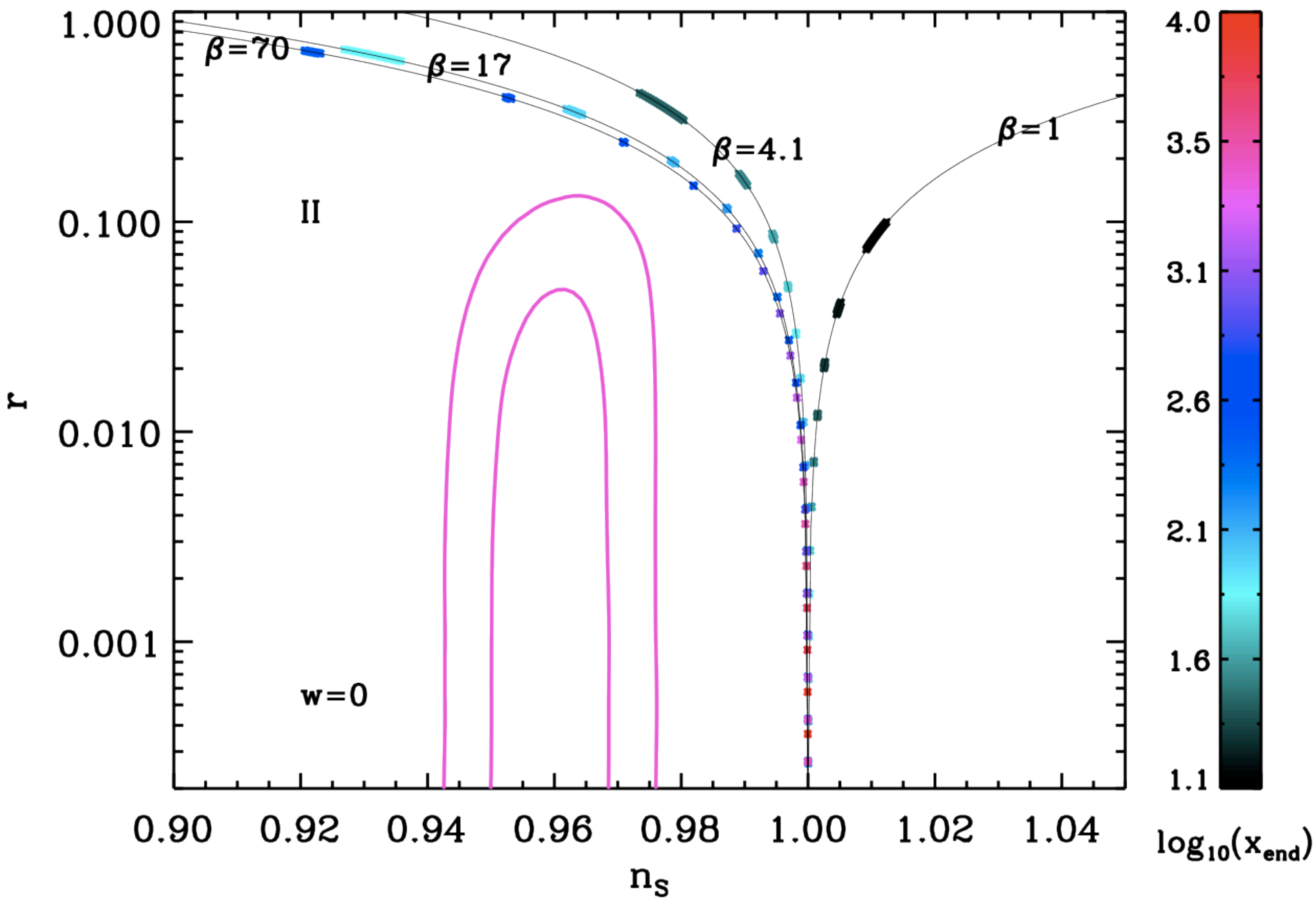

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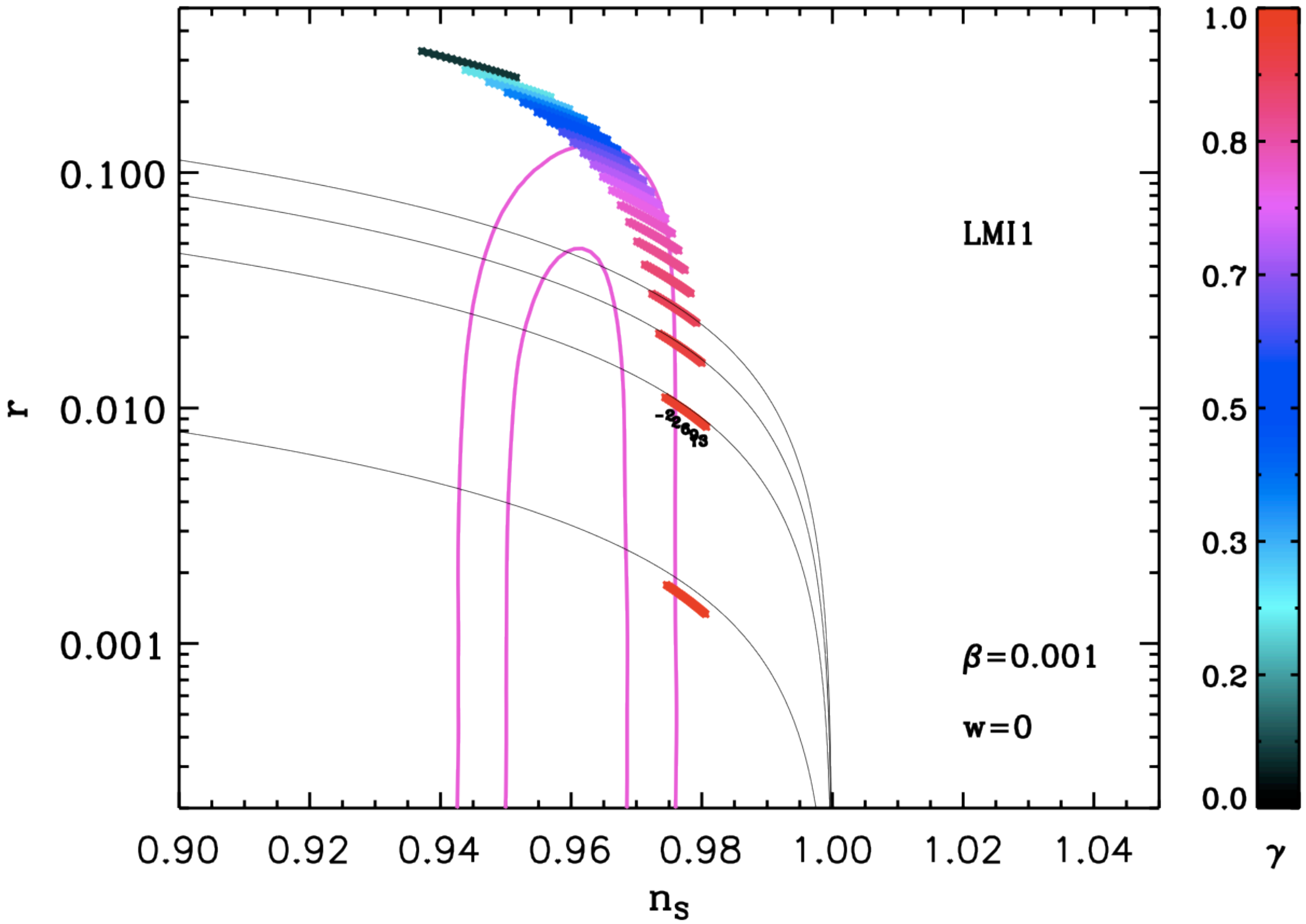

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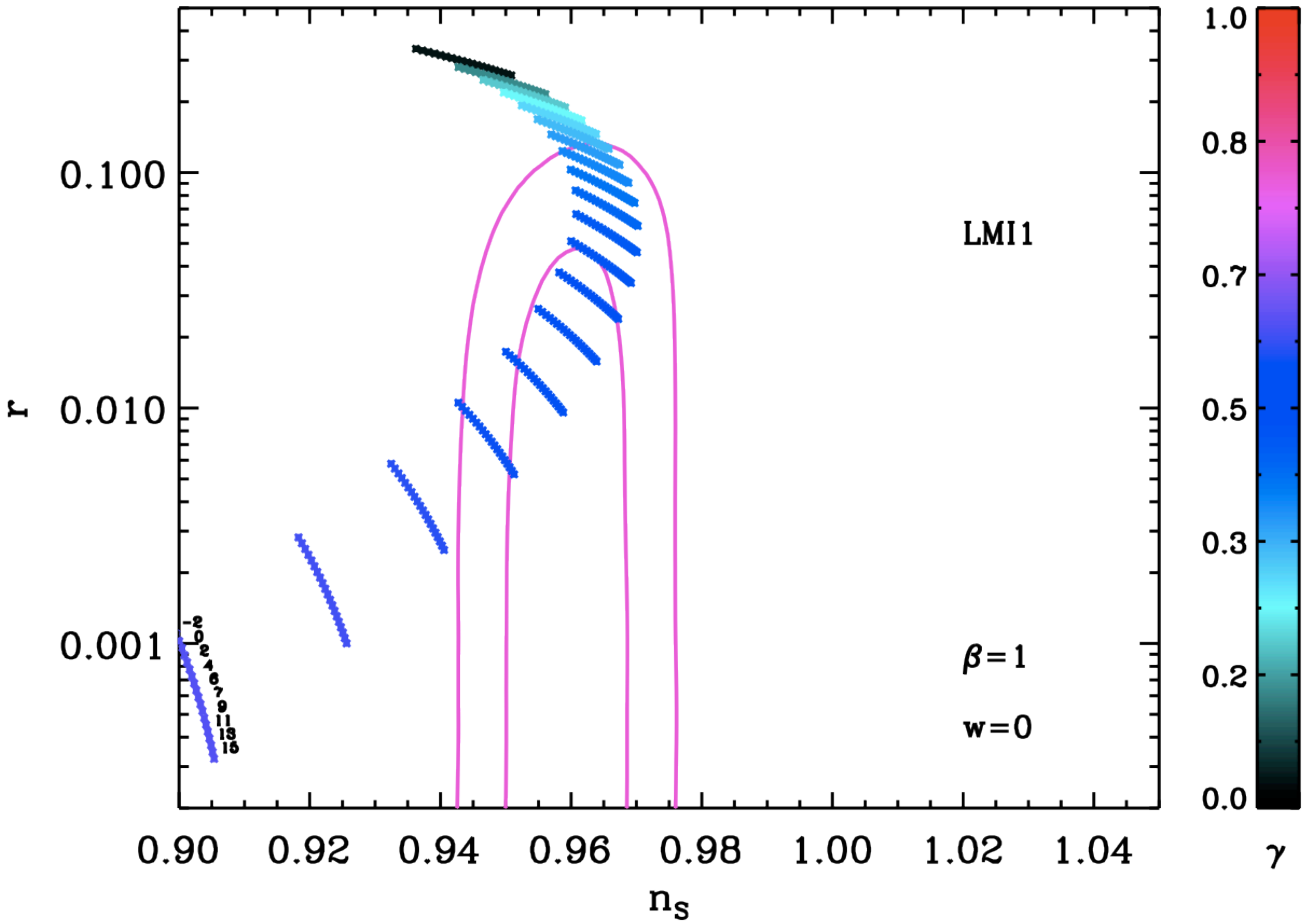

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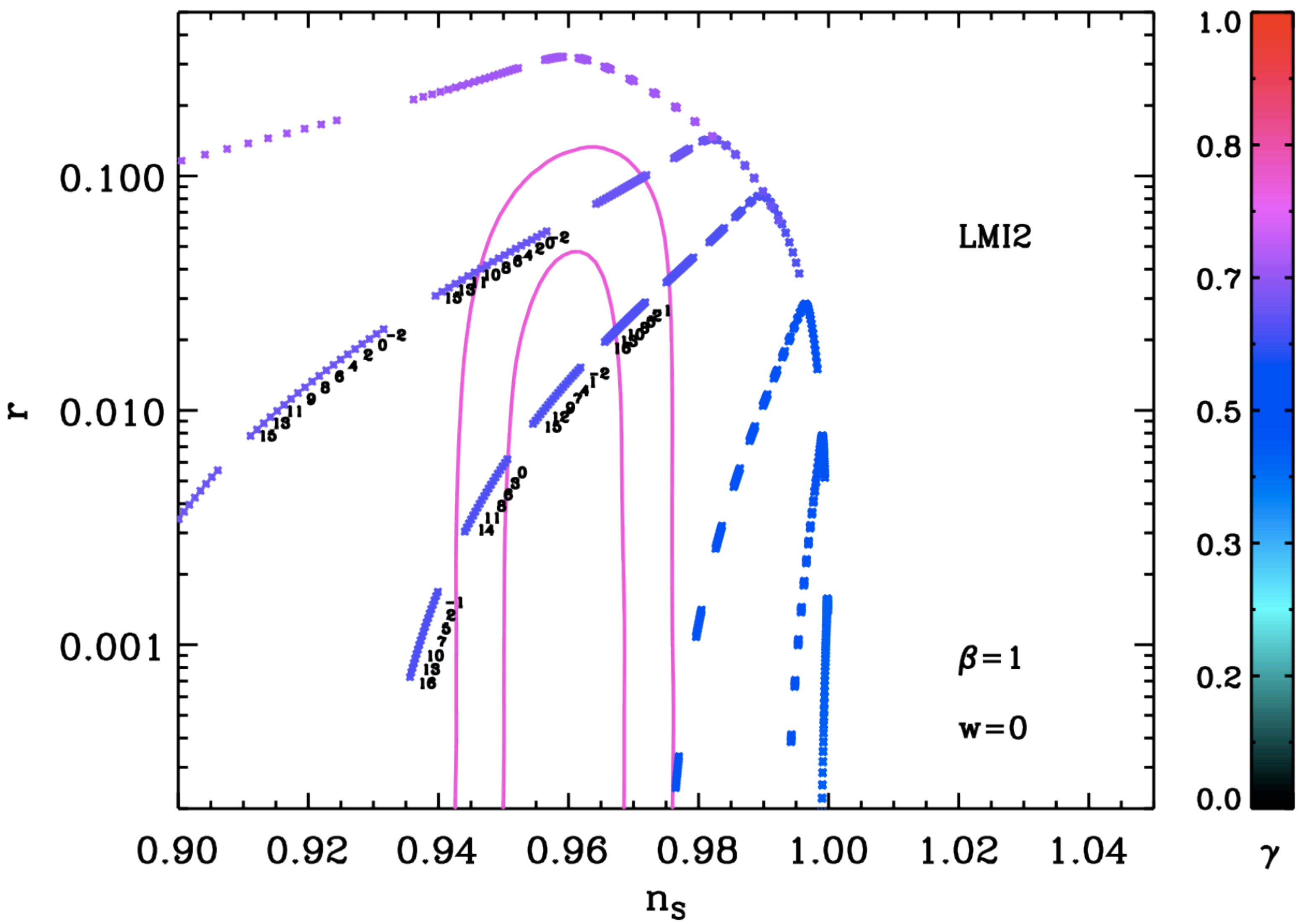

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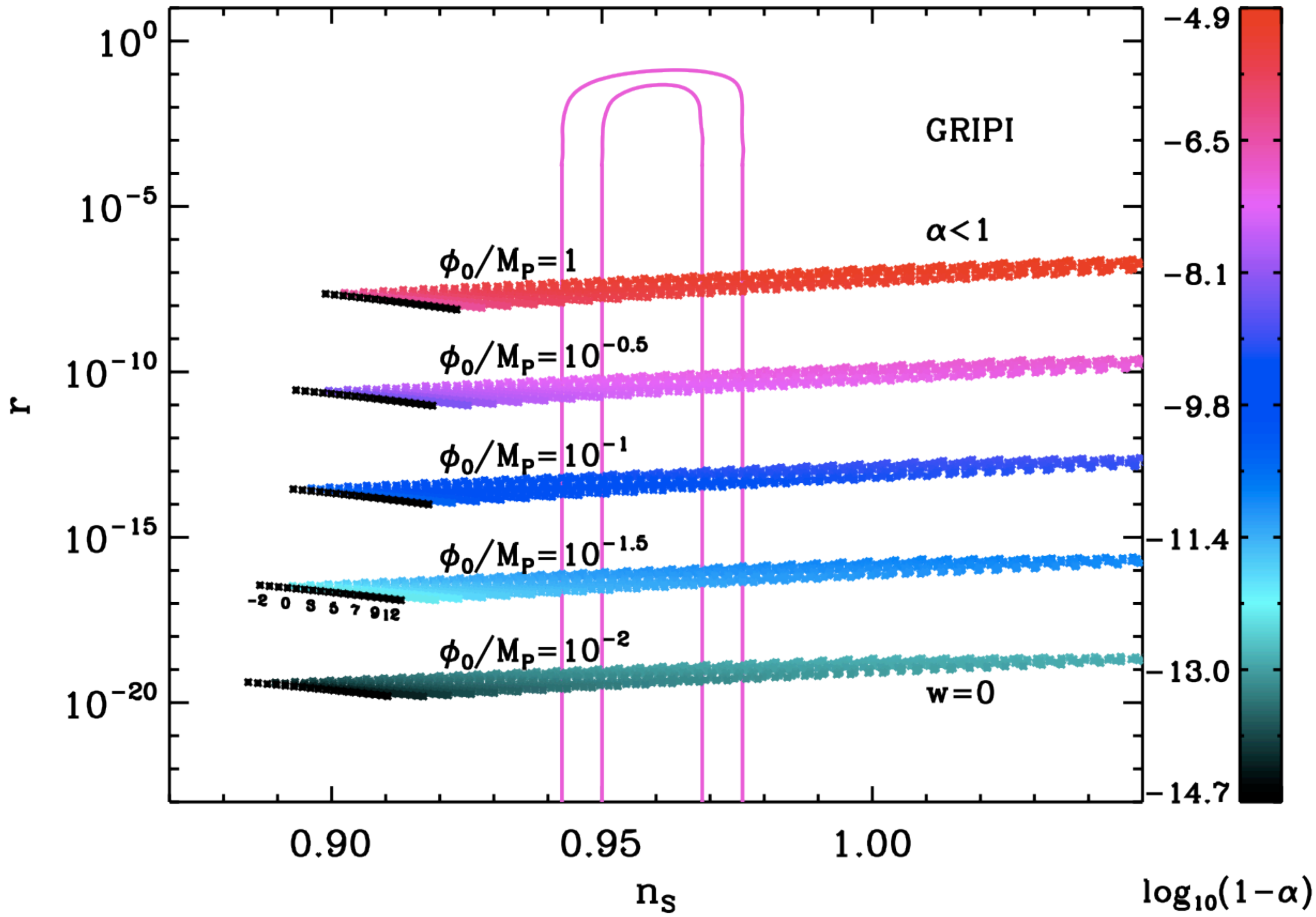

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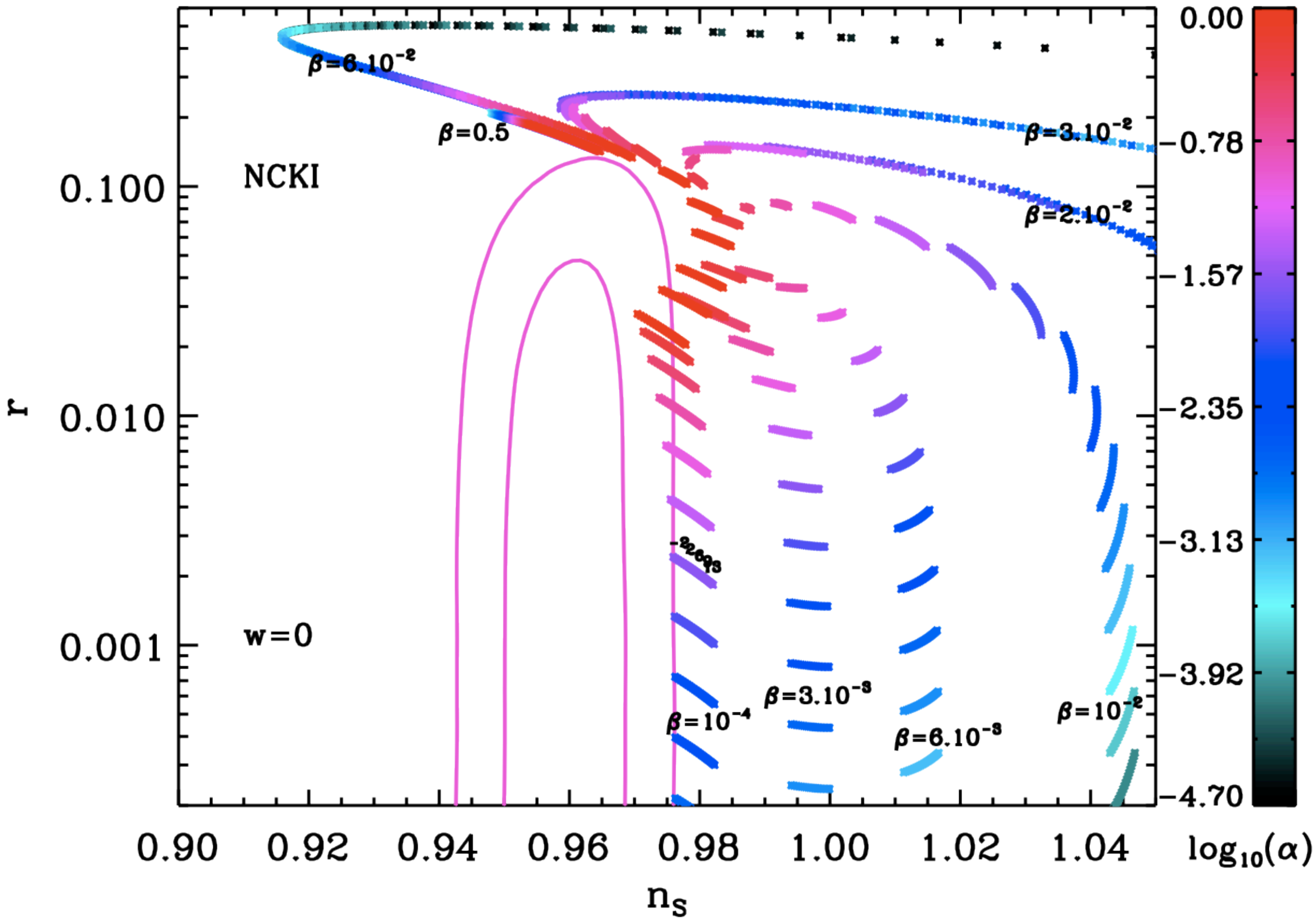

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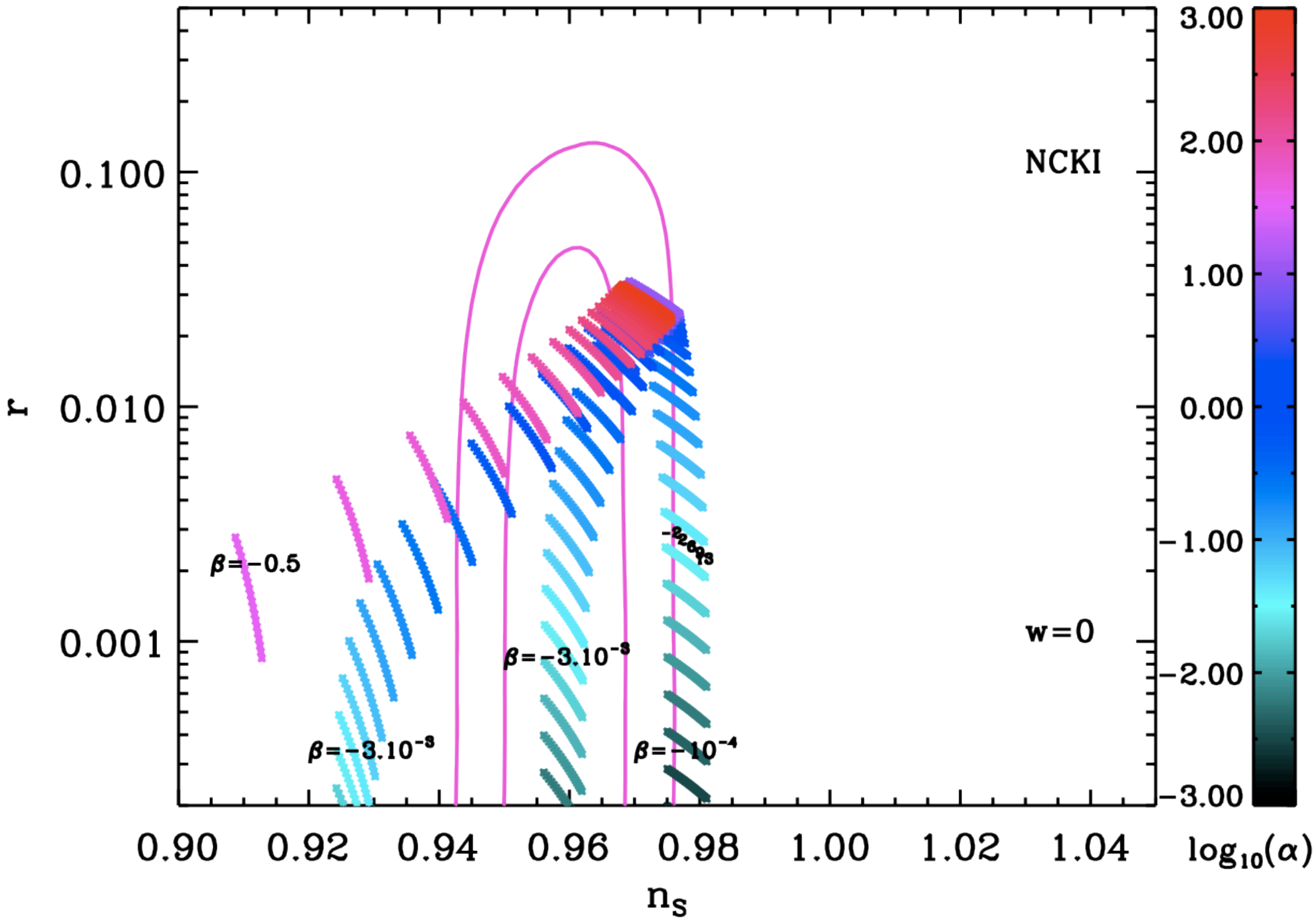

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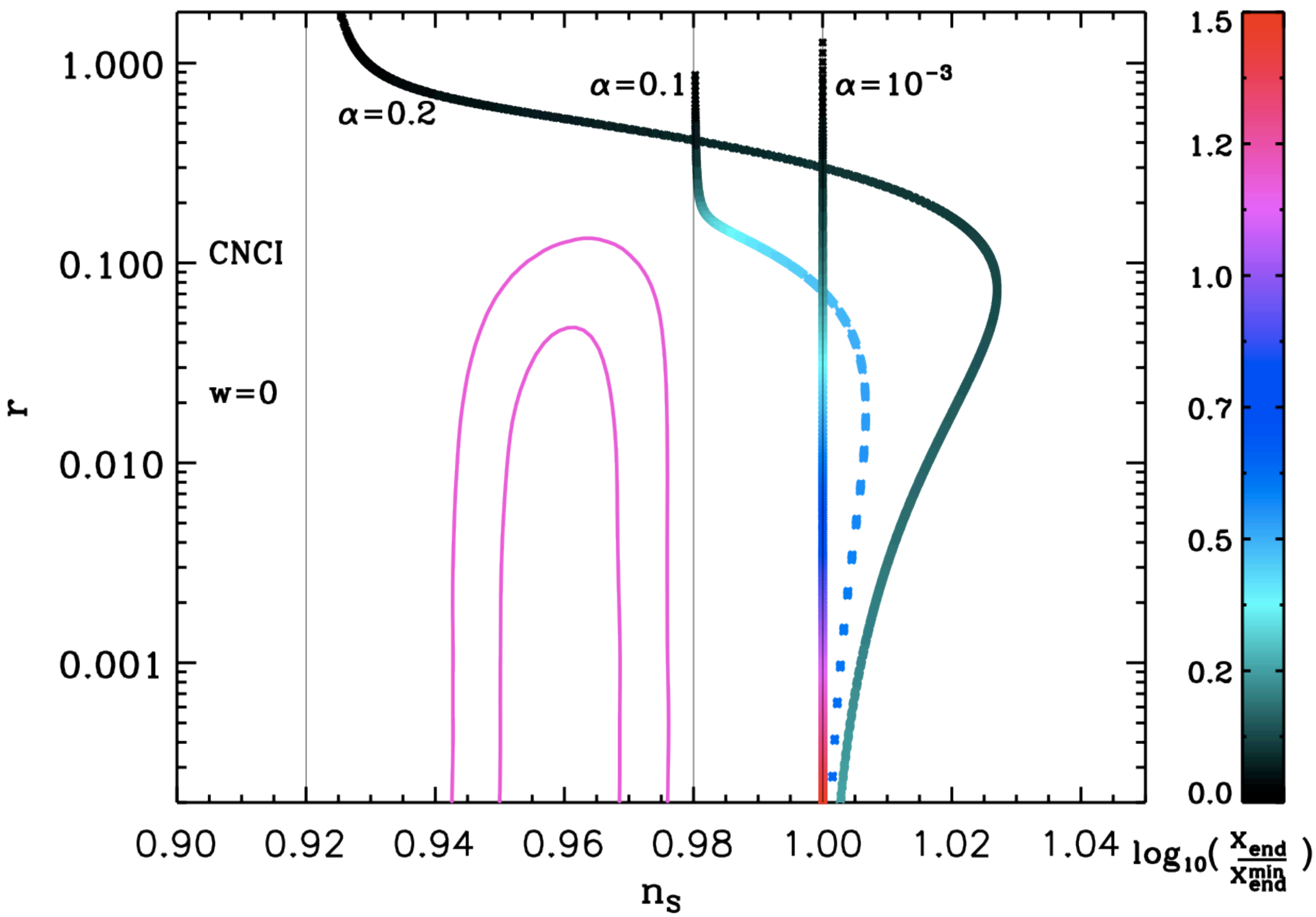

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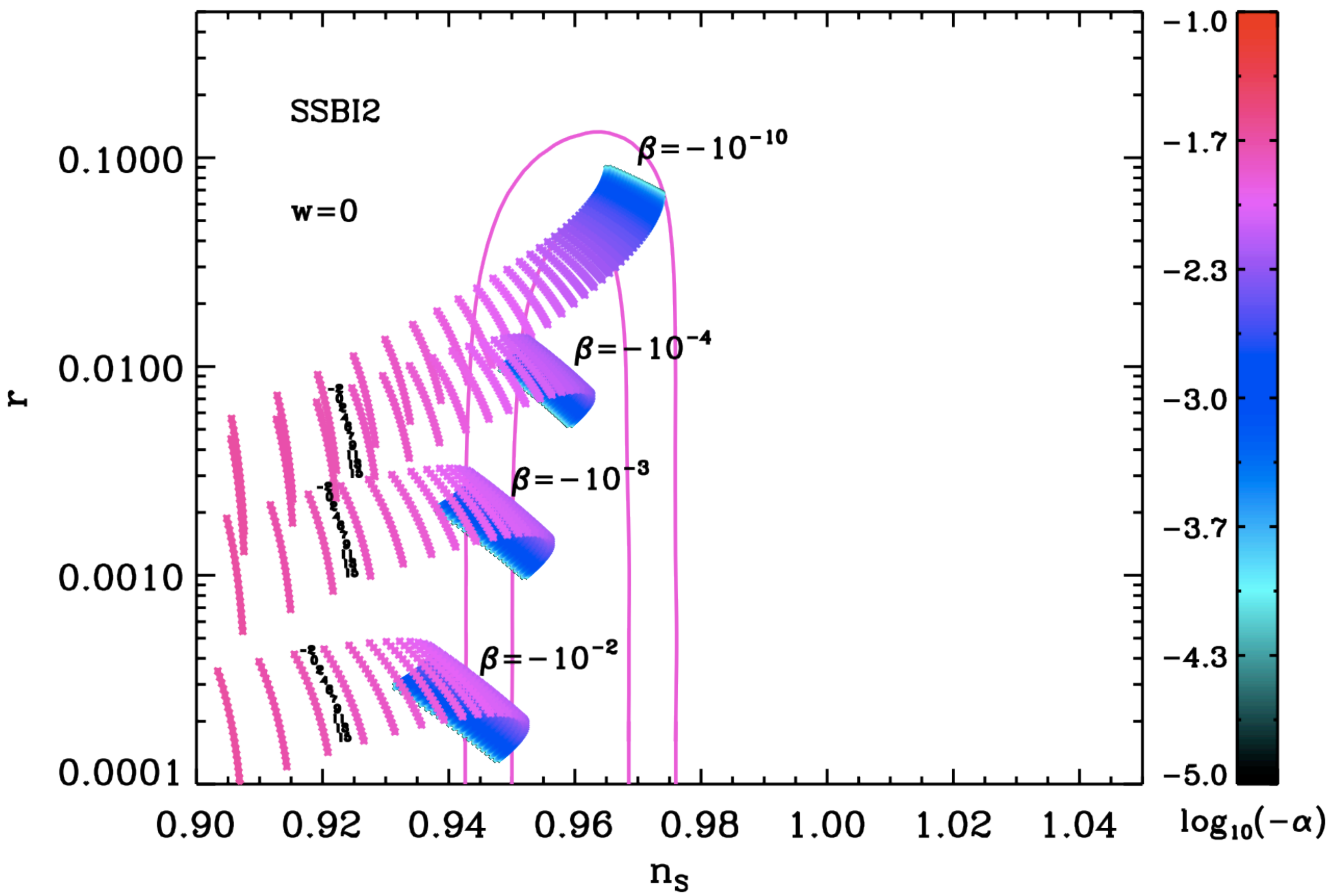

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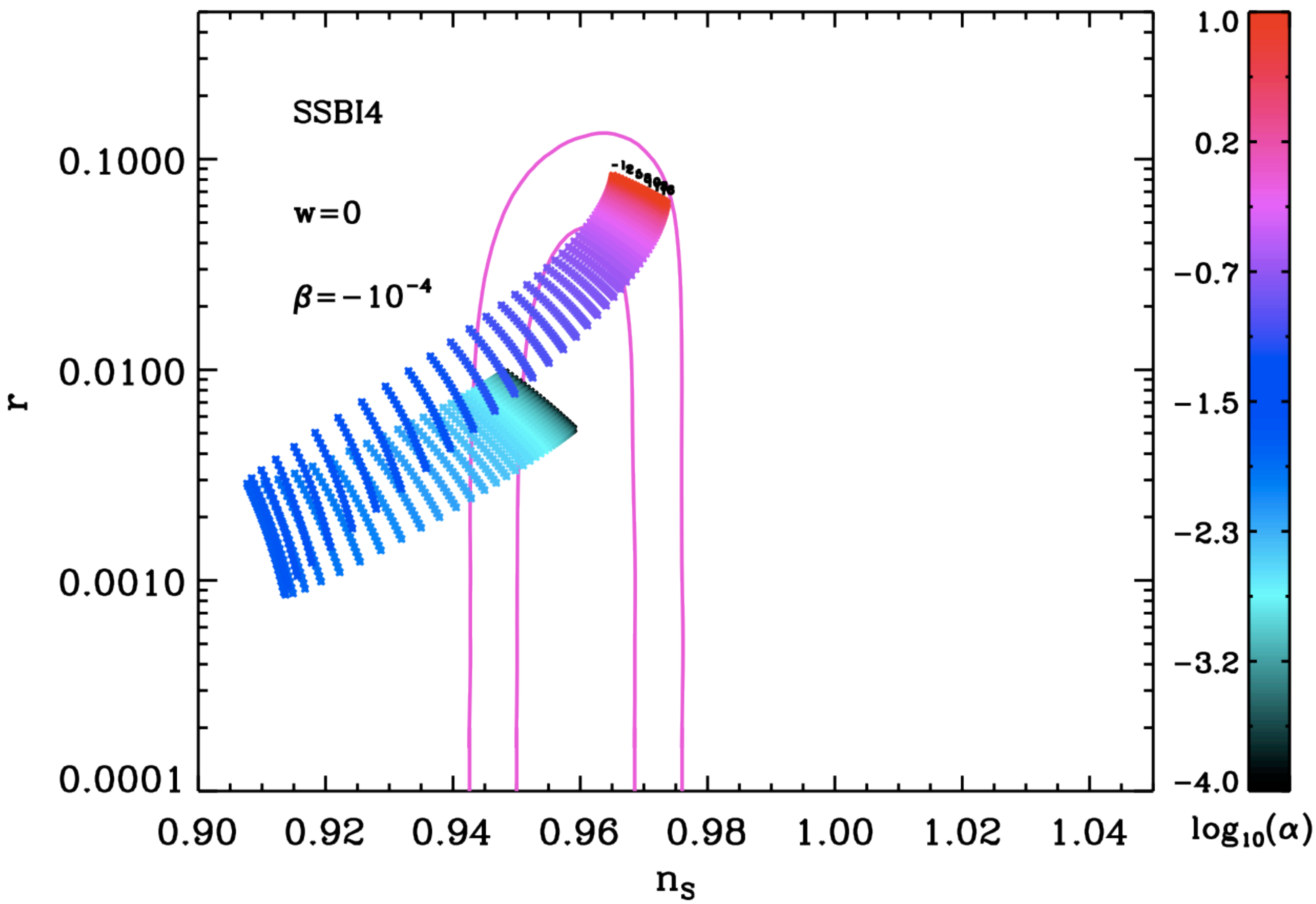

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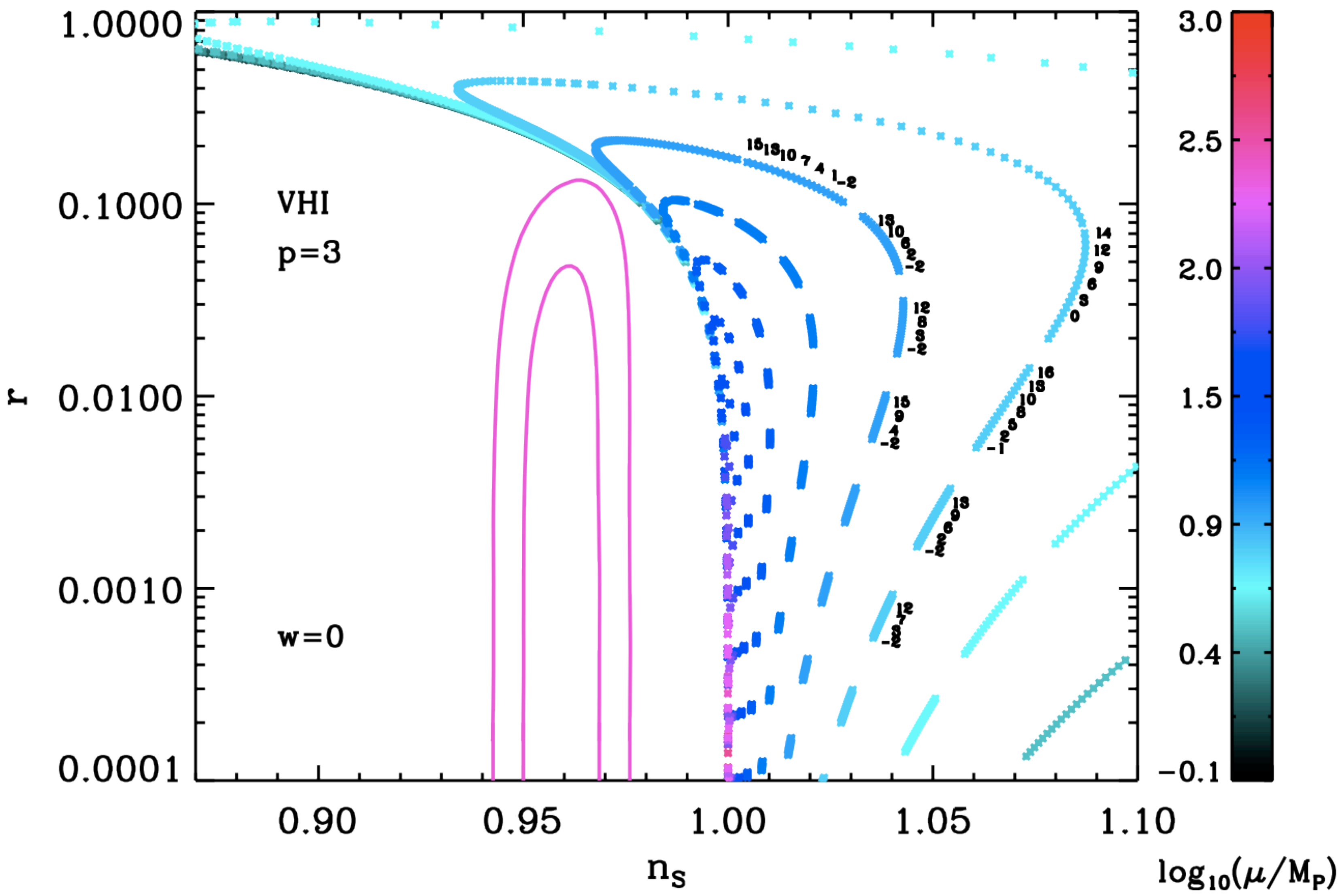

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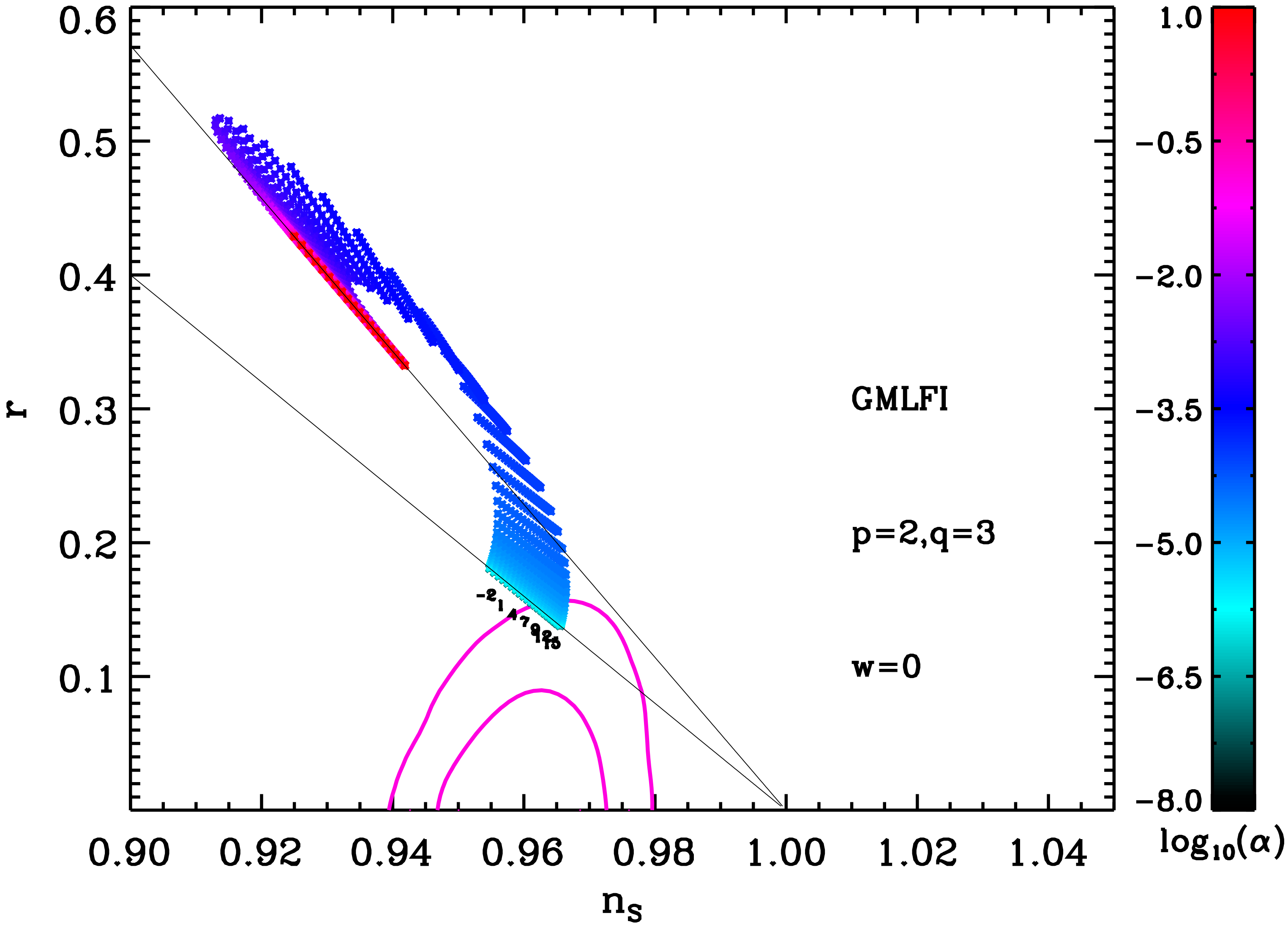
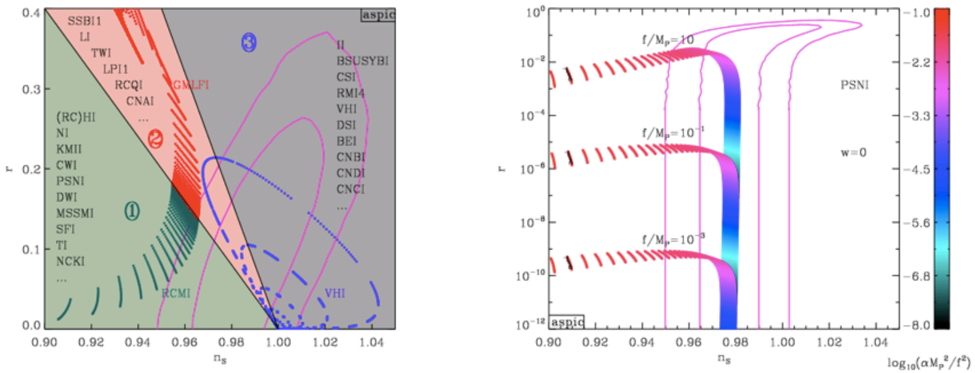


theory.physics.unige.ch/~ringeval/aspic.html

Accurate Slow-roll Predictions for Inflationary Cosmology



Reheating consistent slow-roll predictions for a subset of inflationary models supported by **aspic** (left). The right panel features the Pseudo Natural Inflation (PSNI) predictions. The annotated values show the logarithmic energy scale, log(Ereh/GeV), at which a matter dominated reheating ends (arXiv:1303.3787).

Aspic is a collection of fast modern fortran routines for computing various observable quantities used in Cosmology from definite single field inflationary models. It is distributed as a scientific library and aims at providing an efficient, extendable and accurate way of comparing theoretical inflationary predictions with cosmological data. **Aspic** currently supports 64 models of inflation, and more to come!

By observable quantities, we currently refer to as the Hubble flow functions, up to second order in the slow-roll approximation, which are in direct correspondence with the spectral index, the tensor-to-scalar ratio and the running of the primordial power spectrum. The **aspic** library also provides the field potential, its first and second derivatives, the energy density at the end of inflation, the energy density at the end of reheating, and the field value (or e-fold value) at which the pivot scale crossed the Hubble radius during inflation. All these quantities are computed in a way which is consistent with the existence of a reheating phase.

The code is released as a GNU software which compiles itself into both a static and shared library. As the list of inflationary models is always increasing, you are encouraged to add support for any model that would not yet be implemented.

Please, check the MAN file for a complete documentation a

download the source file.

For details, please read the original paper arXiv:1303.3787

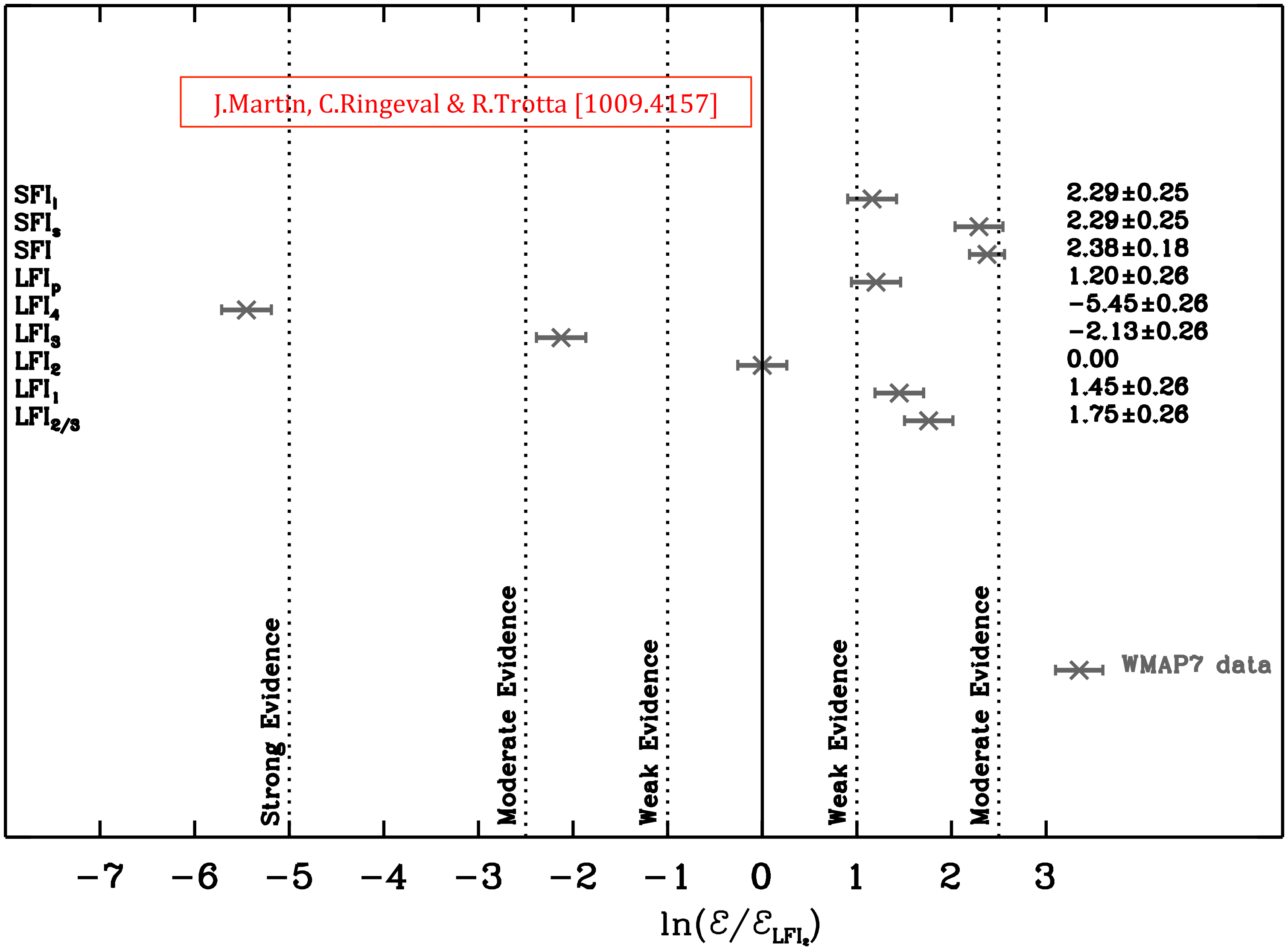
For an exact integration of any inflationary models, without assuming slow-roll, checkout the **fieldinf** code and library.

Last modif 03/2013

How to quantify how good a model fits the data ?

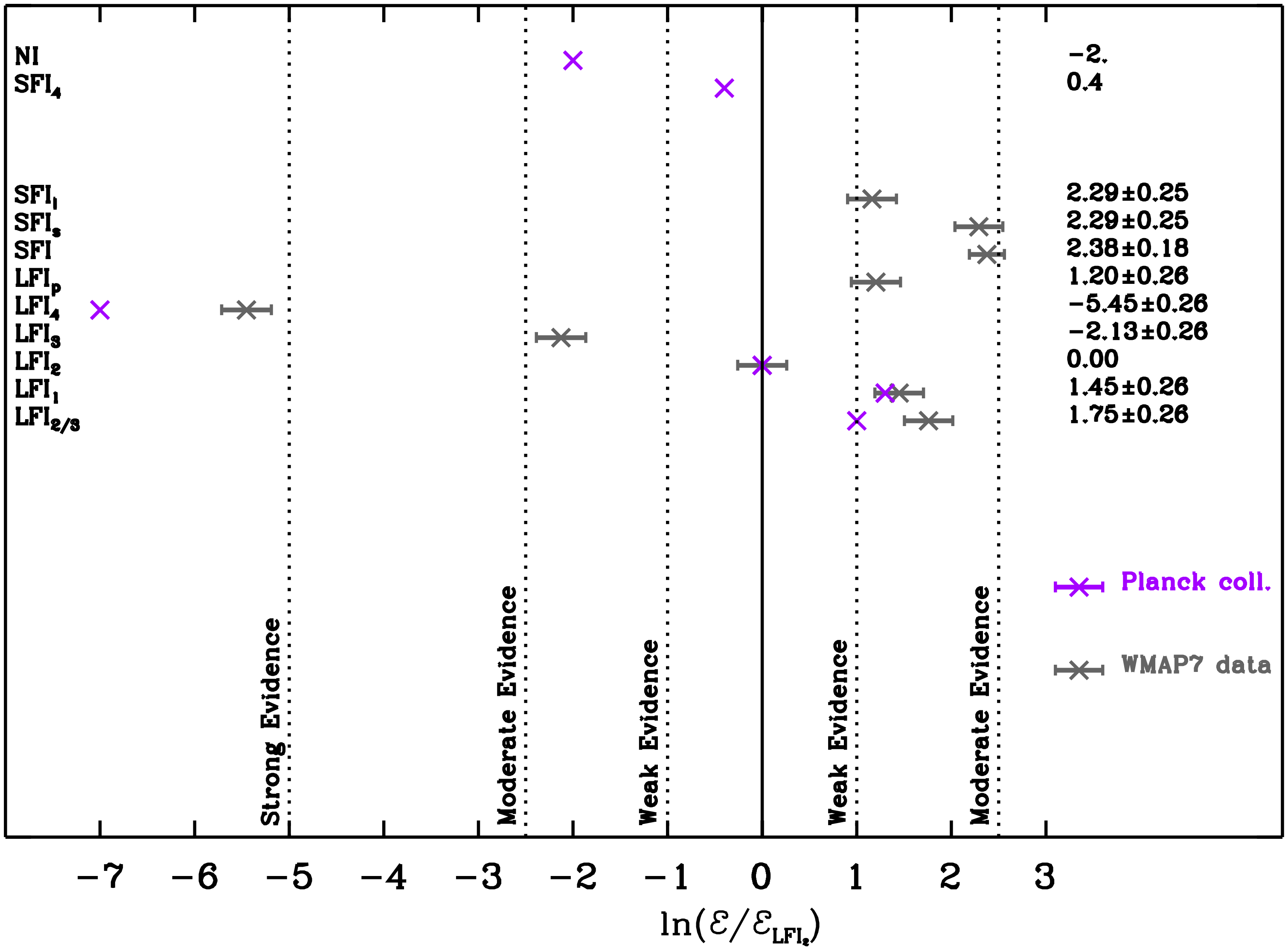
Preliminary Results

BEST MODELS



Preliminary Results

BEST MODELS



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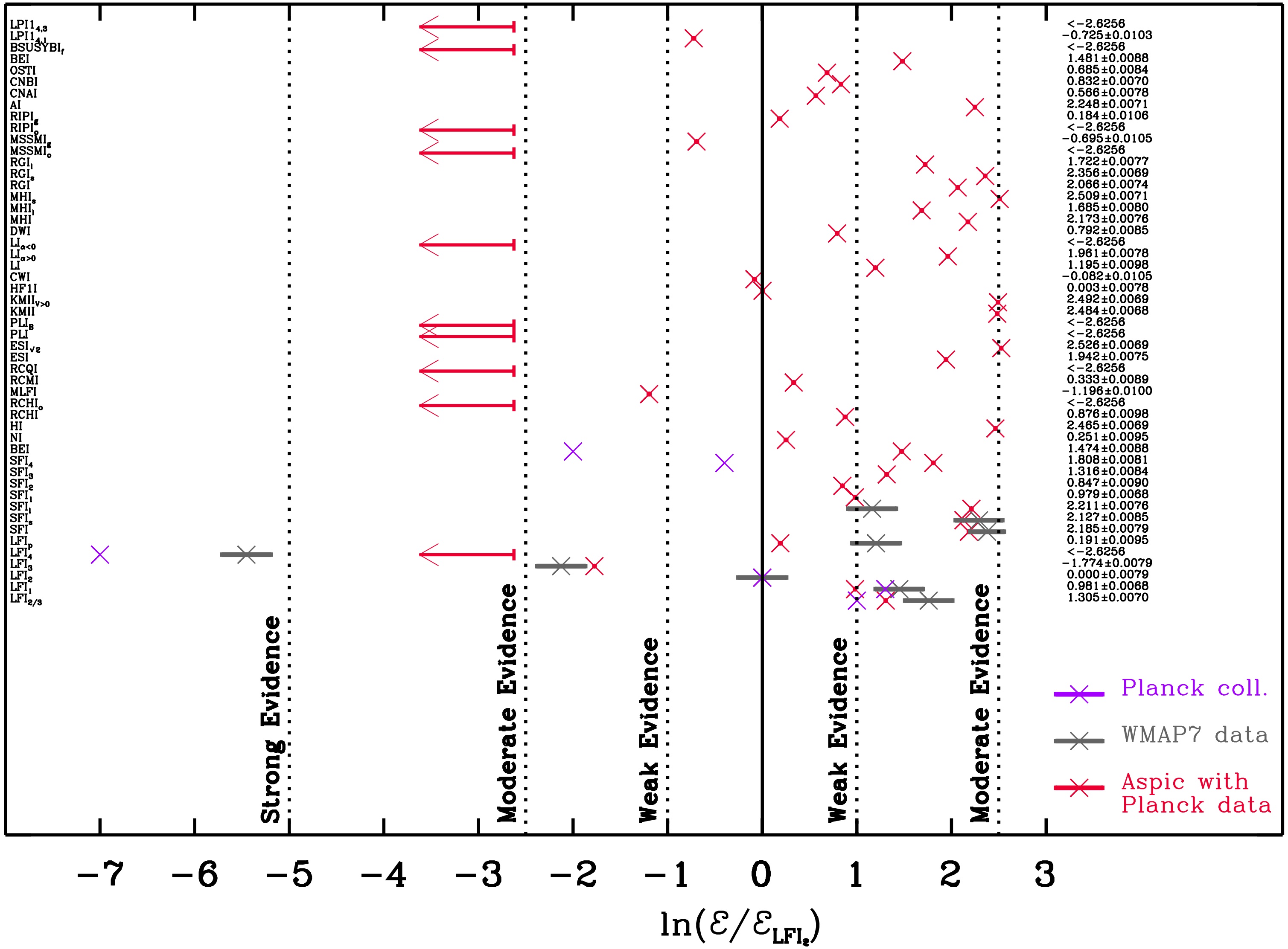


Typological Classification

$$\frac{\mathrm{d}\left(\dot{\phi}^2/2\right)}{\mathrm{d}t} = H\frac{\dot{\phi}^2}{2}\left(\epsilon_2 - 2\epsilon_1\right)$$

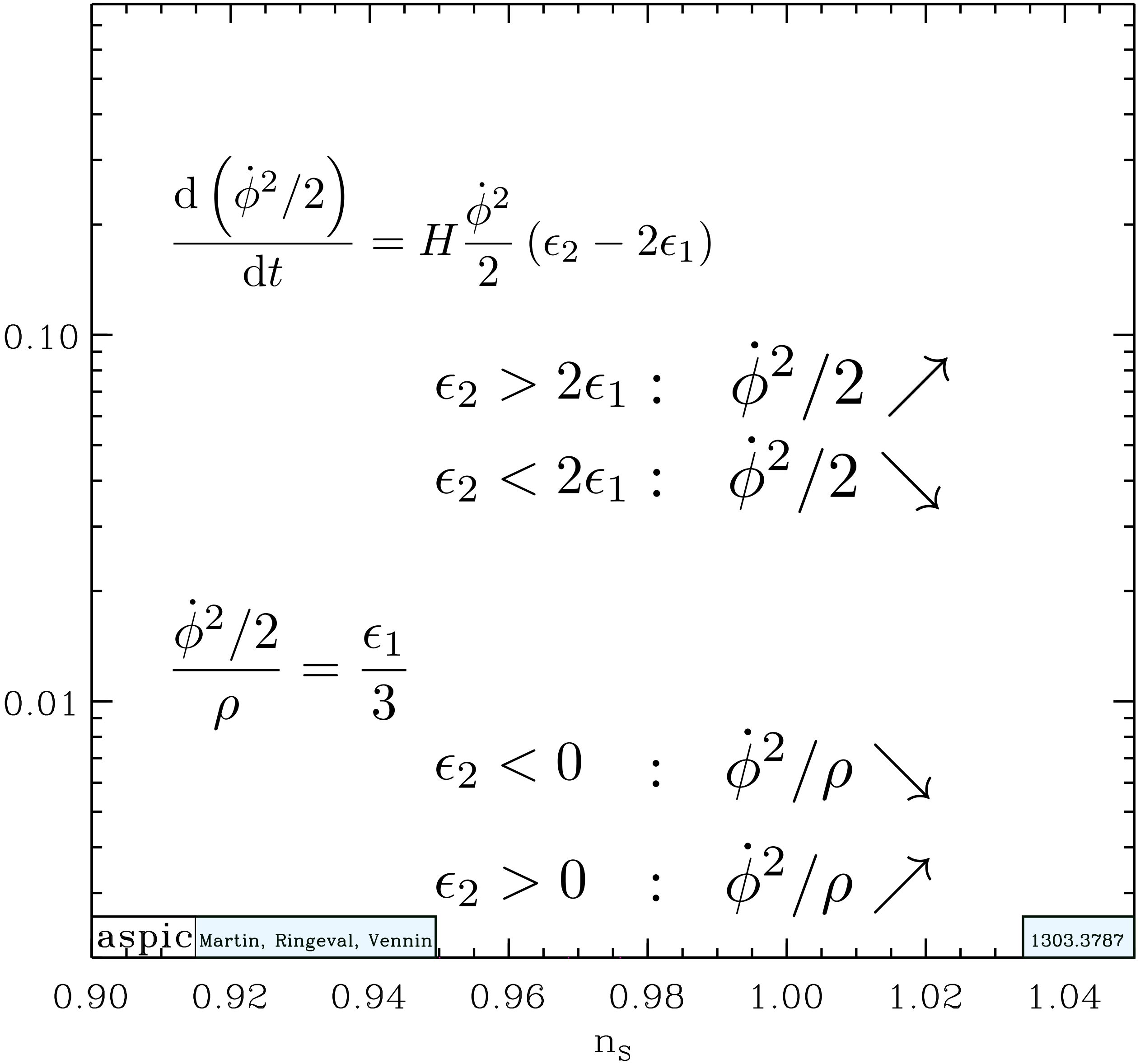
$$\epsilon_2 > 2\epsilon_1 : \quad \dot{\phi}^2/2 \nearrow$$

$$\epsilon_2 < 2\epsilon_1 : \quad \dot{\phi}^2/2 \searrow$$

$$\frac{\dot{\phi}^2/2}{\rho} = \frac{\epsilon_1}{3}$$

$$\epsilon_2 < 0 \quad : \quad \dot{\phi}^2/\rho \searrow$$

$$\epsilon_2 > 0 \quad : \quad \dot{\phi}^2/\rho \nearrow$$



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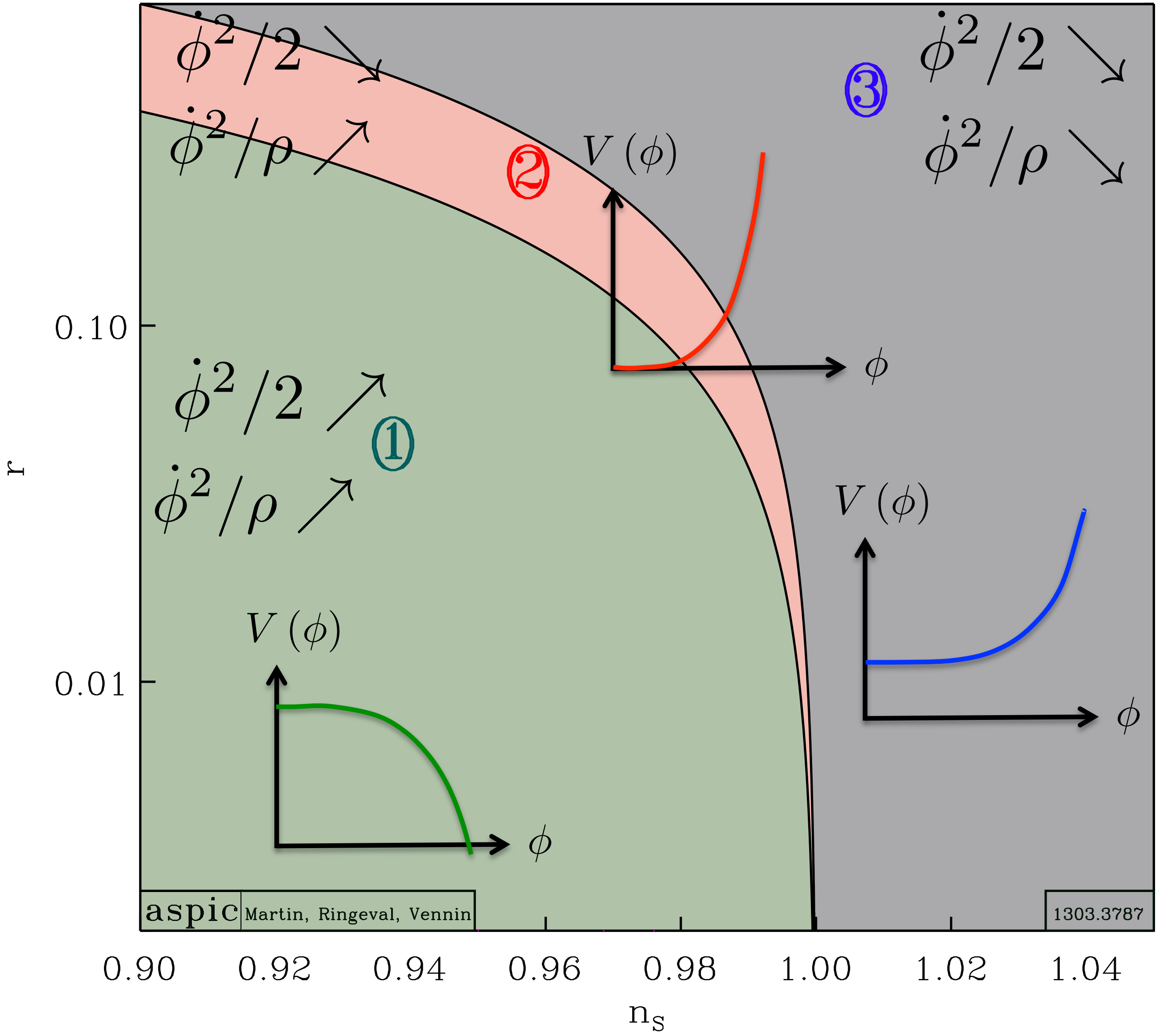

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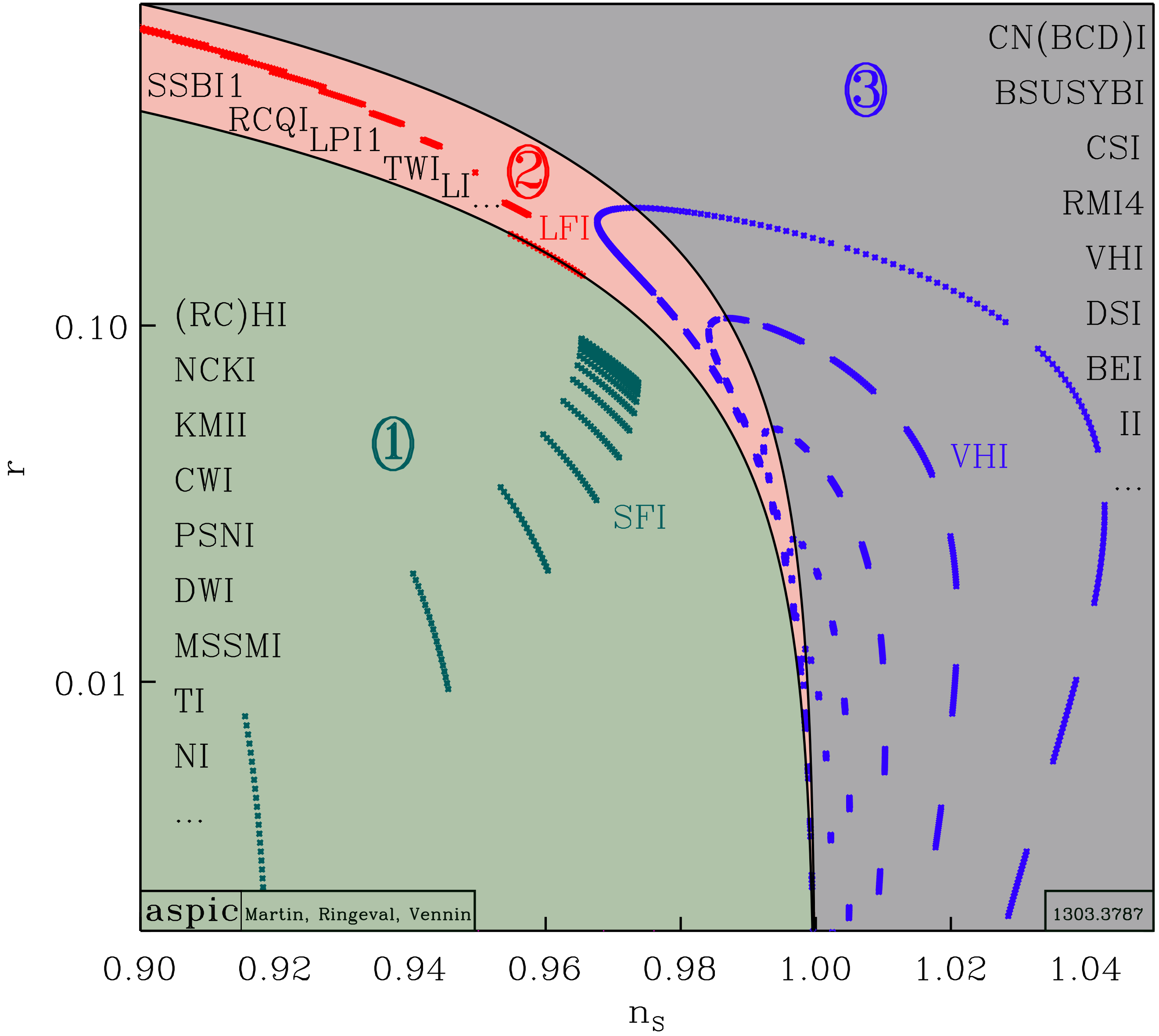

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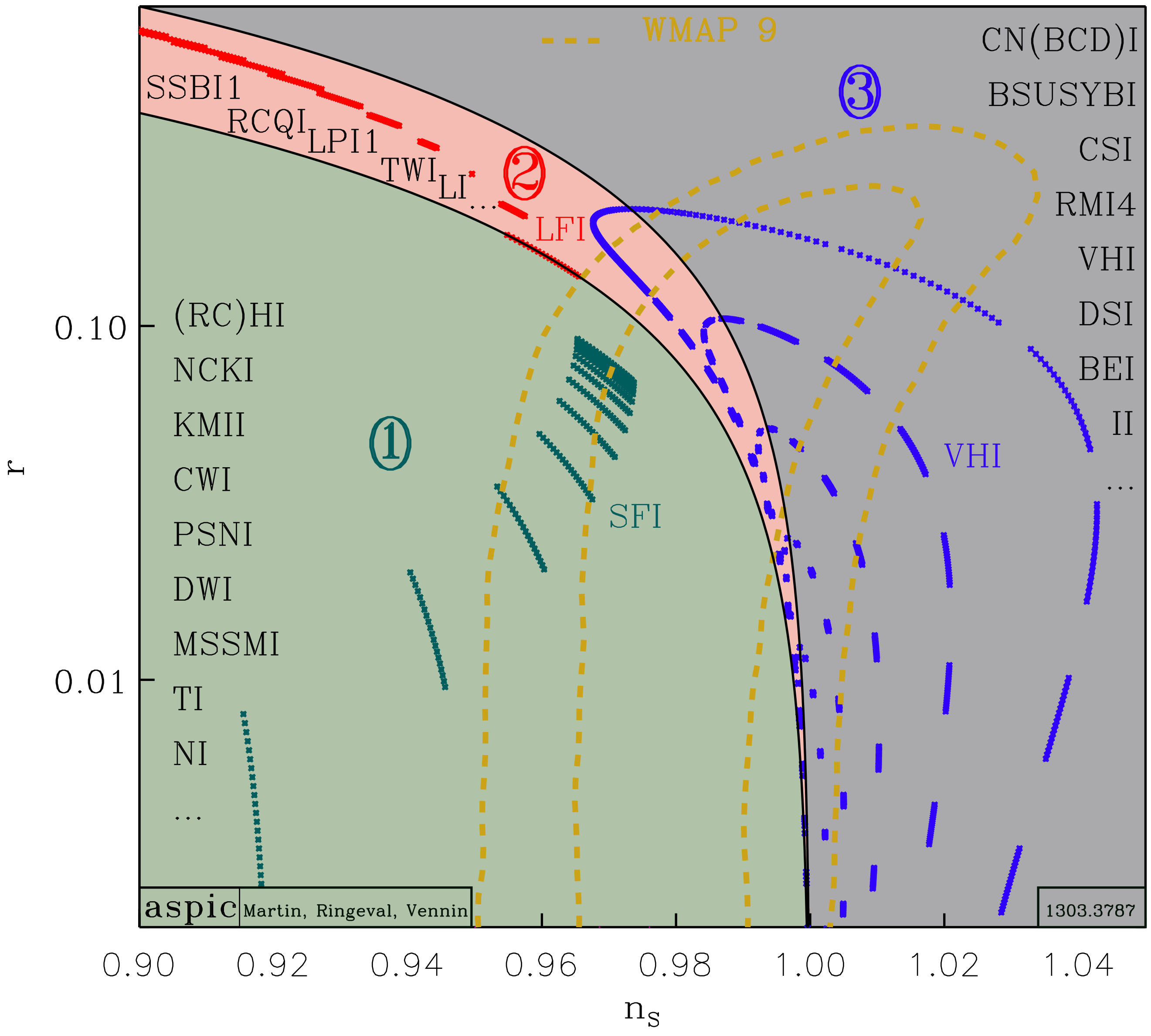

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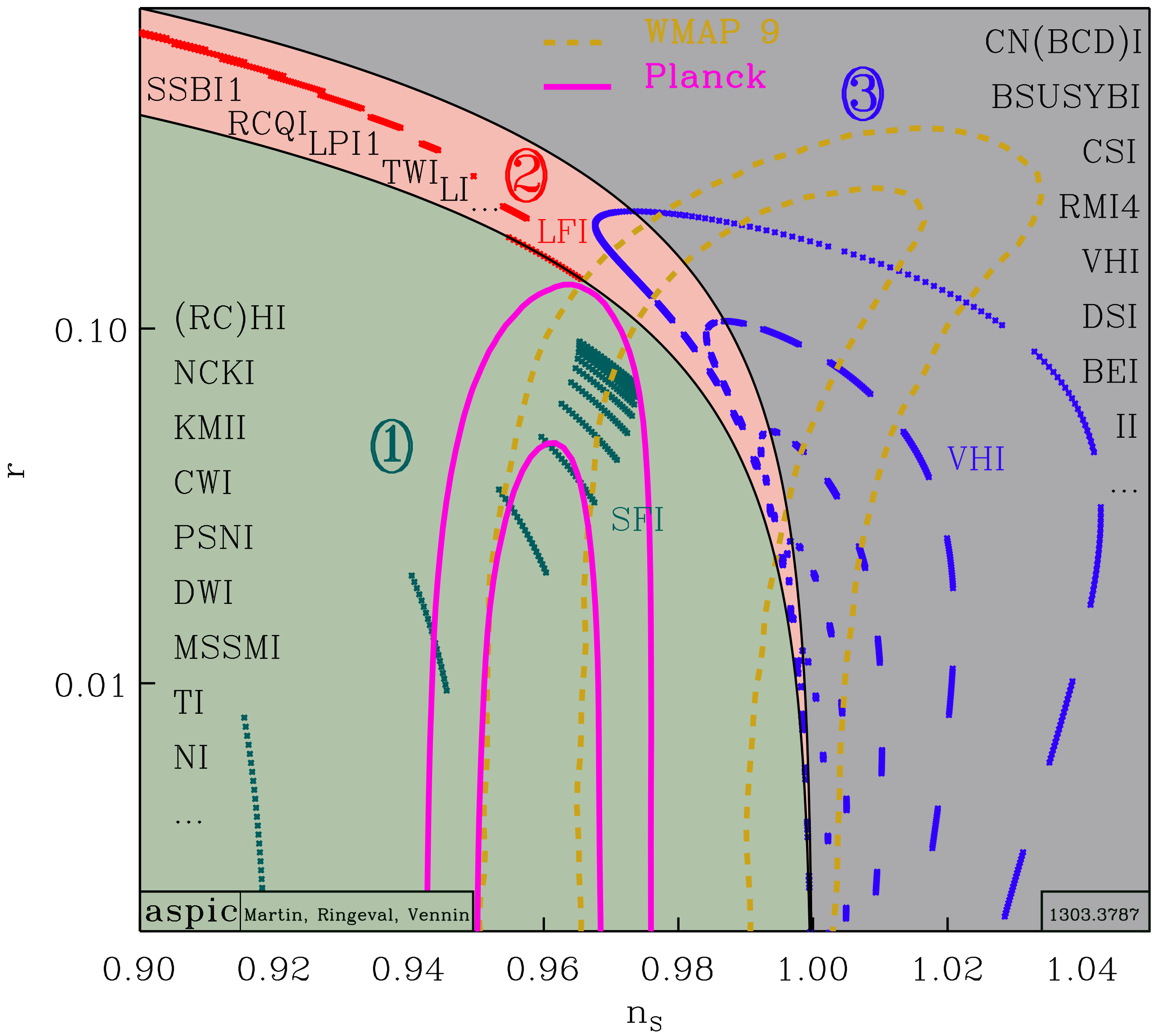

Typological Classification

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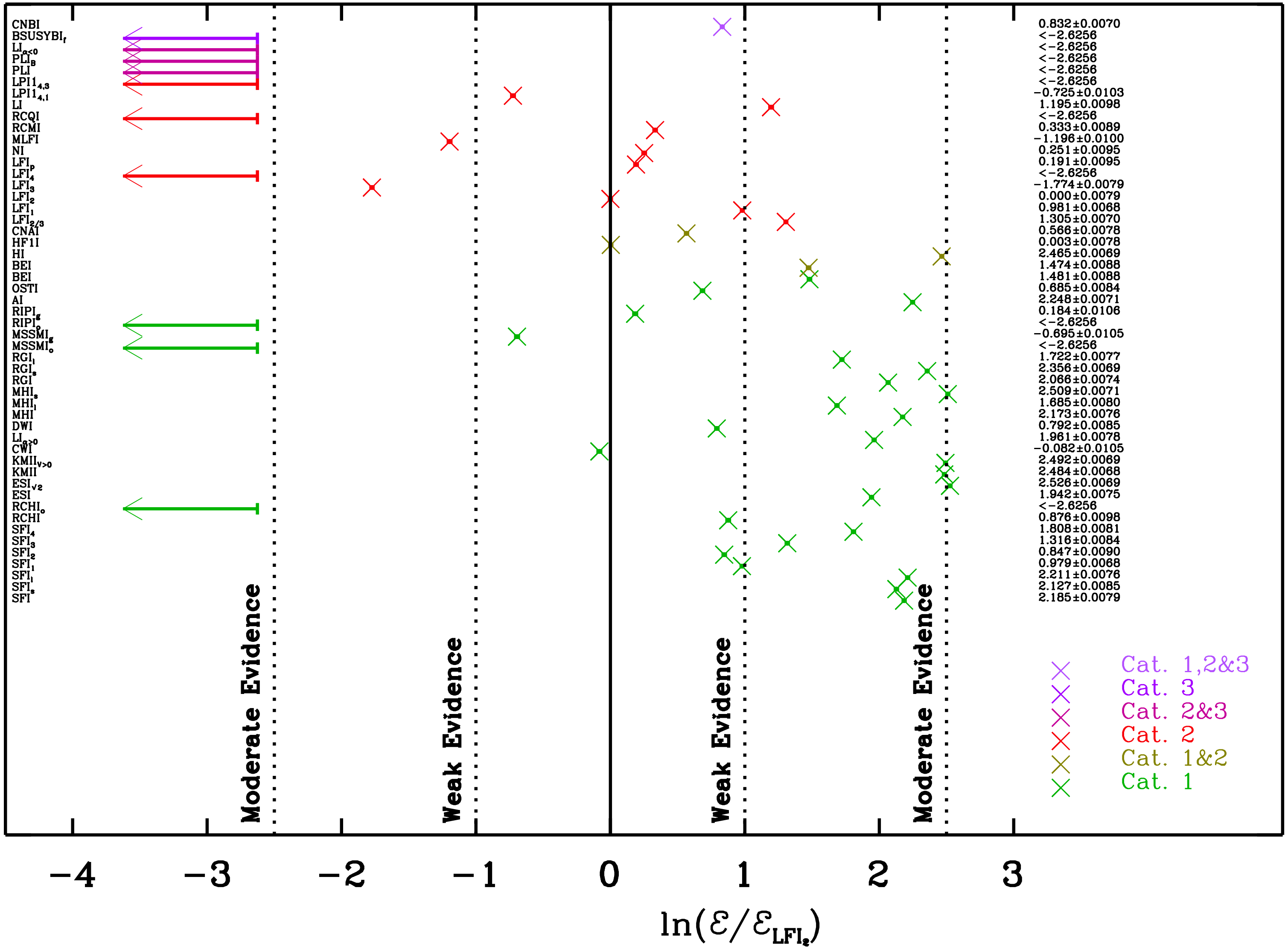


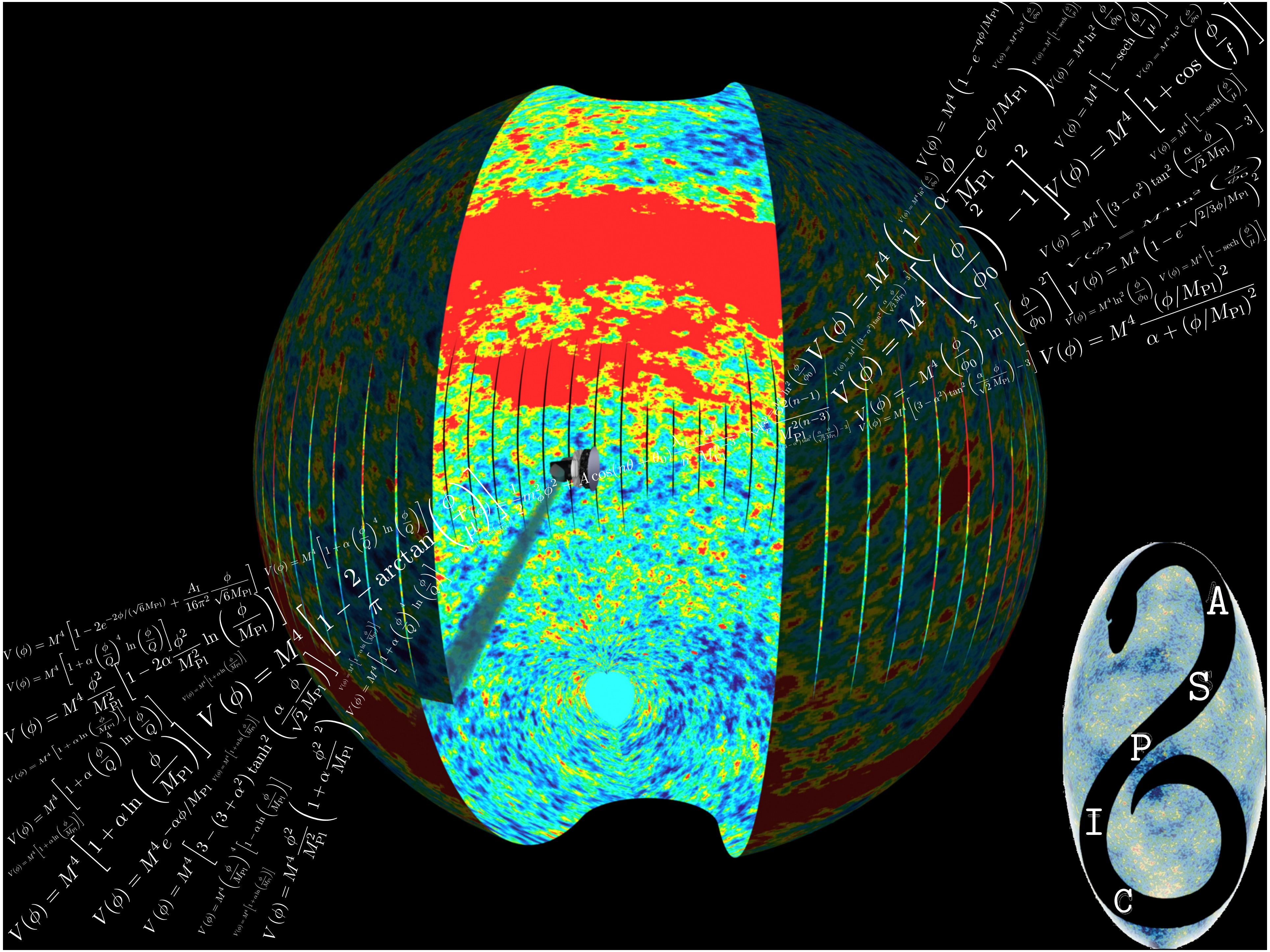
And now what?

What the first results show:

- Some models clearly are « ruled out » by the data
- The best models of inflation lie in the first category

What should be obtained very soon:

- A complete model ranking [in prep]
- A comparison among various categories: **models statistics** [in prep]:
  - phenomenological / theory bases
  - inflationary energy scale
  - higher energy embedding theory: SUGRA/SUSY/STRING/…
  - etc …
- Constraints on the reheating [in prep]
- etc …



$V(\phi) = M^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$
$V(\phi) = M^4 \frac{(\phi/M_{Pl})^2}{\alpha + (\phi/M_{Pl})^2}$
$V(\phi) = M^4 \ln^2\left(\frac{\phi}{\phi_0}\right)$
$V(\phi) = M^4\left(1 - e^{-\sqrt{2/3}\phi/M_{Pl}}\right)^2$
$V(\phi) = M^4\left[1 - \frac{2}{\pi}\arctan\left(\frac{\phi}{\mu}\right)\right]$
$V(\phi) = M^4\left[1 + \alpha\ln\left(\frac{\phi}{M_{Pl}}\right)\right]$
$V(\phi) = M^4 e^{-\alpha\phi/M_{Pl}}$
A
S
P
I
C

Back-up slides

Prior Specification on the parameters

Well specified parameters ➔ Flat Priors

Unknown order of magnitude ➔ Jeffrey Priors

A phenomenological model example: « Intermediate Inflation »

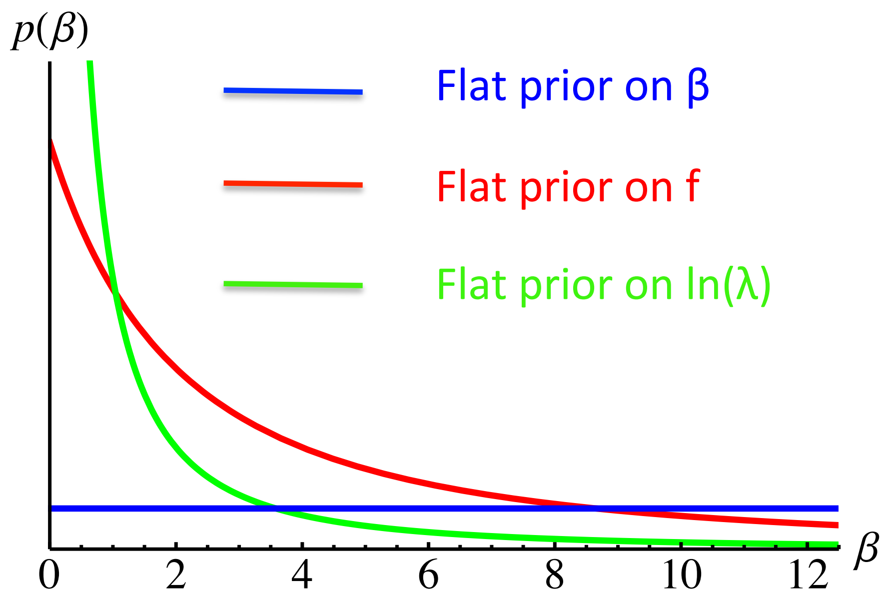
$$\rho + p = \gamma \rho^{\lambda}$$

$$f = 2\frac{1-\lambda}{1-2\lambda}$$

$$a(t) \propto \exp\left(A\, t^{f}\right)$$

$$\beta = 4\left(\frac{1}{f} - 1\right)$$

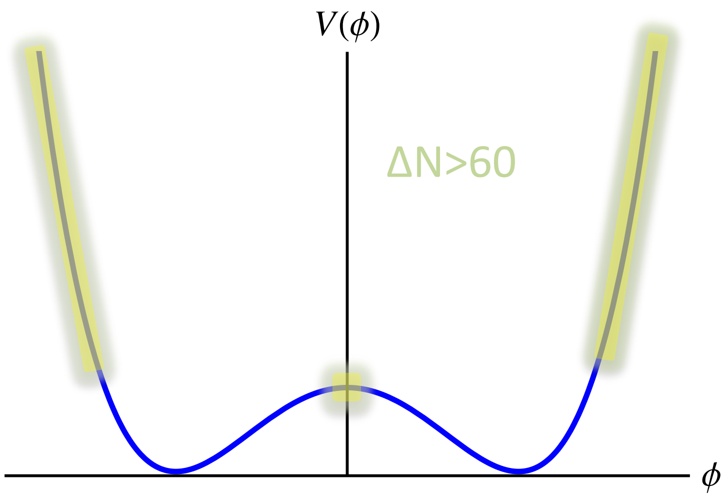
$$V(\phi) = M^4 \left[\left(\frac{\phi}{M_{\rm Pl}}\right)^{-\beta} - \frac{\beta^2}{6}\left(\frac{\phi}{M_{\rm Pl}}\right)^{-\beta-2}\right]$$



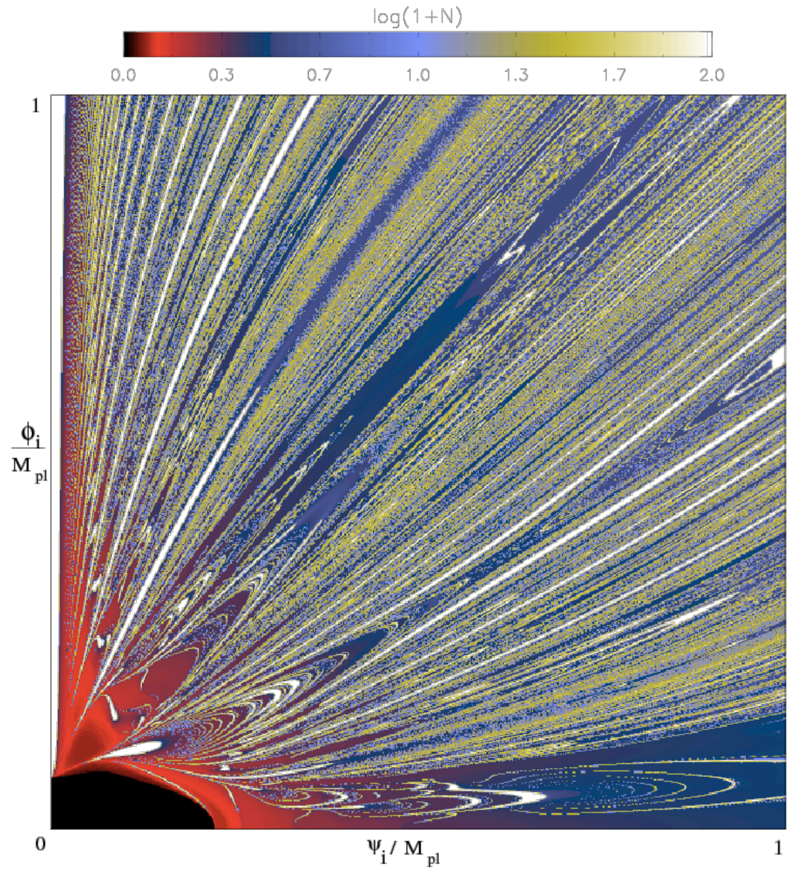
Prior Specification
on the models

Theoretical Grounds?

Initial Conditions Problem?



[Ijjas, Steinhardt & Loeb]



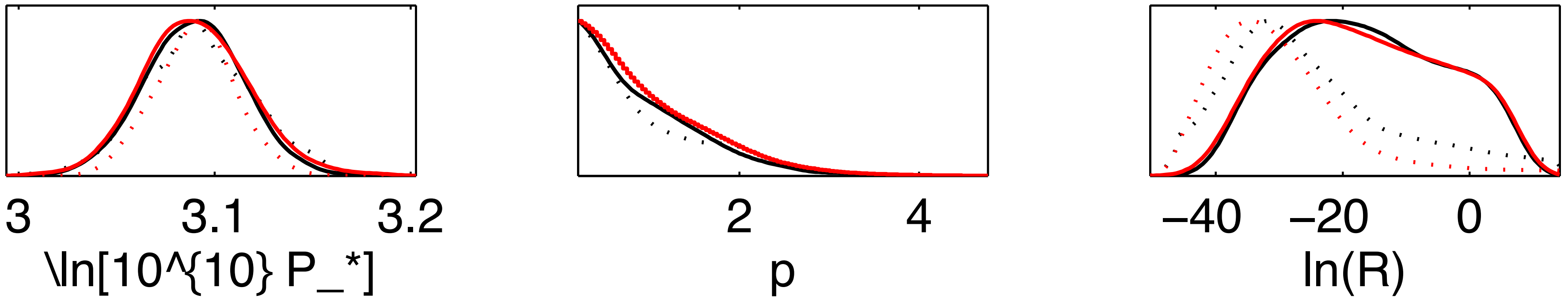
[Clesse, Ringeval & Rocher]

Homogenization / Isotropization?

Matzner, Piran, etc...

Comparison with exact calculation

$$V\left(\phi\right) = M^4 \left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^p$$

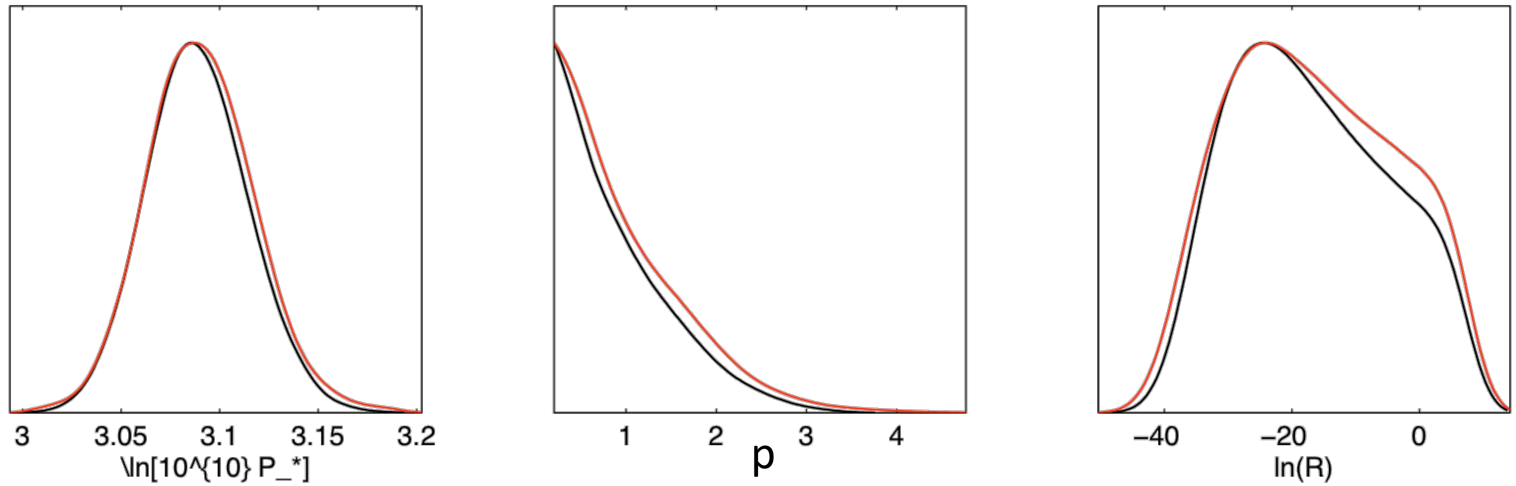


Exact calculation (integrating the full equations of motion for the background and for the perturbations)

ASPIC results

Comparison with exact calculation

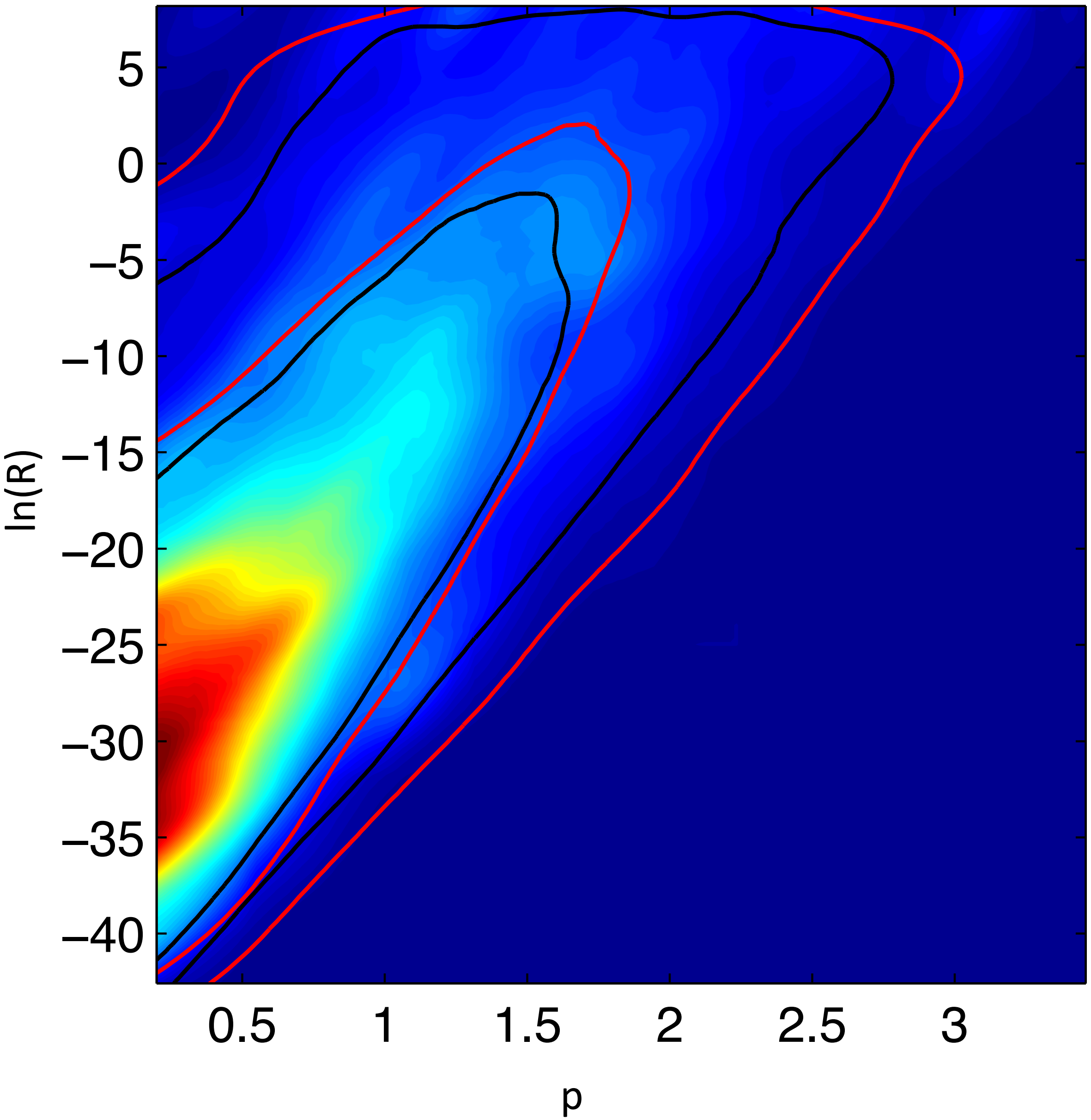
$$V\left(\phi\right)=M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p$$



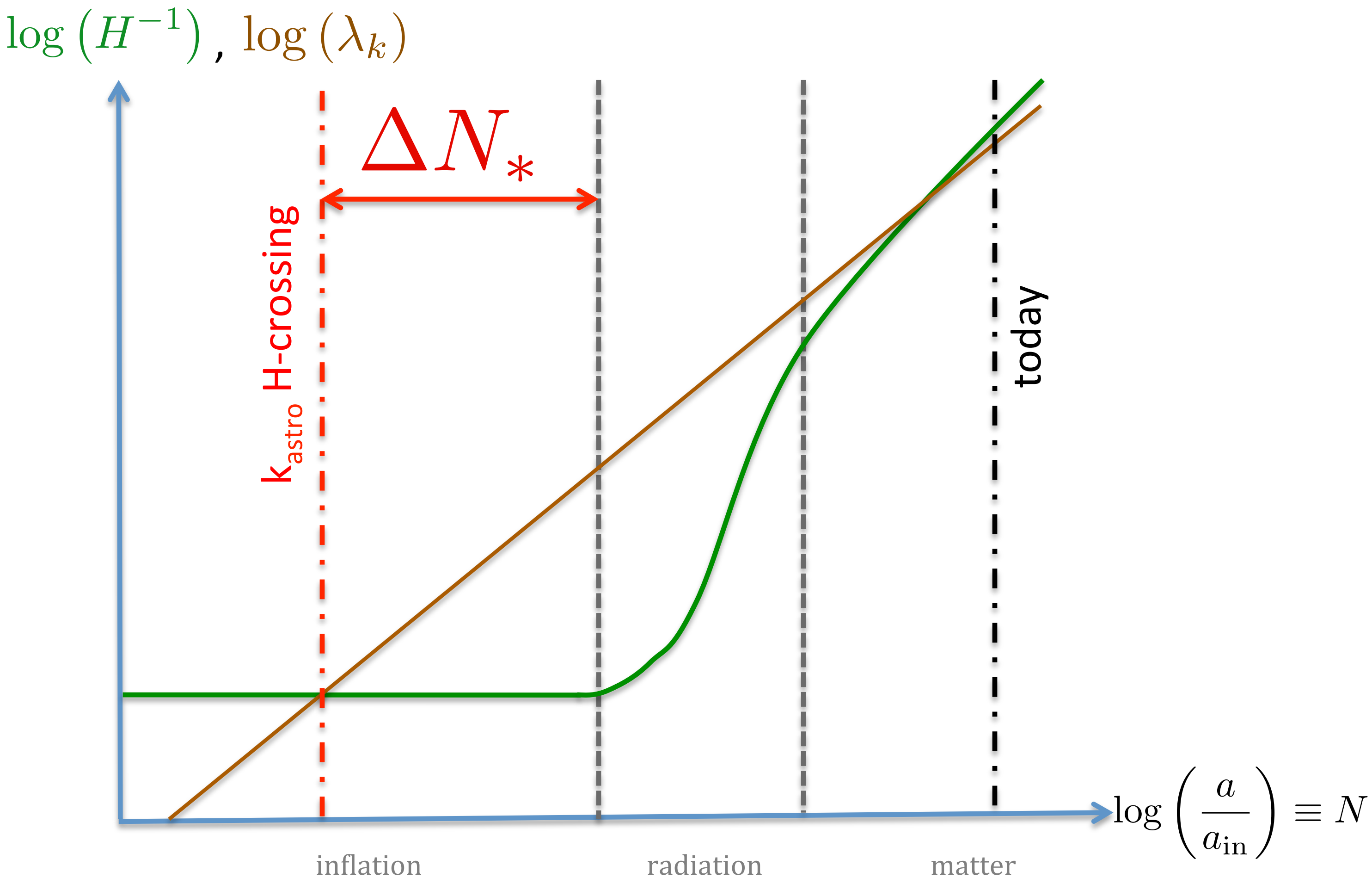
Exact calculation (integrating the full equations of motion for the background and for the perturbations)

ASPIC results

Comparison with exact calculation

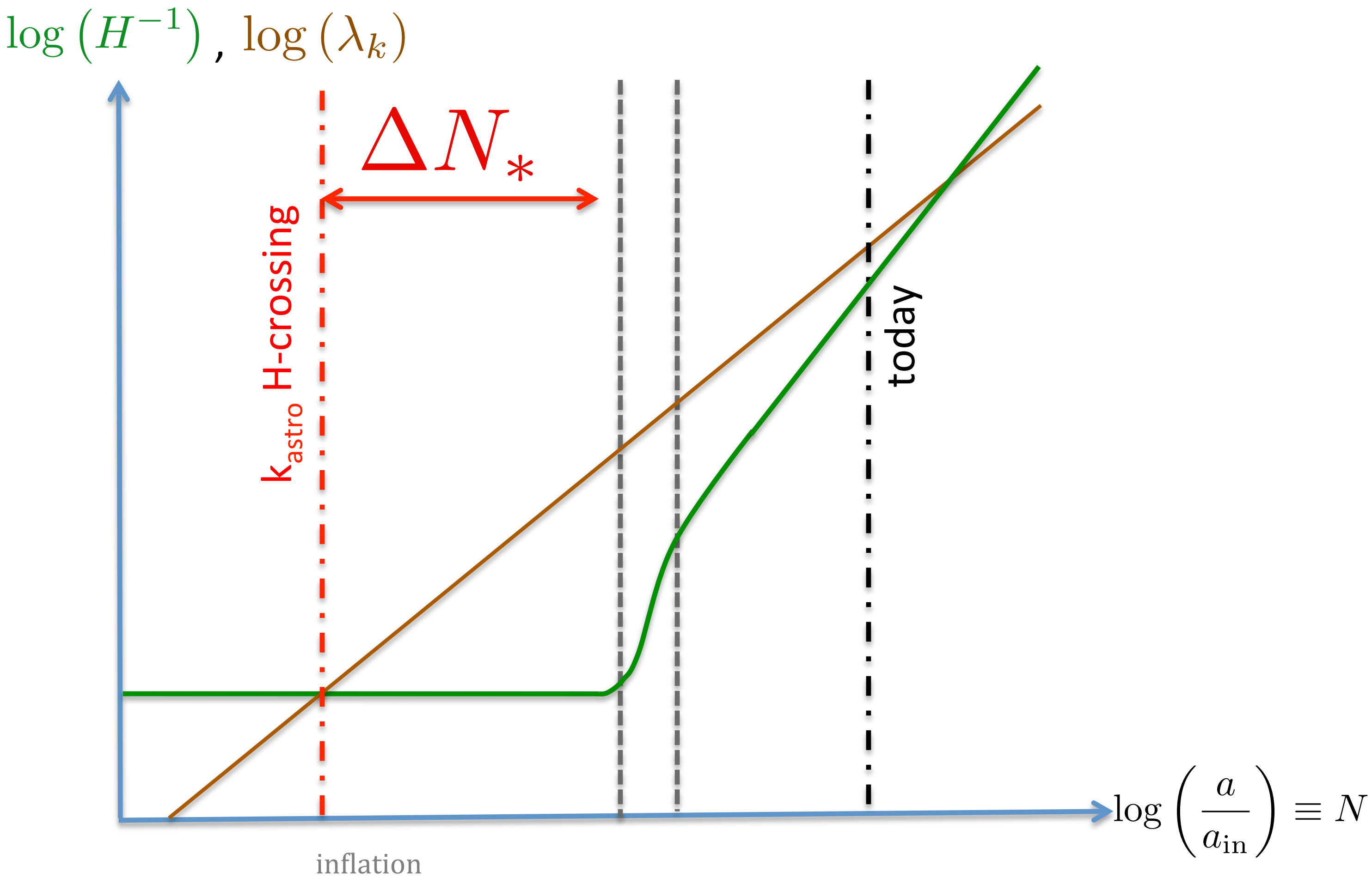


ΔN_* fixed by the thermal subsequent history



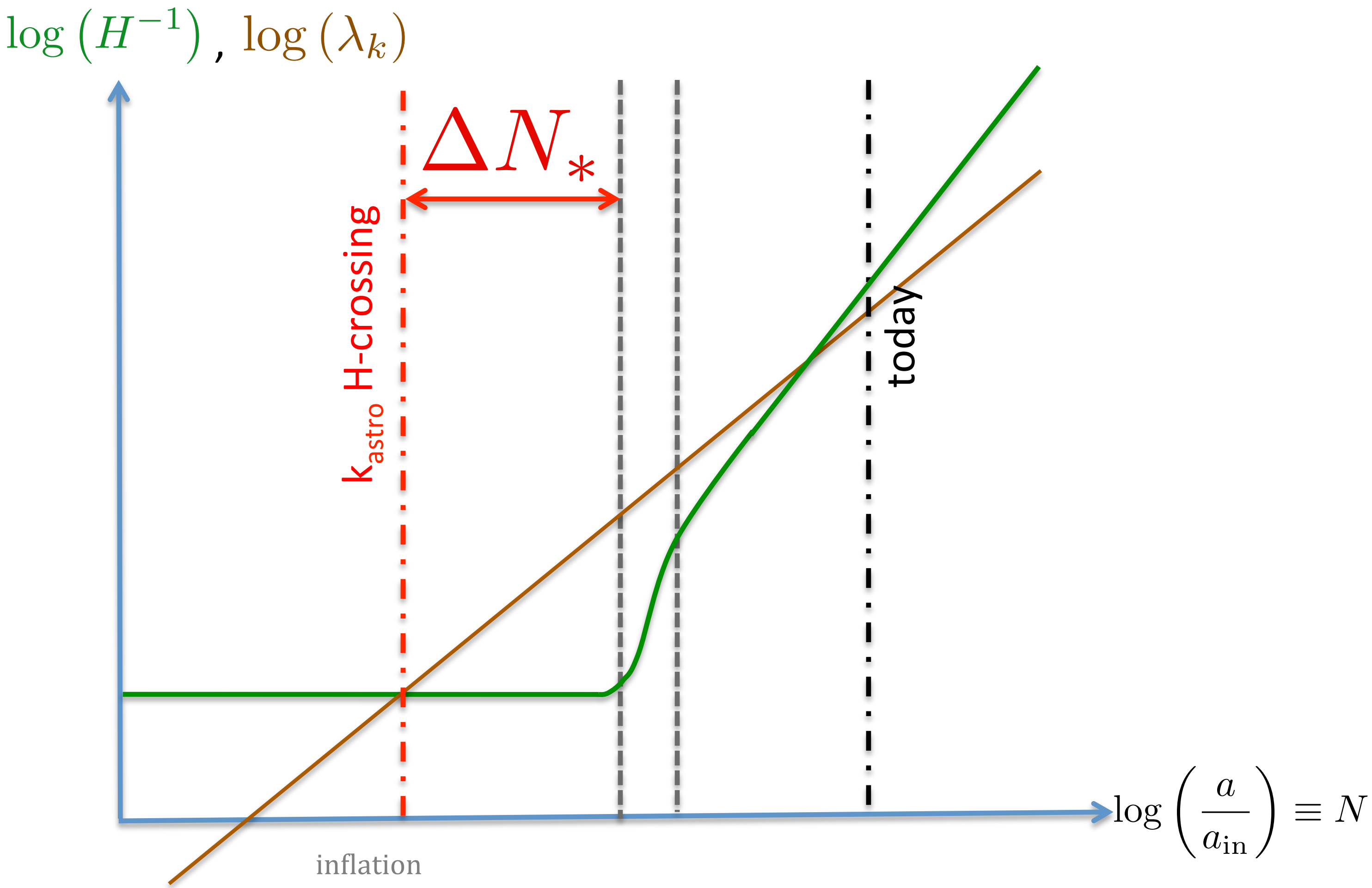
ΔN* fixed by the thermal subsequent history

changing radiation era lasting …



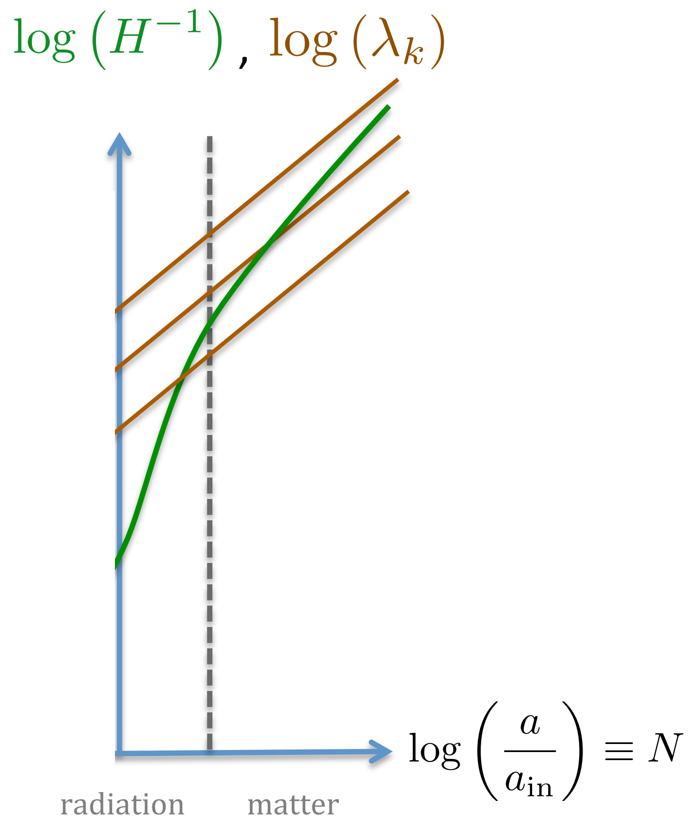
ΔN_* fixed by the thermal subsequent history

changing radiation era lasting …



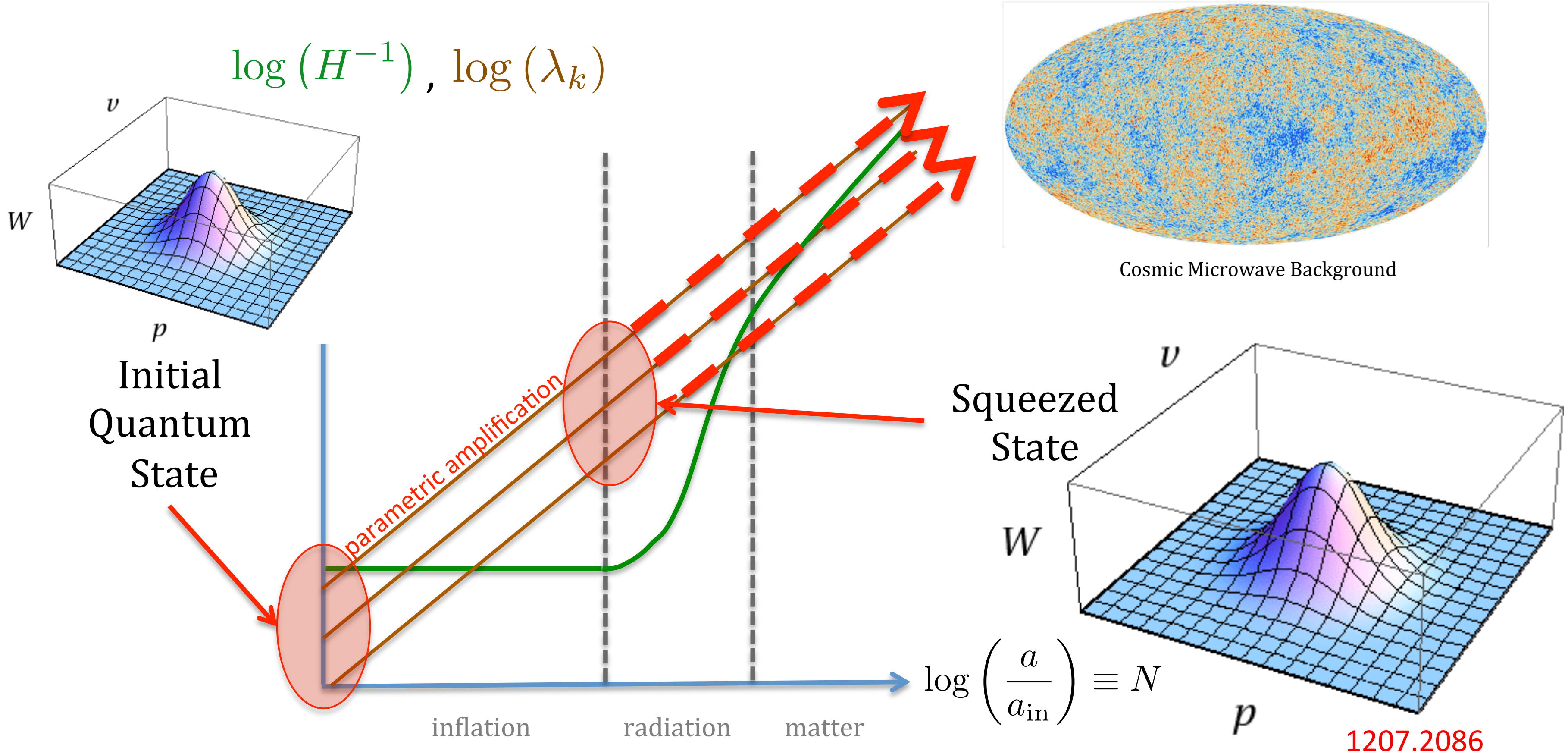
Cosmological Inflation

Solves the Horizon Problem



Cosmological Inflation

Quantized fluctuations evolved over an expanding background



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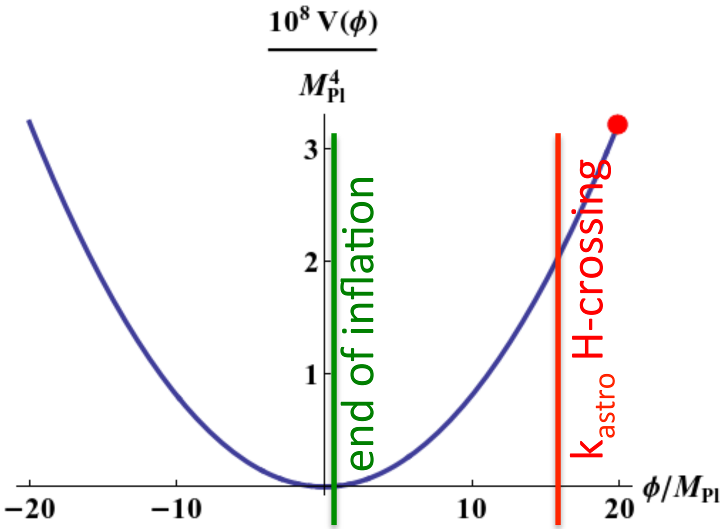
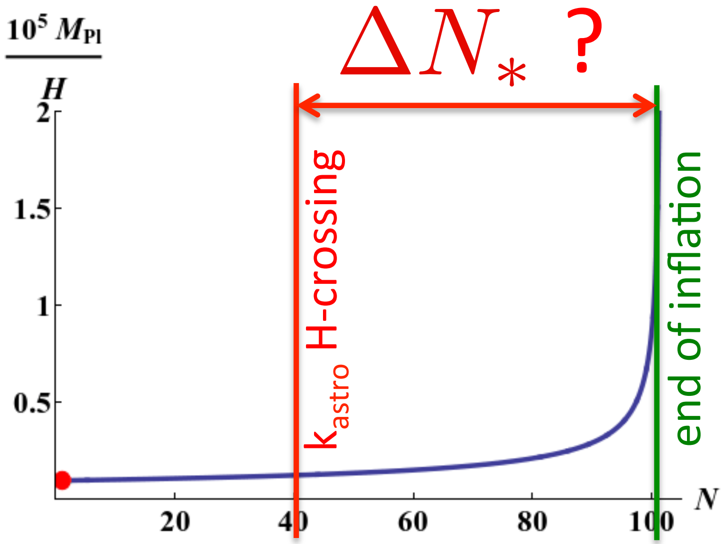
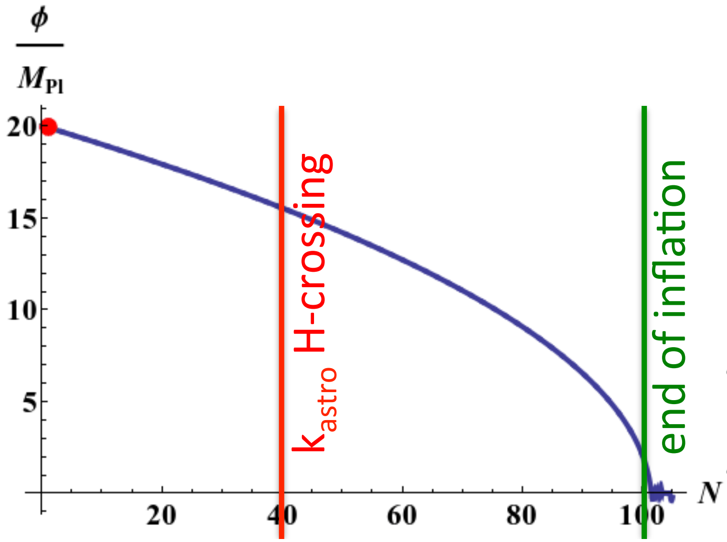
J.Martin, VV & P.Peter

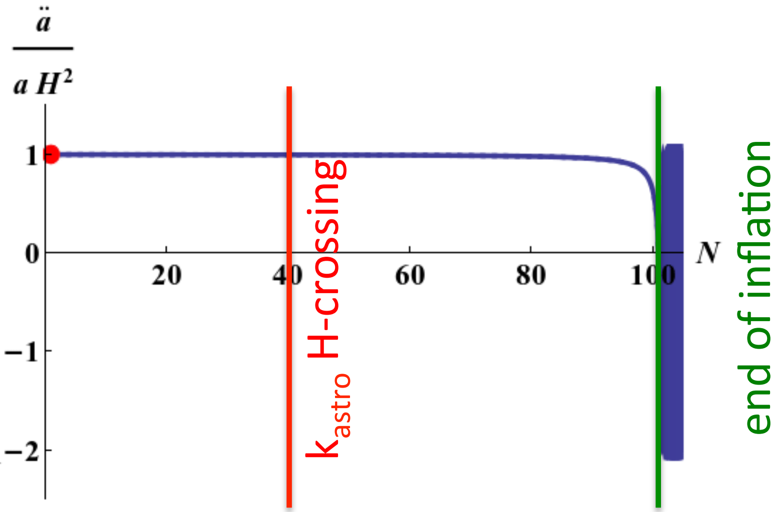
Inflationary Dynamics



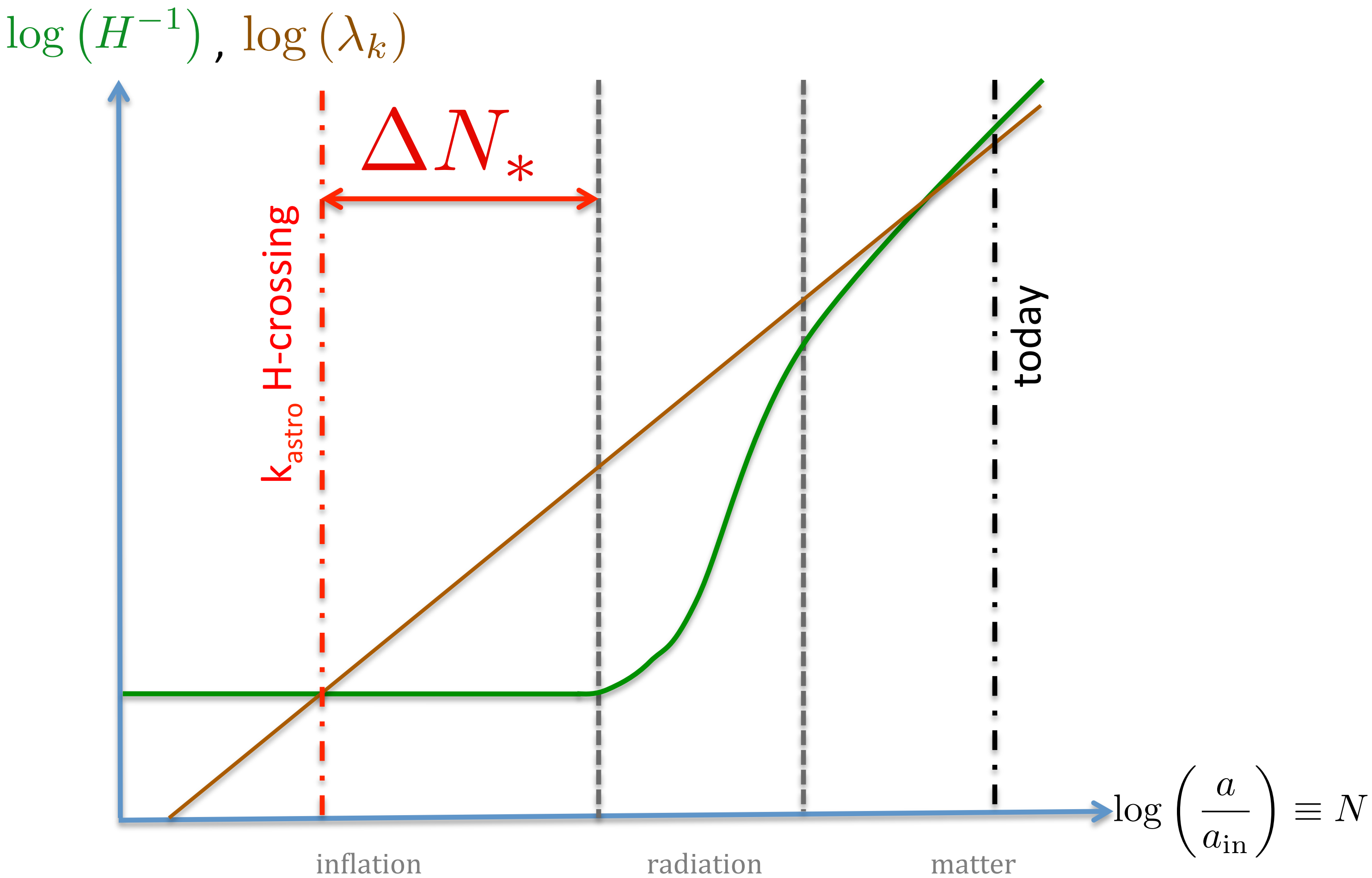


An example: « large field inflation » $V(\phi) = \frac{m^2}{2}\phi^2$



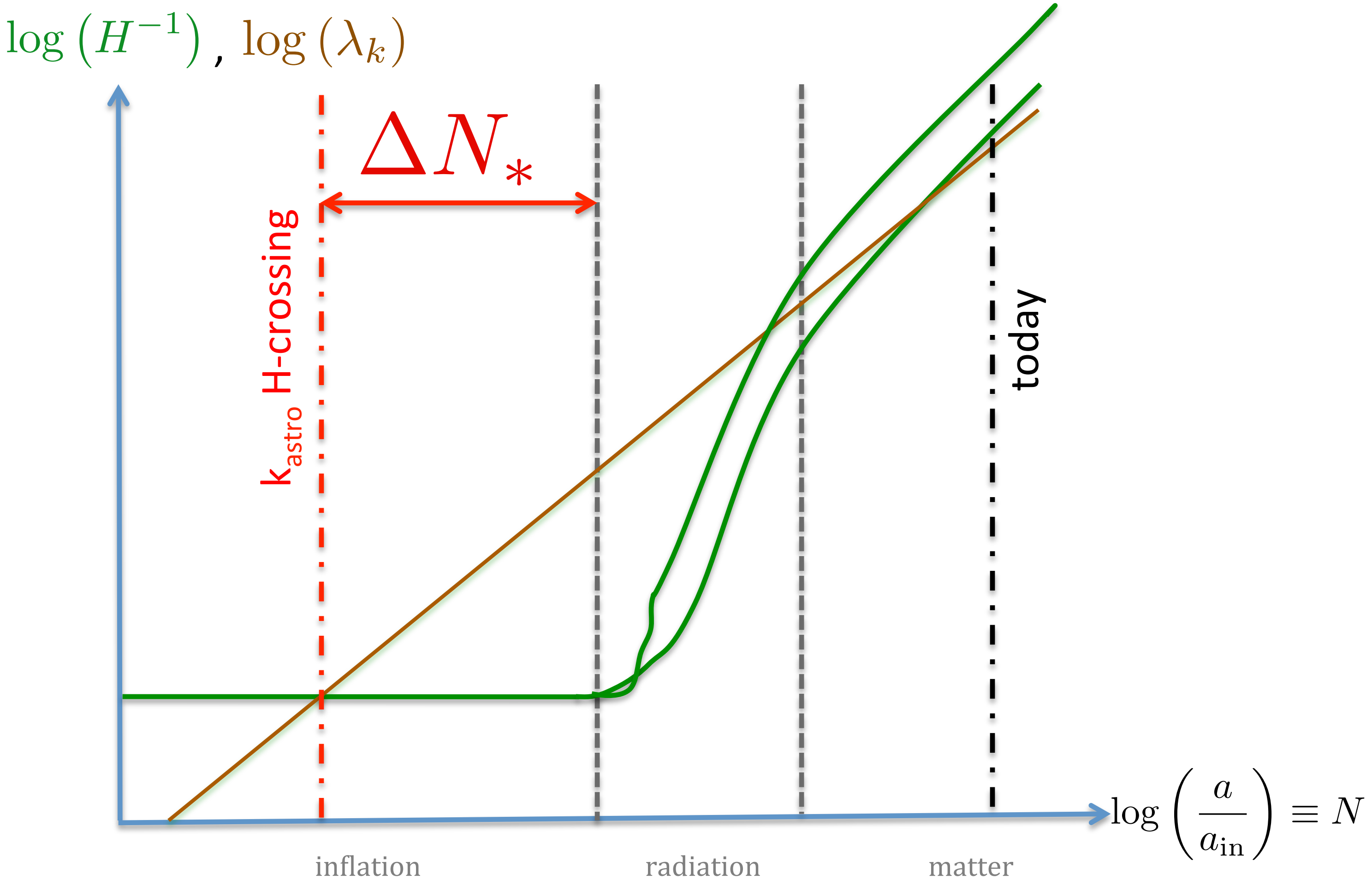


ΔN_* fixed by the thermal subsequent history



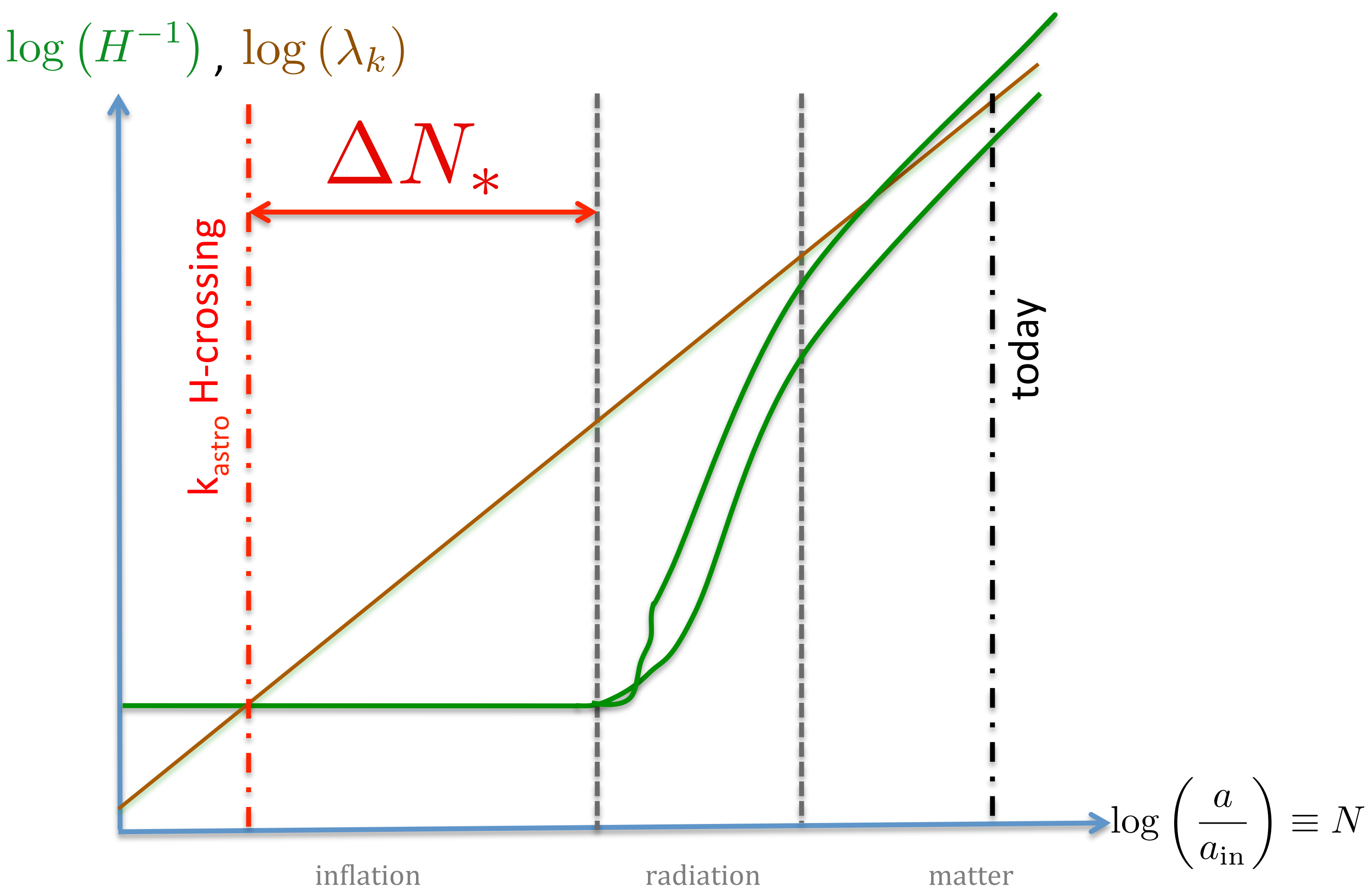
ΔN_* fixed by the thermal subsequent history

changing reheating history…



ΔN_* fixed by the thermal subsequent history

changing reheating history...



ΔN_* fixed by the thermal subsequent history

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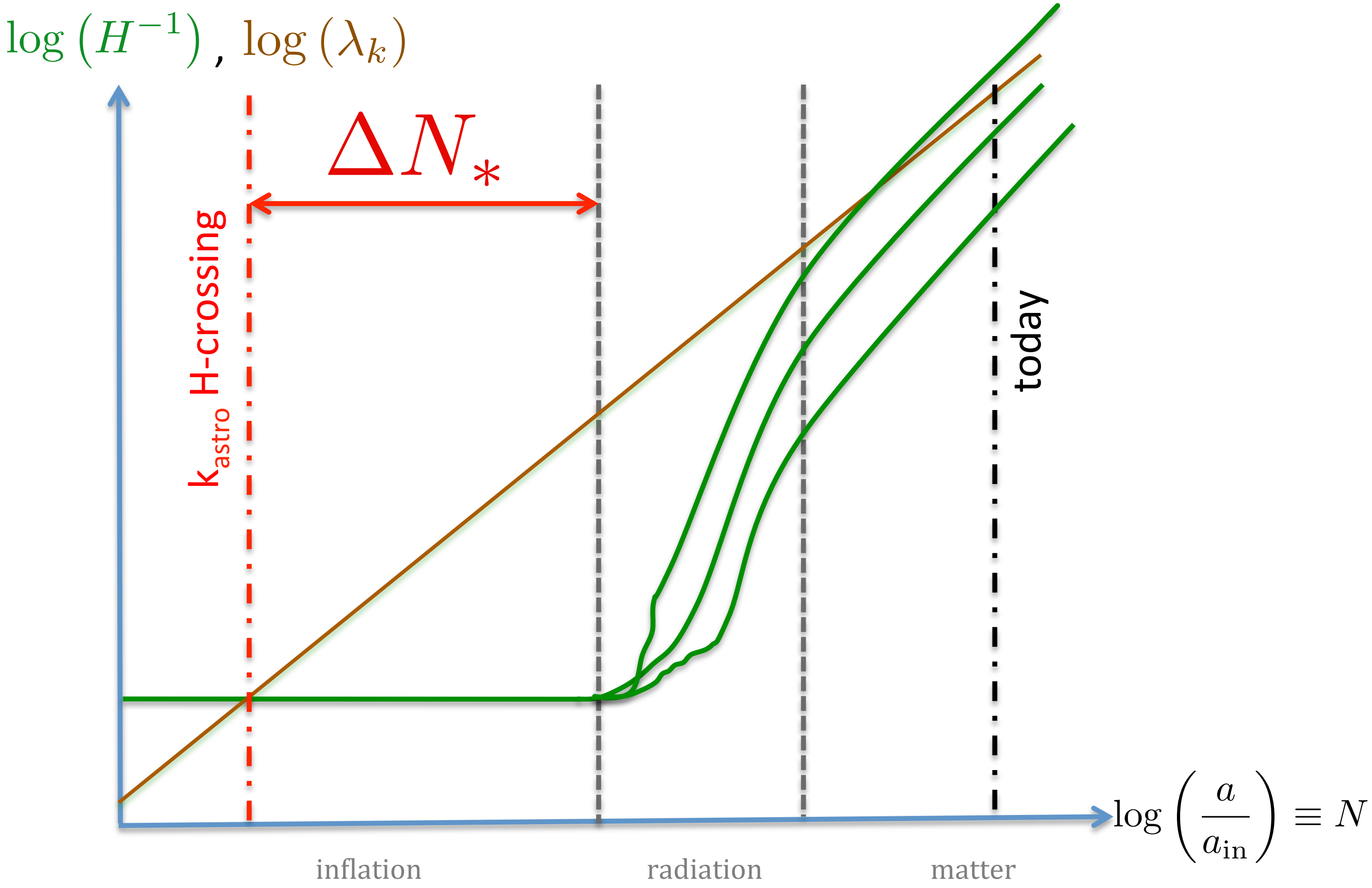
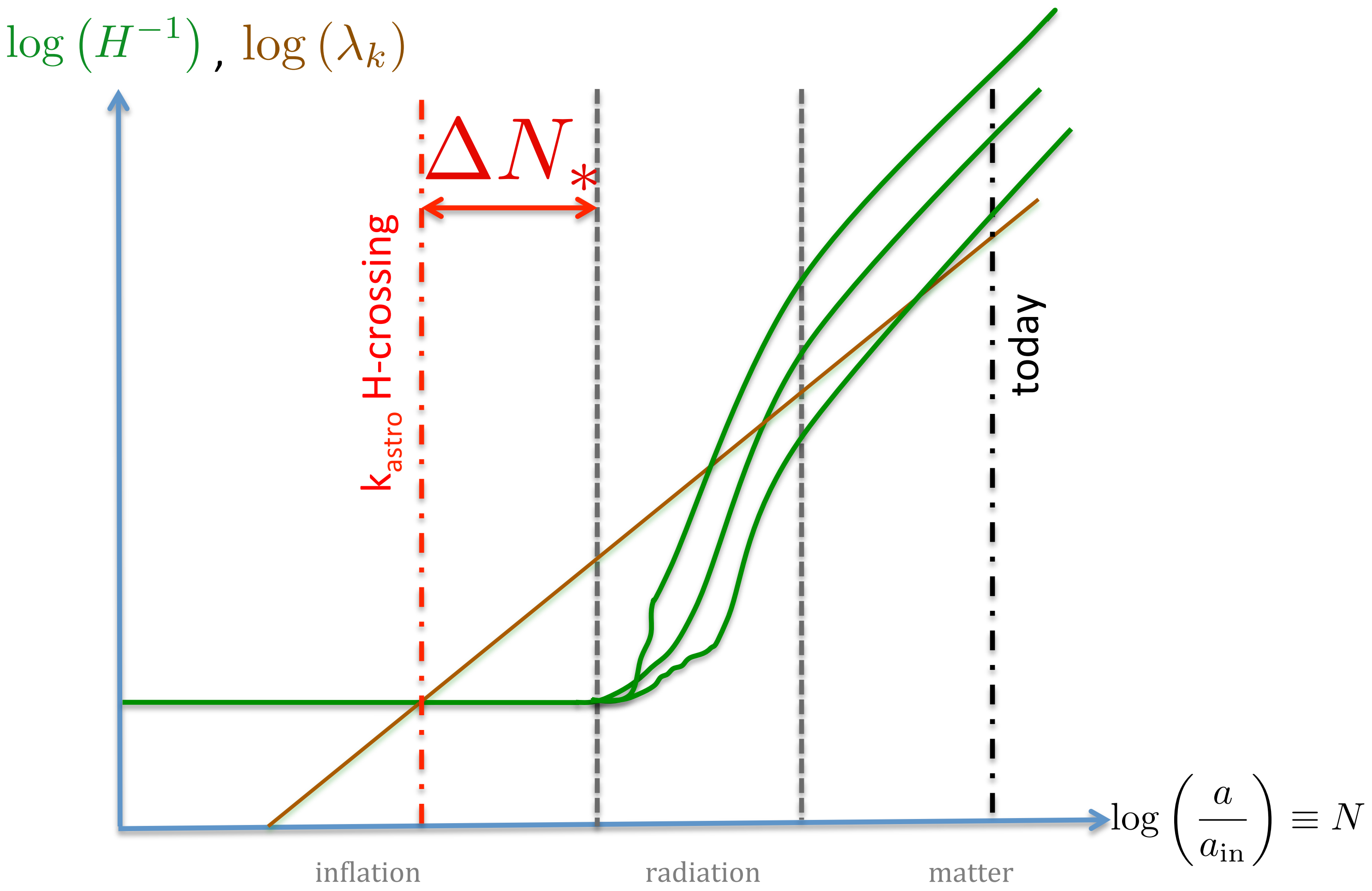


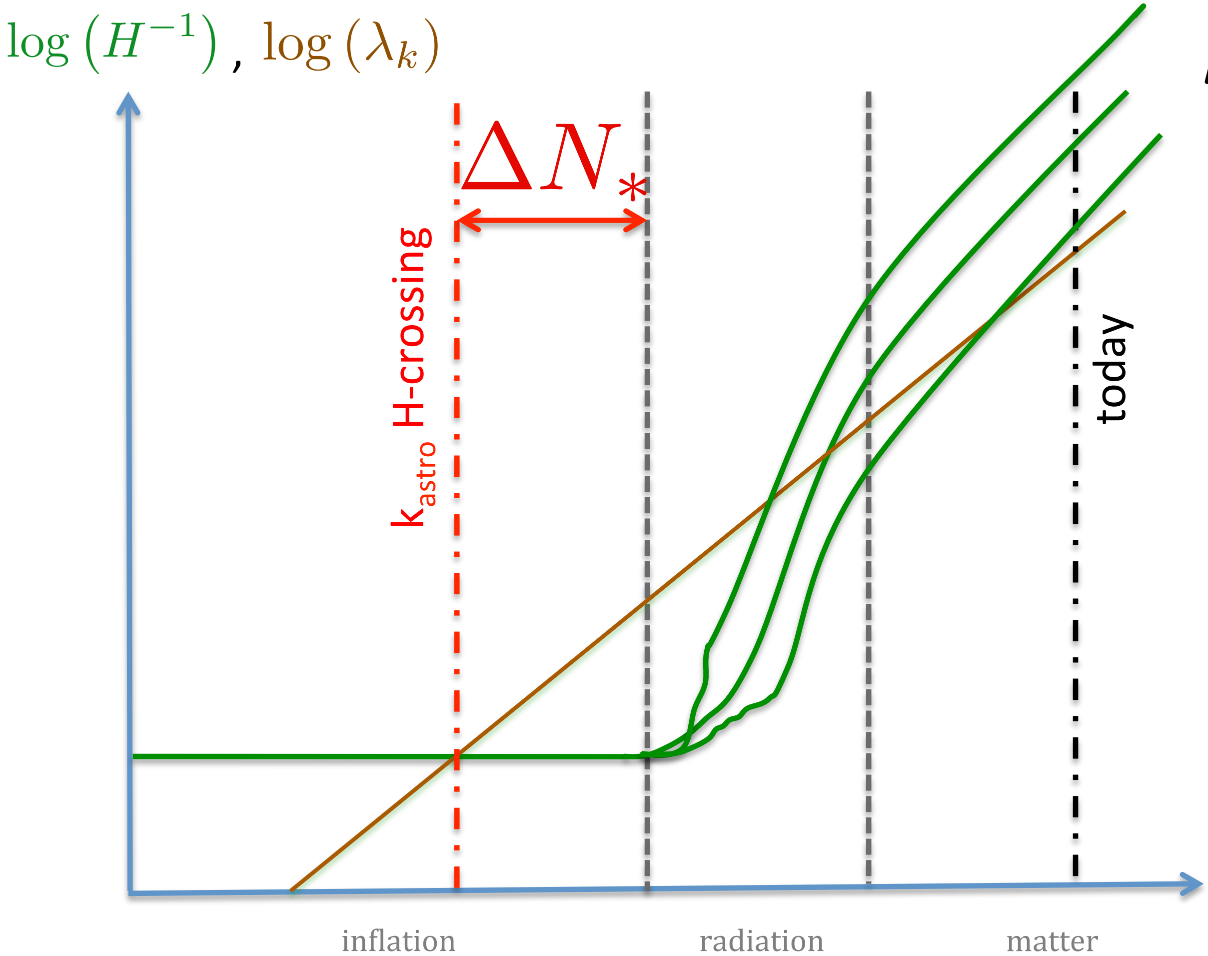
ΔN_* fixed by the thermal subsequent history

changing reheating history…

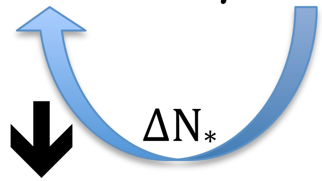


ΔN_* fixed by the thermal subsequent history

$$\Delta N_* \left(\bar{\rho}_{\rm reh}, \bar{w}_{\rm reh}\right)$$

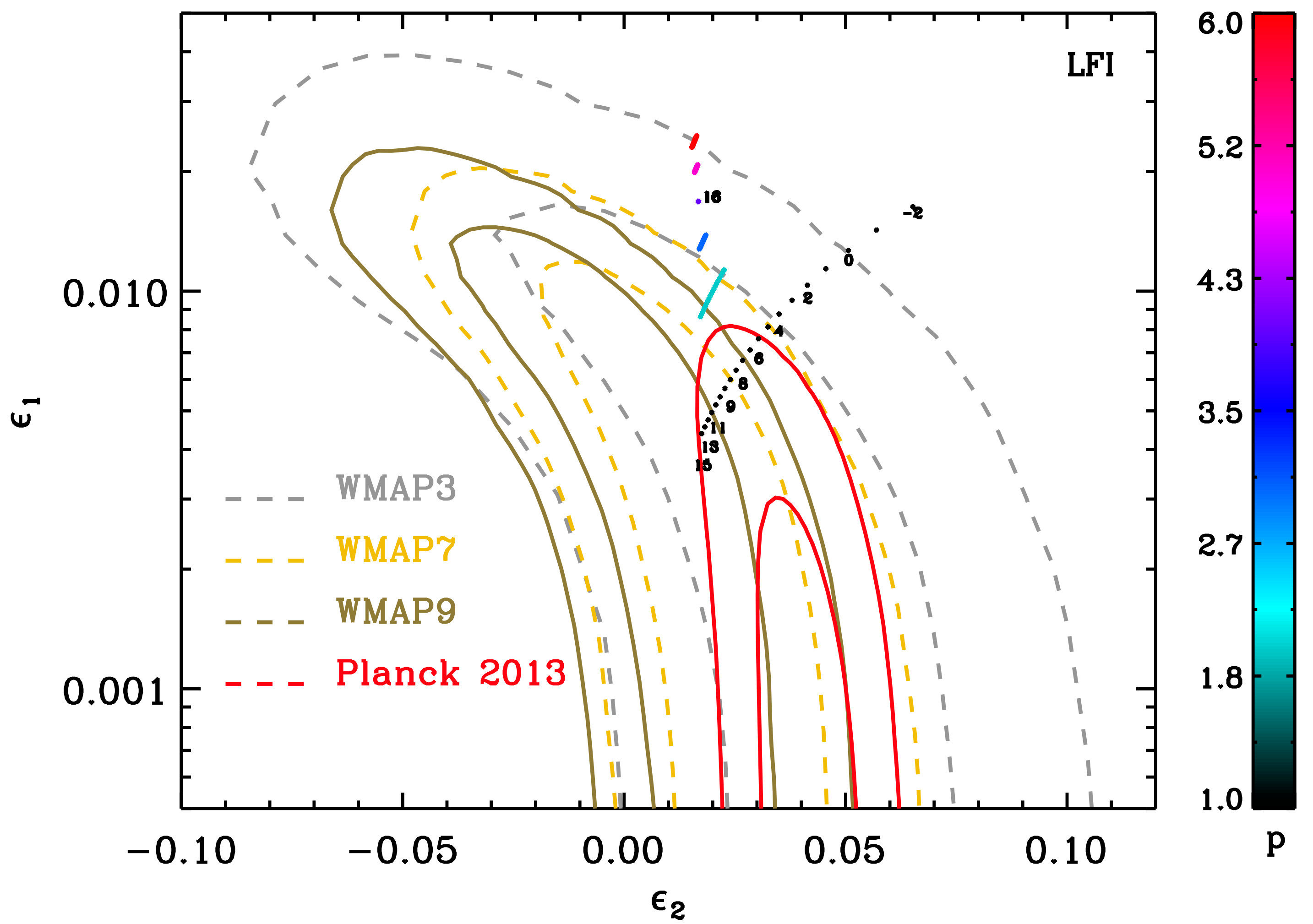


$$\rho_{\rm nuc} < \bar{\rho}_{\rm reh} < \rho_{\rm end}$$



restrains ΔN_* in some range

ΔN$_*$ fixed by the thermal subsequent history



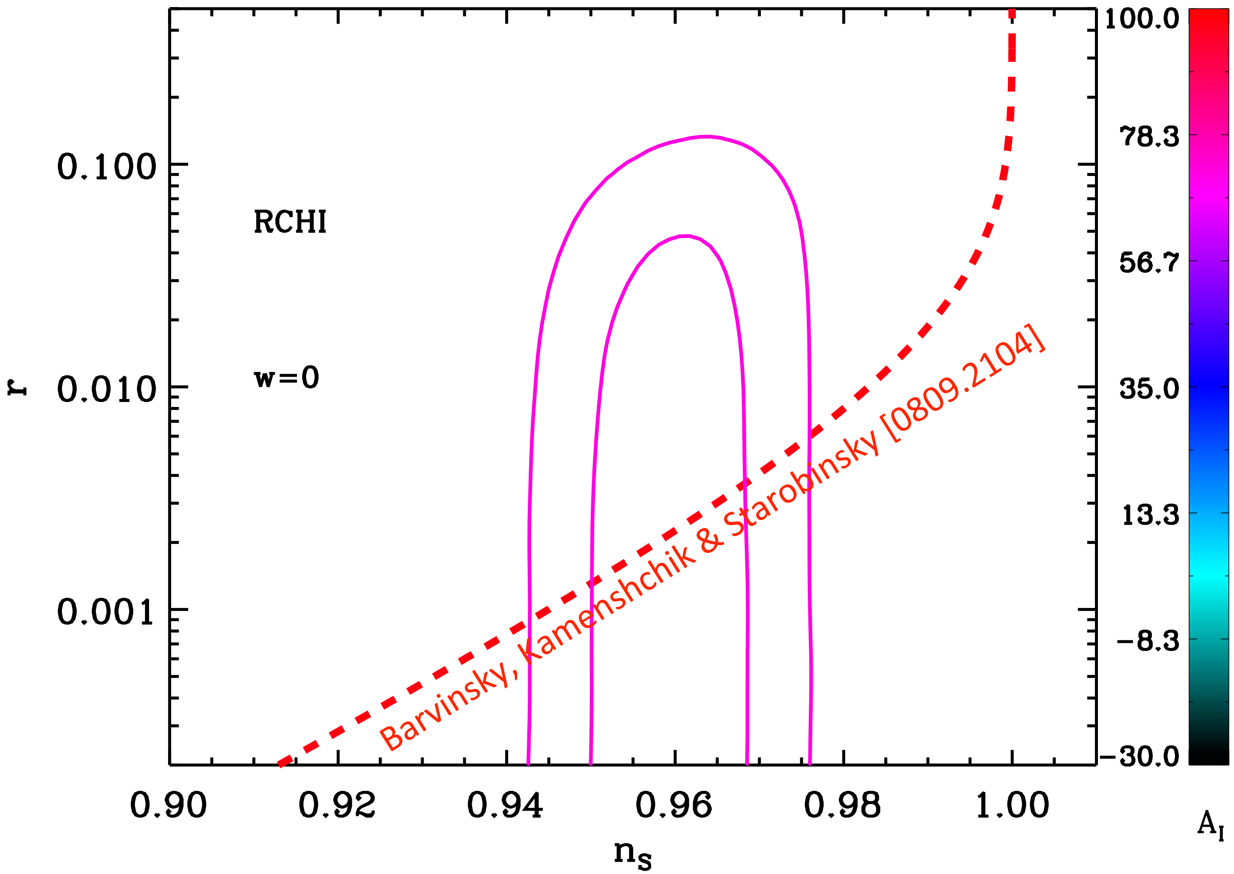
Increased Observational Constraints

	n_S $(\pm 1\sigma)$	r (95%CL)	α_S $(\pm 1\sigma)$	f_{nl}^{local} $(\pm 1\sigma)$	$\mathcal{I}/\mathcal{R}$ (95%CL)
COBE 2	1.21 ± 0.57				
COBE 4	1.20 ± 0.3				
WMAP 1	1.20 ± 0.11	<0.81	-0.077 ± 0.05	40 ± 49	$<32\%$
WMAP 3	0.984 ± 0.029	<0.65	-0.055 ± 0.03	30 ± 42	
WMAP 5	0.960 ± 0.013	<0.43	-0.037 ± 0.028	51 ± 30	$<16\%$
WMAP 7	0.968 ± 0.012	<0.36	-0.034 ± 0.026	32 ± 21	$<13\%$
WMAP 9	0.9608 ± 0.008	<0.13	-0.019 ± 0.025	37.2 ± 19.9	$<15\%$
Planck 2013	0.9603 ± 0.007	<0.11	-0.013 ± 0.009	2.7 ± 5.8	$<3.6\%$

Need for exact Formulas

Example: Radiatively Corrected Higgs Inflation

$$V = M^4\left[1 - 2e^{-2\phi/(\sqrt{6}M_{\mathrm{Pl}})} + \frac{A_{\mathrm{I}}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\mathrm{Pl}}}\right]$$



$V \sim M^4$ in

- Inflationary Dynamics
- Φ_{end}
- $\Delta N_* = 60$

Need for exact Formulas

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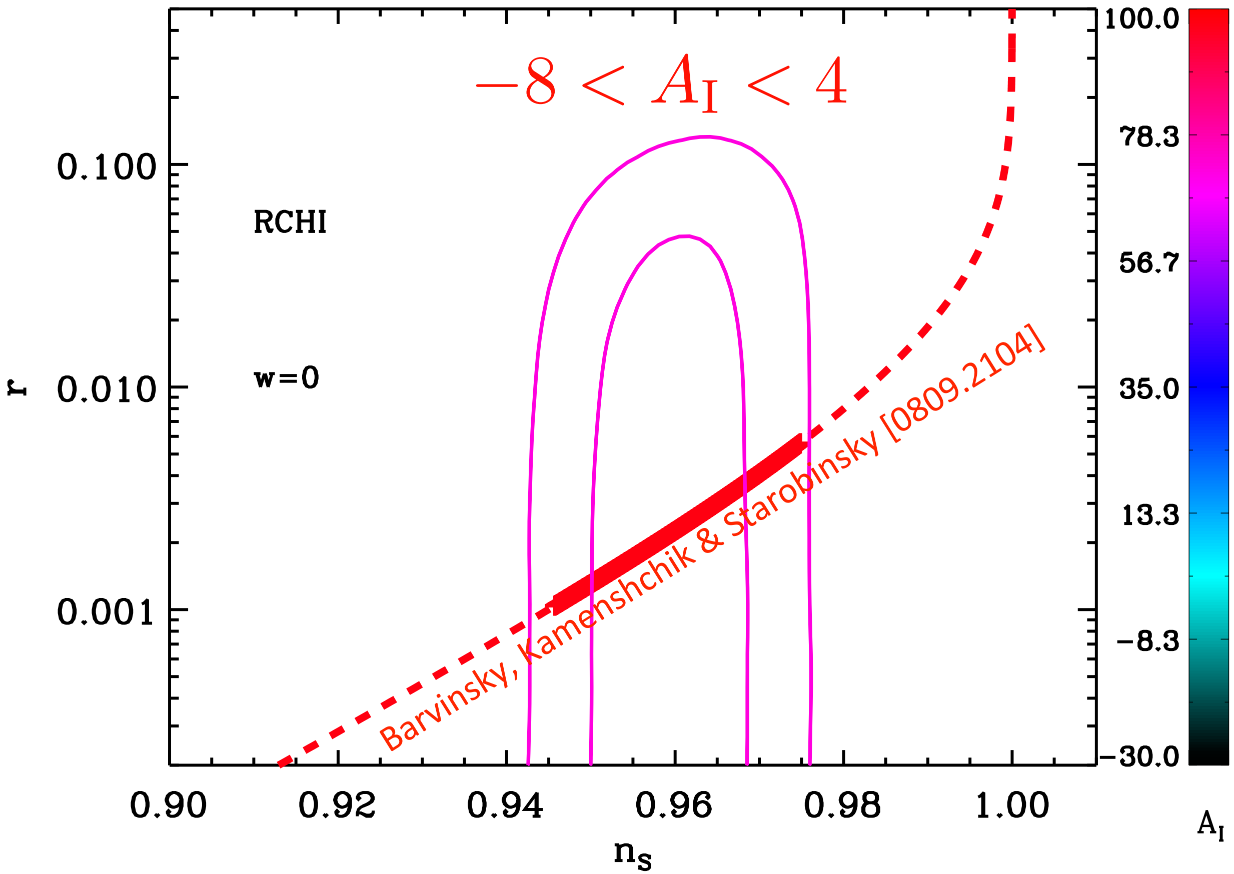

$V \sim M^4$ in

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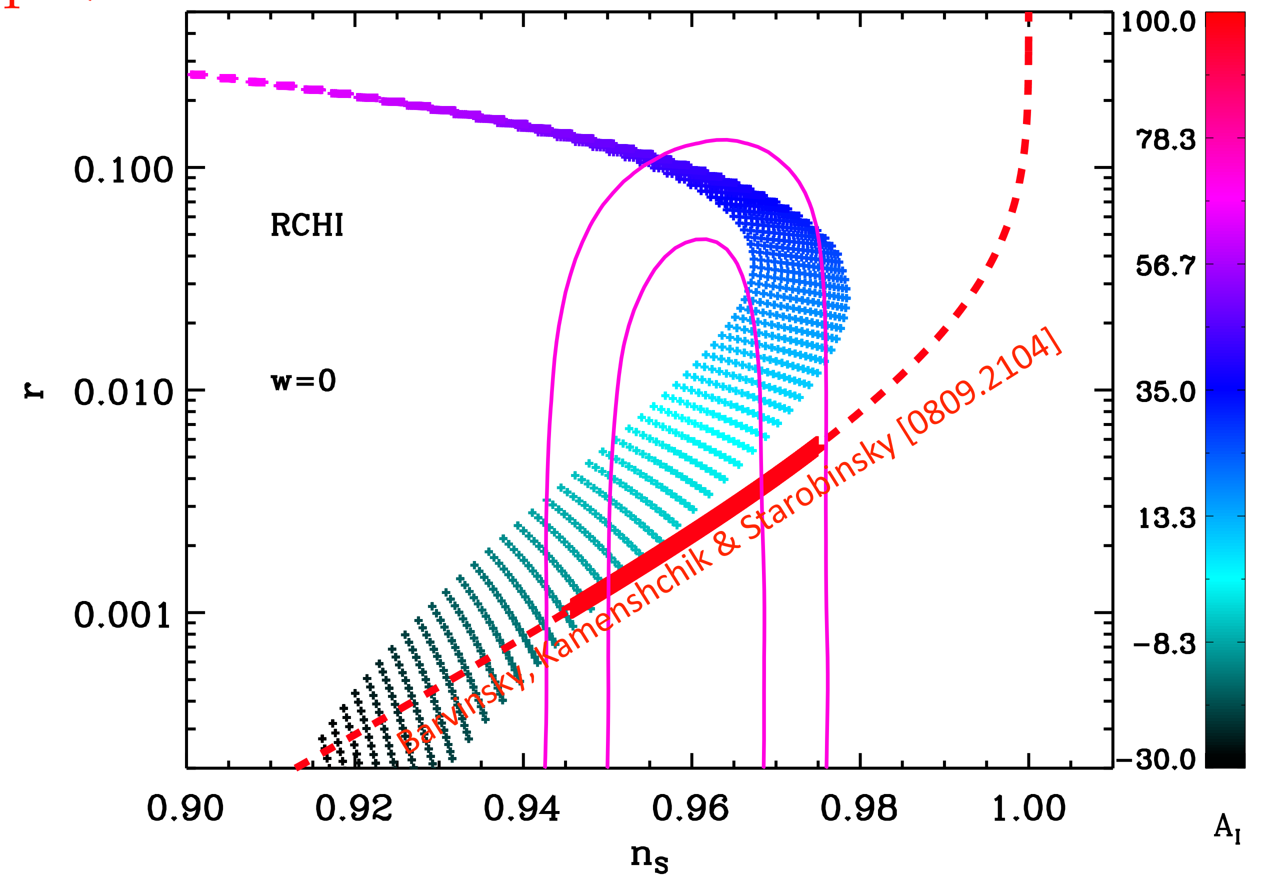
Example: Radiatively Corrected Higgs Inflation

$$V = M^4 \left[1 - 2e^{-2\phi/(\sqrt{6}M_{\rm Pl})} + \frac{A_{\rm I}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\rm Pl}}\right]$$

$-8 < A_{\rm I} < 4$



$V \sim M^4$ in

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- ~~$\Phi_{\rm end}$~~
- ~~$\Delta N_* = 60$~~

Need for exact Formulas

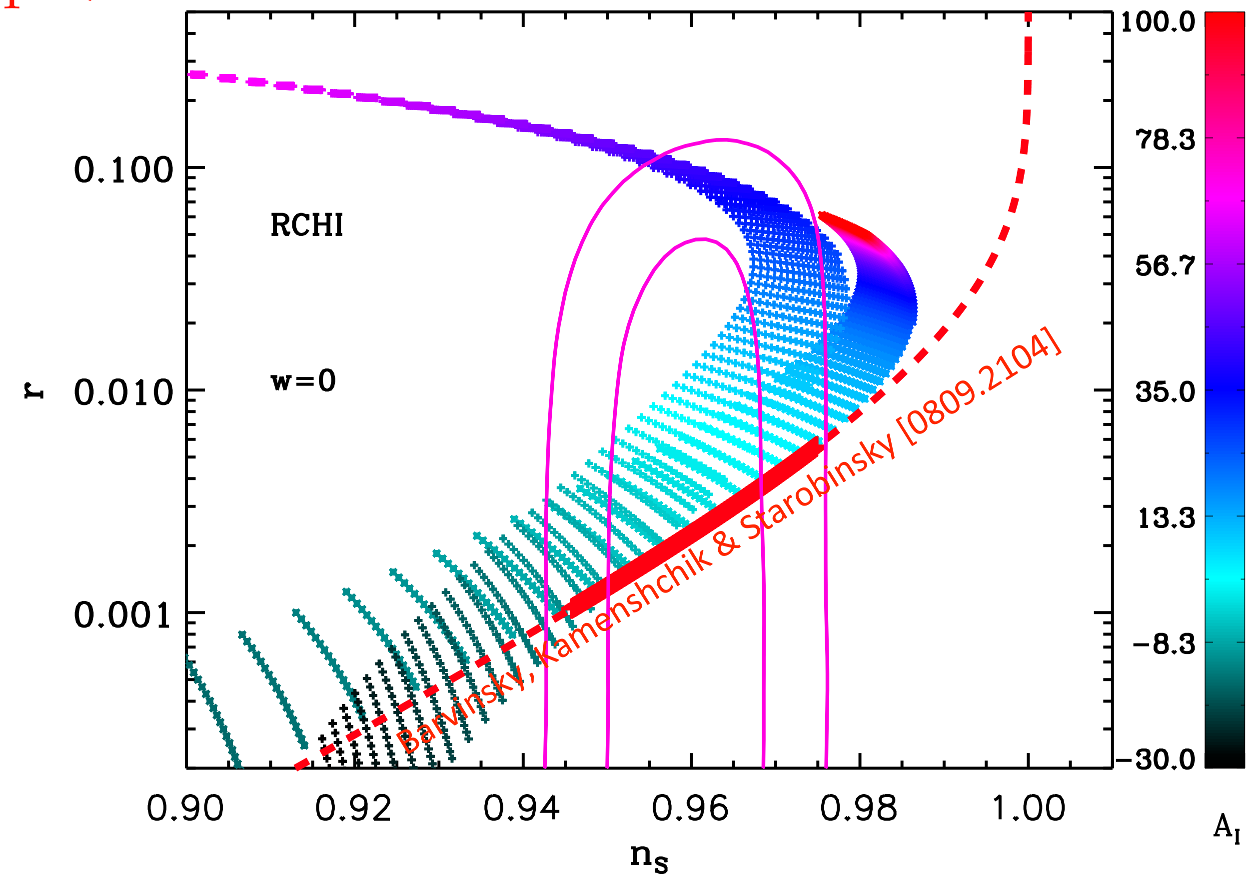
Example: Radiatively Corrected Higgs Inflation

$$V = M^4\left[1 - 2e^{-2\phi/(\sqrt{6}M_{\rm Pl})} + \frac{A_{\rm I}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\rm Pl}}\right]$$

Particle Physics:

$-48 < A_{\rm I} < -20$

$-8 < A_{\rm I} < 4$



$V \sim M^4$ in

- ~~Inflationary Dynamics~~
- ~~$\Phi_{\rm end}$~~
- ~~$\Delta N_* = 60$~~

Bayesian Approach

$$P(A \cap B) = P_B(A)P(B) = P_A(B)P(A)$$

$$P_B(A) = \frac{P_A(B)P(A)}{P(B)}$$ Bayes Formula

posterior

evidence

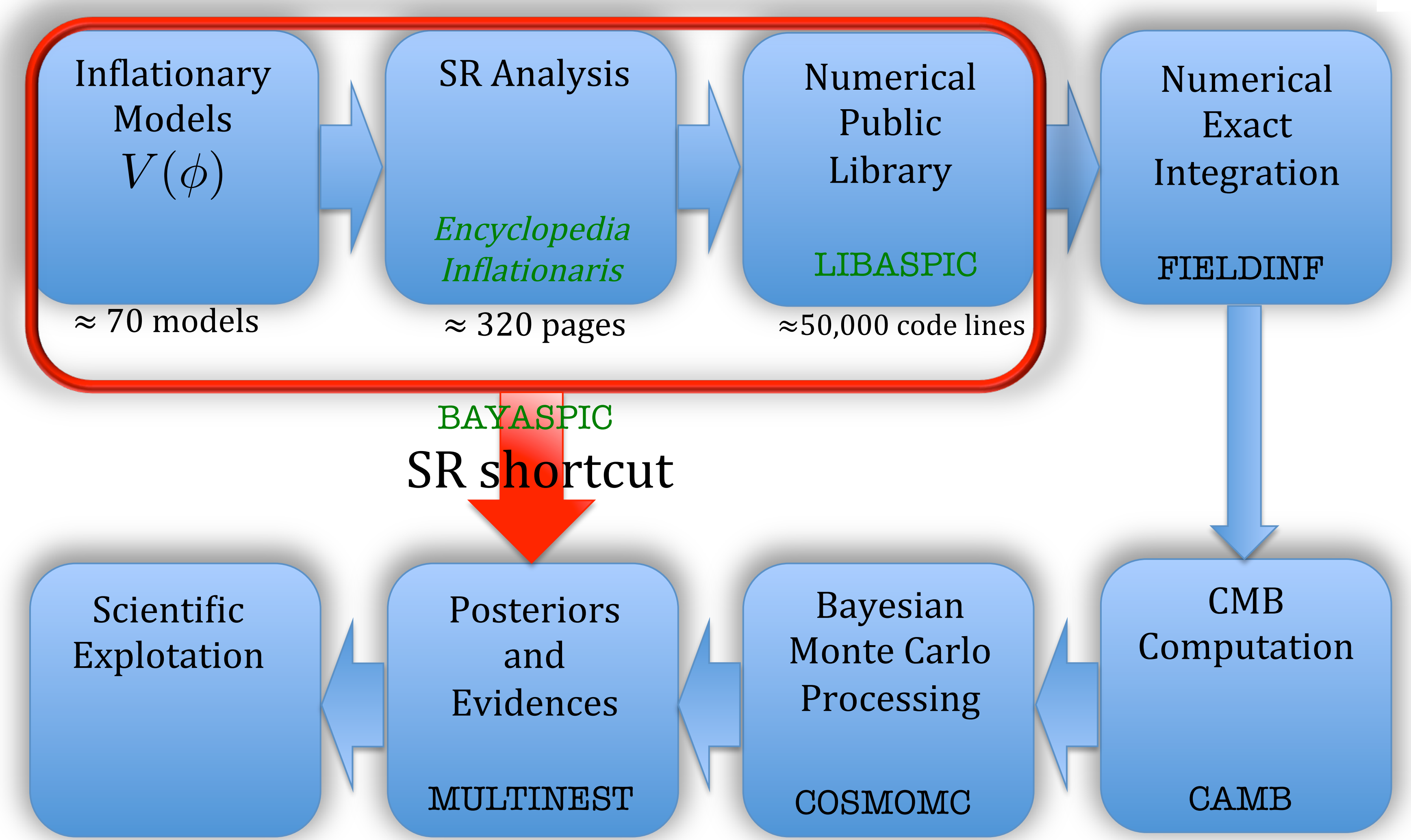
prior

$$P_{\mathrm{data}}(\mathcal{M}) = \frac{P_{\mathcal{M}}(\mathrm{data})P(\mathcal{M})}{P(\mathrm{data})}$$

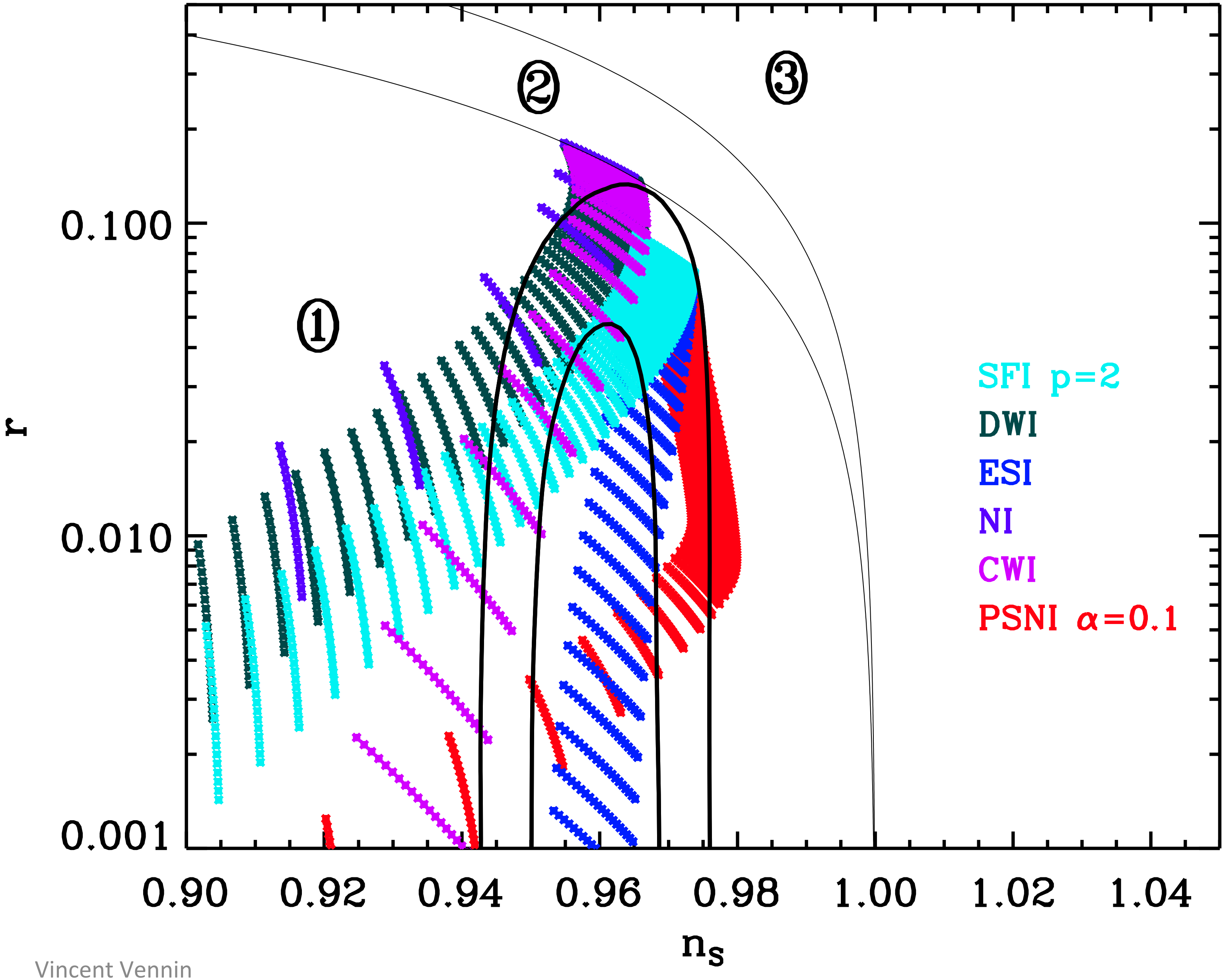
normalization $P(\mathrm{data}) = \int_{\mathcal{M}} P(\mathcal{M})P_{\mathcal{M}}(\mathrm{data})$

Evidence ratios:
updates the relative state of belief in two models

Computational Pipeline: The ASPIC project



Typological Classification



Summary

- Inflation solves the Hot Big Bang problems, and provides a causal mechanism for genrating cosmological perturbations from quantum fluctuations

- Its simplest versions (single scalar field with canonical kinetic term) account for all the observational facts about their statistics

- The acuracy of the data has improved so much that it now allows to distinguish between the models

- This can be achieved by means of semi analytical bayesian computation

- The ASPIC project has developed a publicly available numerical library of slow roll routines for ≈ 70 models, along with an *Encyclopedia Inflationaris*

- It is now providing for the first time the first evidence of these models and should allow to answer the question: **What is the best model of inflation?**

- It should be associated with complementary approaches: model independent calculation, potential reconstruction, etc..