Continuity, deconfinement, and (super)-Yang-Mills theory

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1205.0290, 1212.1238

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the general theme:

while the LHC continues looking for SUSY
- and may or may not see evidence for it -
the development I will describe is an(other)
example of how ideas initially studied in string
theory and supersymmetry improve our
understanding of non-SUSY gauge dynamics

our case:

it is well known that Yang-Mills theories, when "heated up" undergo a confinement-deconfinement transition - from models, lattice data, and, more recently, heavy-ion collisions at RHIC.

Thermal partition function is (without fermions):

$Z(\beta) = \mathrm{tr}[e^{-\beta H}],$ $\beta = 1/T = \text{radius of } S^1$ $\mathbb{R}^3 \times S^1$

↕

L - size of S[1]

$\mathbb{R}^3$ Λ^{-1} $\mathbb{R}^4$

high-T: low-T: L

Quark Gluon Plasma Hadronic Confined Phase

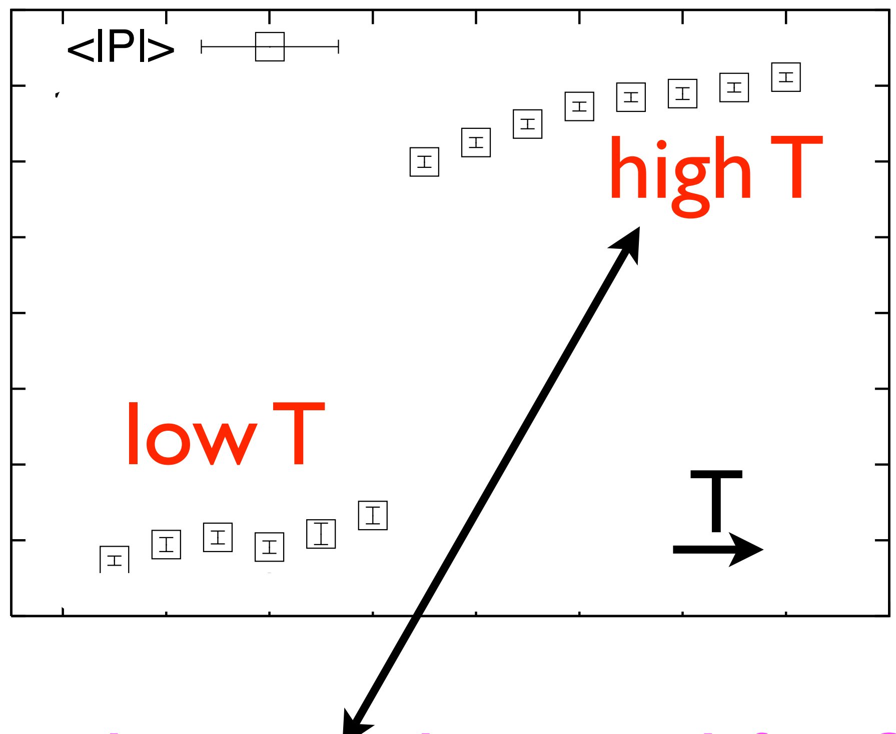
Transition occurs at temperatures of order the strong scale of the theory. It is, thus, hard to study by analytical means. Numerical experiment - lattice - works... Theoretically interesting and experimentally relevant problem.

Consider (for now, pure) Yang-Mills theory on $\mathbb{R}^3 \times S^1_L$

L - size of S^1

$$\Omega = \exp\left[i \int A_4 dx_4\right]$$

$\langle \mathrm{tr}\, \Omega \rangle$ ↑



T>>T_c behavior has been understood for 30 years

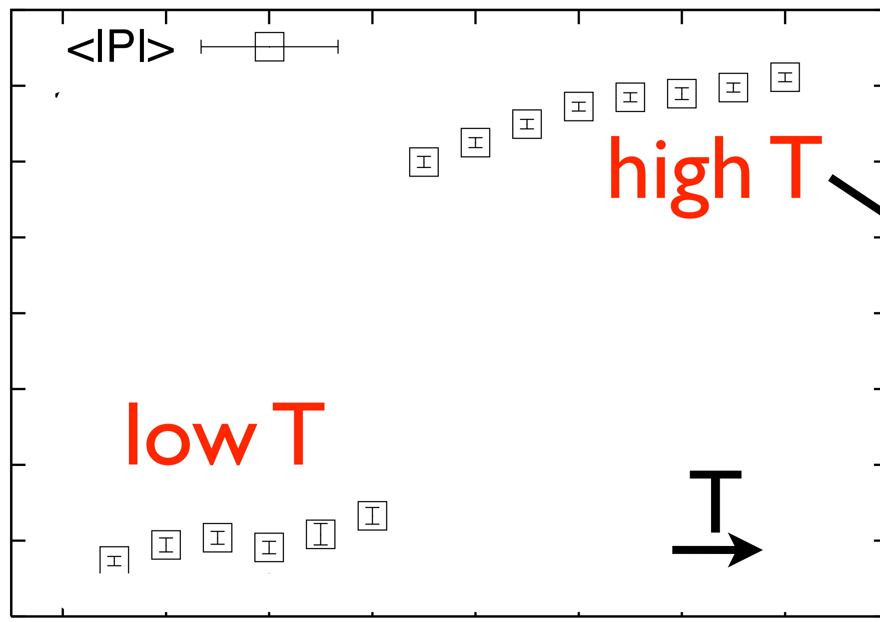
[Gross, Pisarski, Yaffe, 1981]

High-T perturbation theory good, gives one-loop V(pert), which favors center-broken vacuum.

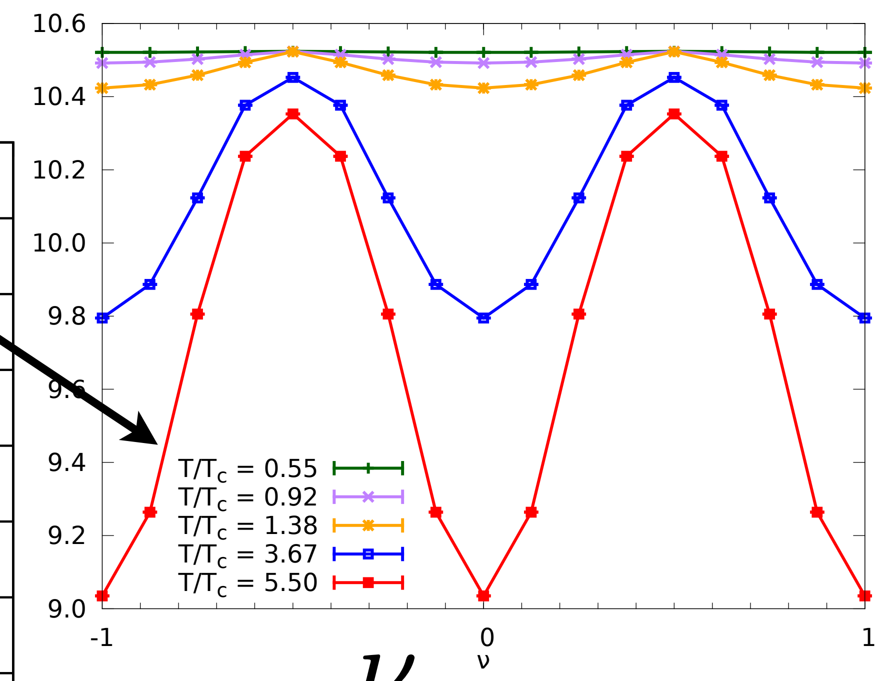
Consider (for now, pure) Yang-Mills theory on $\mathbb{R}^3 \times S^1_L$

$$\Omega = \exp\left[i \int A_4 dx_4\right]$$

$\langle \mathrm{tr}\ \Omega \rangle$ ↑



e.g., Diakonov, Gattringer,Shadler, 1205.4768
SU(2) 40x40x40x30 lattice calcul. of V



minima at T/Tc=5.5 correspond exactly to V_pert, coinciding eigenvalues $\frac{1}{2}\langle \mathrm{tr}\ \Omega \rangle$ = 1

Polyakov loop unitary, eigenvalues $e^{\pm i\pi\nu}$ (SU(2)) lie on unit circle

$$V_{\mathrm{pert.}}(\Omega) = -\frac{2}{\pi^2\beta^4}\sum_{n=1}^{\infty}\frac{1}{n^4}|\mathrm{tr}\Omega^n|^2(1+O(g^2)),$$

Consider (for now, pure) Yang-Mills theory on $\mathbb{R}^3 \times S^1_L$

T>>Tc behavior has been understood for 30 years.
As for lower T:

Gross, Pisarski, Yaffe, 1981:

"***It is hardly surprising that we cannot explore the transition, as the temperature is lowered, from the unconfined to the confined phase using solely weak coupling techniques***"

Nonetheless, it is of interest to find examples where one could study deconfinement by reliable analytical techniques (*"why bother?"*):

- understanding an analytically calculable regime is always good, likely to give insight into important aspects of the physics
- pushing a calculable regime to/beyond borders of its validity can be useful (and fun); resulting models can be compared, e.g. with lattice - (e.g., work of Shuryak, Sulejmanpasic)

Consider (for now, pure) Yang-Mills theory on $\mathbb{R}^3 \times S^1_L$

Nonetheless, it is of interest to find examples where one could study deconfinement by reliable analytical techniques (I do not include models in my list below, as these are not my topic today; see e.g.: Pisarski et al./ Diakonov, Petrov/ Zhitnitnsky, Parnachev/ Shuryak, Sulejmanpasic-Faccioli/... FRG approach to deconf. I know nothing about!)

Several ways to do this have been found in the past 30 years:

1. Gauge-gravity duality [many, after Witten 1998, ...]

pro: semiclassical string theory provides a weak-coupling description of strongly-coupled gauge theory

deconfinement=Hawking-Page

con: comes with extra baggage - non decoupling KK modes; no asymptotic freedom;
useful macroscopically; microscopic connection(?)

2. $S^1 \times S^3$ compactifications [Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk, 2003-5]

S^1: thermal; S^3: non-thermal

pro: at small S^3, a weakly coupled matrix model

low-T: Vandermonde repulsion of EVs

high-T: pert. attraction of Polyakov loop EVs

con: thermodynamic limit means large-N transition only

These authors rejected the possibility of finding a weak-coupling transition at infinite volume...

Consider (for now, pure) Yang-Mills theory on $\mathbb{R}^3 \times S^1_L$

Nonetheless, it is of interest to find examples where one could study deconfinement by reliable analytical techniques...

Several ways to do this have been found in the past 30 years:

3. $R^2 \times S^1 \times S^1$ compactifications

$\updownarrow$ non-thermal

$\leftrightarrow$ thermal

[Simic, Unsal 2010
Unsal 2012

Anber, EP, Unsal 2011
Anber, Collier, EP 2012]

"deformed" pure-YM

"QCD(adj)" = YM with many massless adjoint Weyl fermion

pro: at small S^1, map 4d thermal gauge theory to a 2d spin system - "affine" XY spin models related to cond. mat. systems: e.g., 2d triangular lattice crystal melting for SU(3)(adj) (or more general new stat-mech models)

con: abelian (de-)confinement, L< infinite - **nonetheless (I think) fascinating systems:** 2d "gases" of el. and m. charged particles, with Aharonov-Bohm interactions, inheriting the symmetries of their respective 4d gauge theories and showing a deconfinement transition [not all understood!]

>>> talk by Mohamed Anber

FINALLY, THE TOPIC OF THIS TALK!

[EP, Schaefer, Unsal 1205.0290, 1212.1238]

4. $R^3 \times S^1$ compactifications of super YM with small $m_{gaugino}$

↕

(non-) thermal

pro:
con: ***DIY!***

FINALLY, THE TOPIC OF THIS TALK!

[EP, Schaefer, Unsal 1205.0290, 1212.1238]

4. $R^3 \times S^1$ compactifications of super YM with small $m_{gaugino}$

↕ (non-) thermal

Let's first flesh out the idea:

Pure SYM on $\mathbb{R}^3 \times S^1_L$ with periodic (supersymmetric) b.c. for gaugino.

i.) No phase transition as L is varied from small to large.

"twisted partition function" [= Witten index]

$$\widetilde{Z}^{\rm SYM}(L) = {\rm tr}\left[e^{-LH}(-1)^F\right]$$ ← periodic fermions

Pure SYM on $\mathbb{R}^3 \times S^1_L$ with periodic (supersymmetric) b.c. for gaugino.

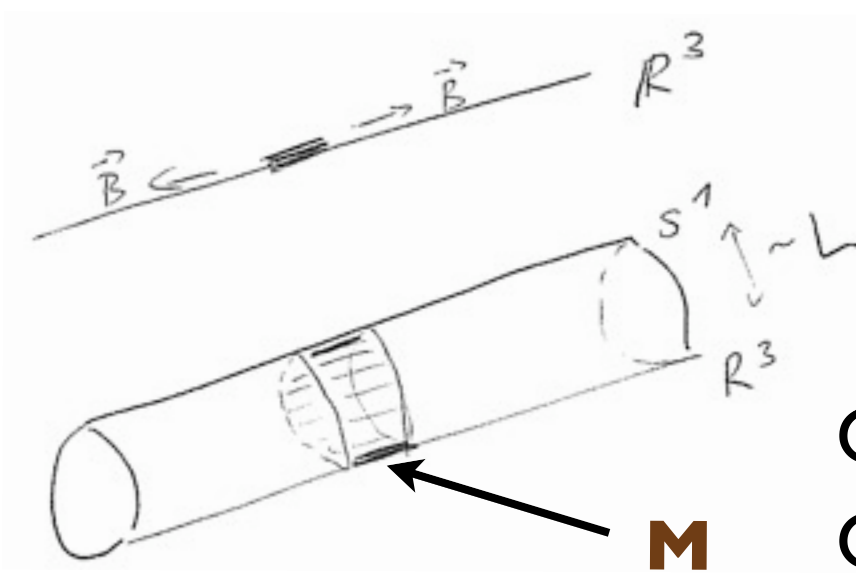
i.) no phase transition as L is varied from small to large.

ii.) N=1 pure SYM in this geometry was studied by

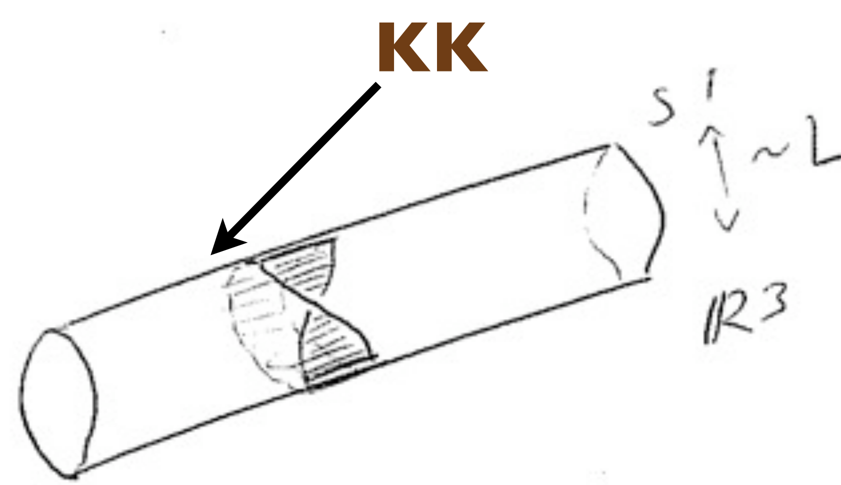
Seiberg, Witten;
Aharony, Hanany, Intriligator, Seiberg, Strassler;
Davies, Hollowood, Khoze - late 1990's

salient points: theory dynamically Abelianizes & preserves center-symmetry,
dynamics semi-classically calculable at small-L; L<<1/(strong scale)

major players: monopole-instanton (M) and twisted (KK) [Piljin Yi, Kimeyong Lee, 1997]



Q_M=1
Q_T=1/2



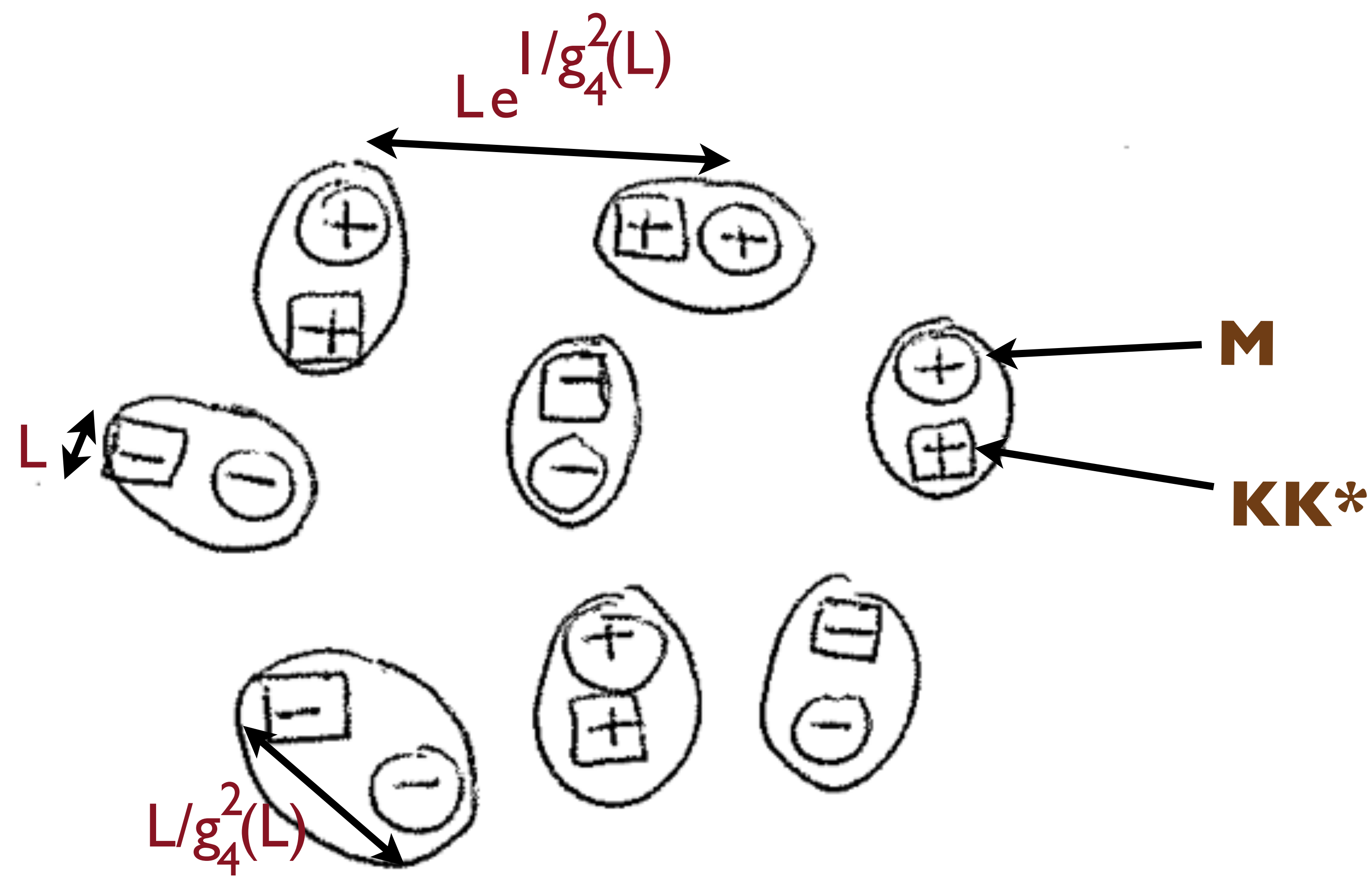
Q_M=-1
Q_T=1/2

Late '90's studies relied heavily on SUSY & string. Unsal, 2007, realized that there's a general mechanism at play, transcending SUSY: theories with Nf massless adjoints confine due to a locally-4d generalization of Polyakov's 3d "Debye screening" by monopole-instantons - the "magnetic bion" mechanism.

Pure SYM on $\mathbb{R}^3 \times S^1_L$ with periodic (supersymmetric) b.c. for gaugino.

4d QCD(adj)
vacuum at small L

(N$_f$ =1 case, i.e. SUSY,
brings in more fun
objects! - to come)



Rich hierarchy of scales in the 4d QCD(adj) bion plasma at small-L ensures the semiclassical calculability. First theory where confinement analytically shown in a locally 4d, continuum, nonsupersymmetric theory. [Unsal 2007; Unsal+one of Shifman, Yaffe, EP, Argyres 2008-]

4d important! - KK monopoles do not exist at zero size circle & theory does not confine at zero L

Furthermore, in softly broken N=2 SYM: "magnetic bion" confinement is continuously connected to 4d Seiberg-Witten confinement by monopole condensation - via Poisson resummation [EP, Unsal - Paris - QCD 2011]

Pure SYM on $\mathbb{R}^3 \times S^1_{\mathrm{L}}$ with periodic (supersymmetric) b.c. for gaugino.

i.) no phase transition as L is varied from small to large.

ii.) at small L, supersymmetric theory confines due to a locally-4d generalization of Polyakov's 3d "Debye screening" due to monopole-instantons - the "magnetic bion" mechanism [Unsal, 2007].
Due to i.), this smoothly connects to 4d limit.

iii.) add gaugino mass "m"

"twisted"-Z still defined but not an index

$$\widetilde{Z}^{\mathrm{SYM}}(L,m) = Z_{\mathcal{B}} - Z_{\mathcal{F}} = \sum_{n\in\mathcal{H}_{\mathcal{B}}} e^{-LE_n} - \sum_{n\in\mathcal{H}_{\mathcal{F}}} e^{-LE_n} .$$

"twisted"-Z different from thermal-Z:

$$Z^{\mathrm{SYM}}(\beta,m) = Z_{\mathcal{B}} + Z_{\mathcal{F}}$$

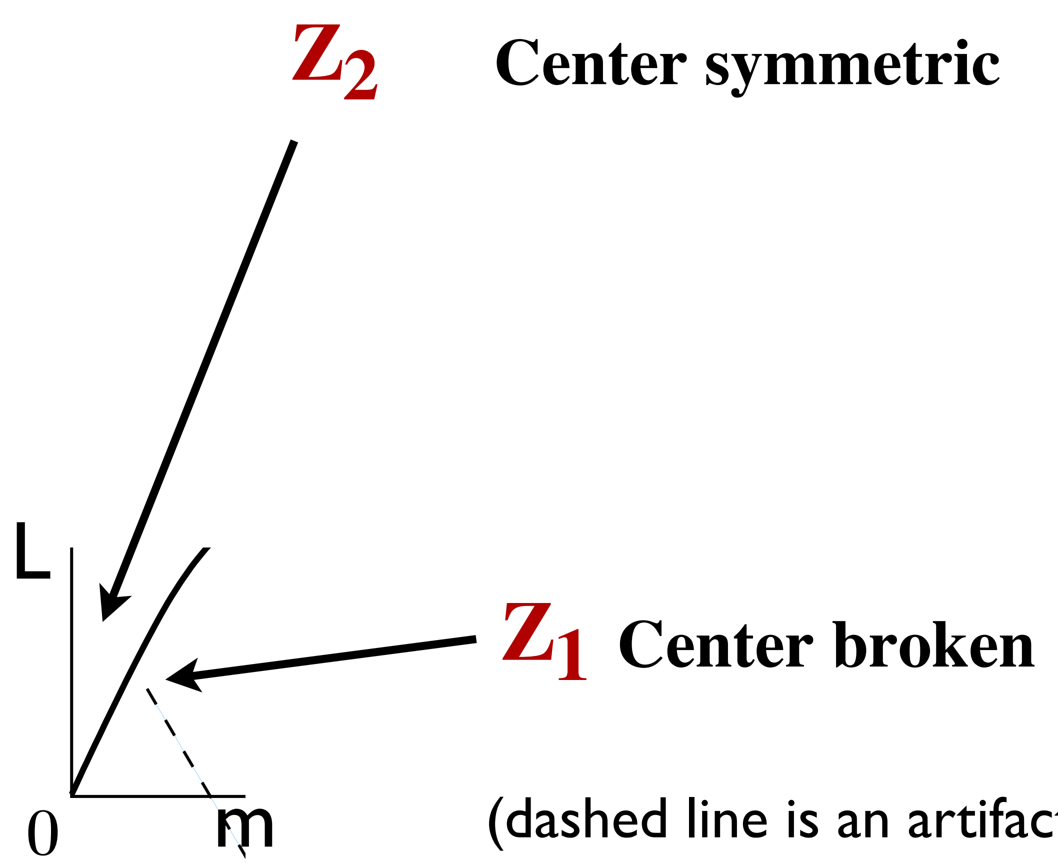
as "m" becomes large, fermions decouple and twisted-Z approaches the pure-YM thermal-Z:

$$\widetilde{Z}^{\mathrm{SYM}}(L,m)\Big|_{m\to\infty} \Longrightarrow Z^{\mathrm{YM}}(\beta) = \mathrm{tr}[e^{-\beta H}] , \qquad \beta \equiv L$$

Pure SYM on $\mathbb{R}^3 \times S^1_L$ with periodic (supersymmetric) b.c. for gaugino.

iv.) at small L, the mass-deformed SYM theory has an interesting phase structure - depending on the order of limits as m, L --> 0, there is a center-symmetry breaking phase transition

[already noted in Unsal, Yaffe 2010]

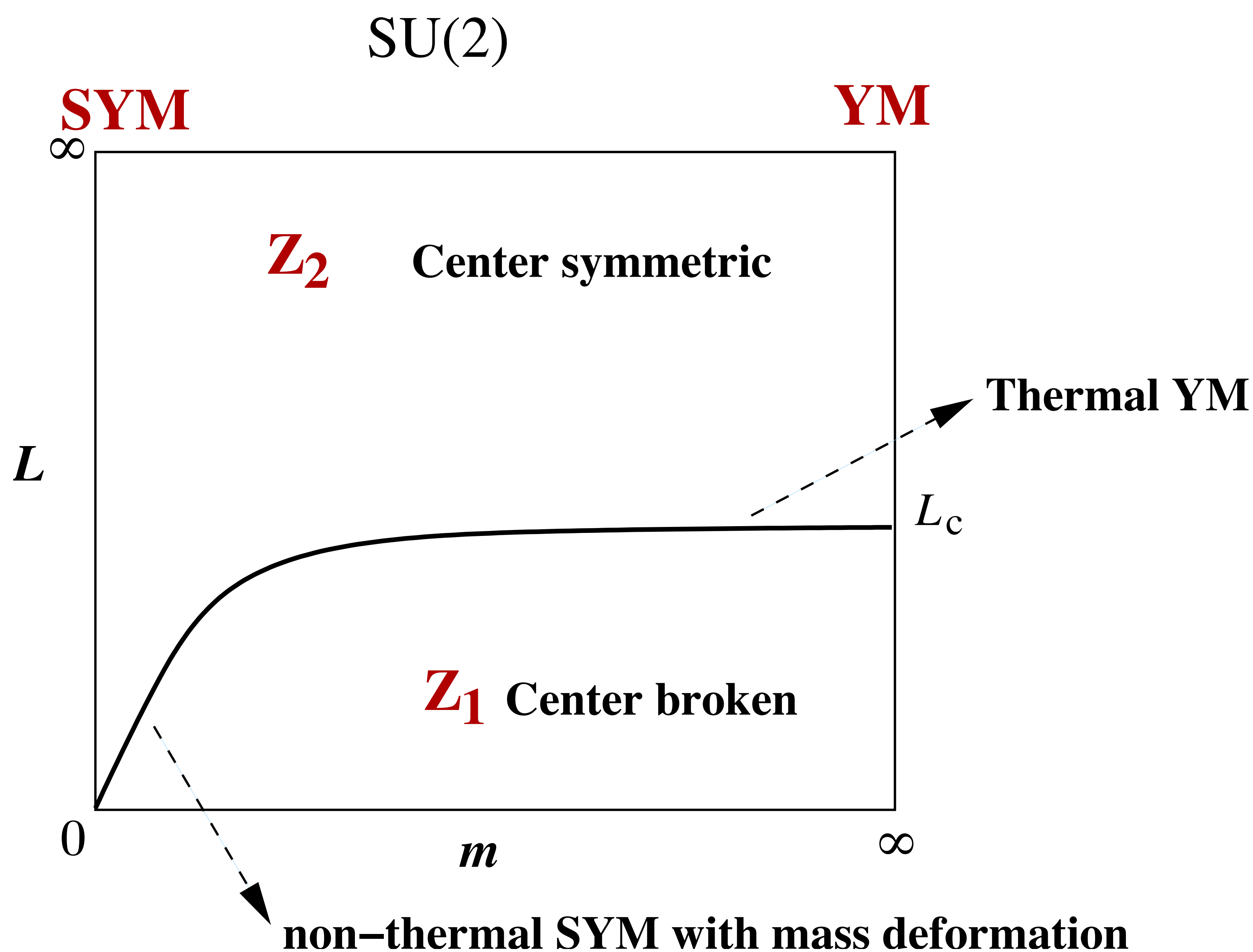


(dashed line is an artifact of plot)

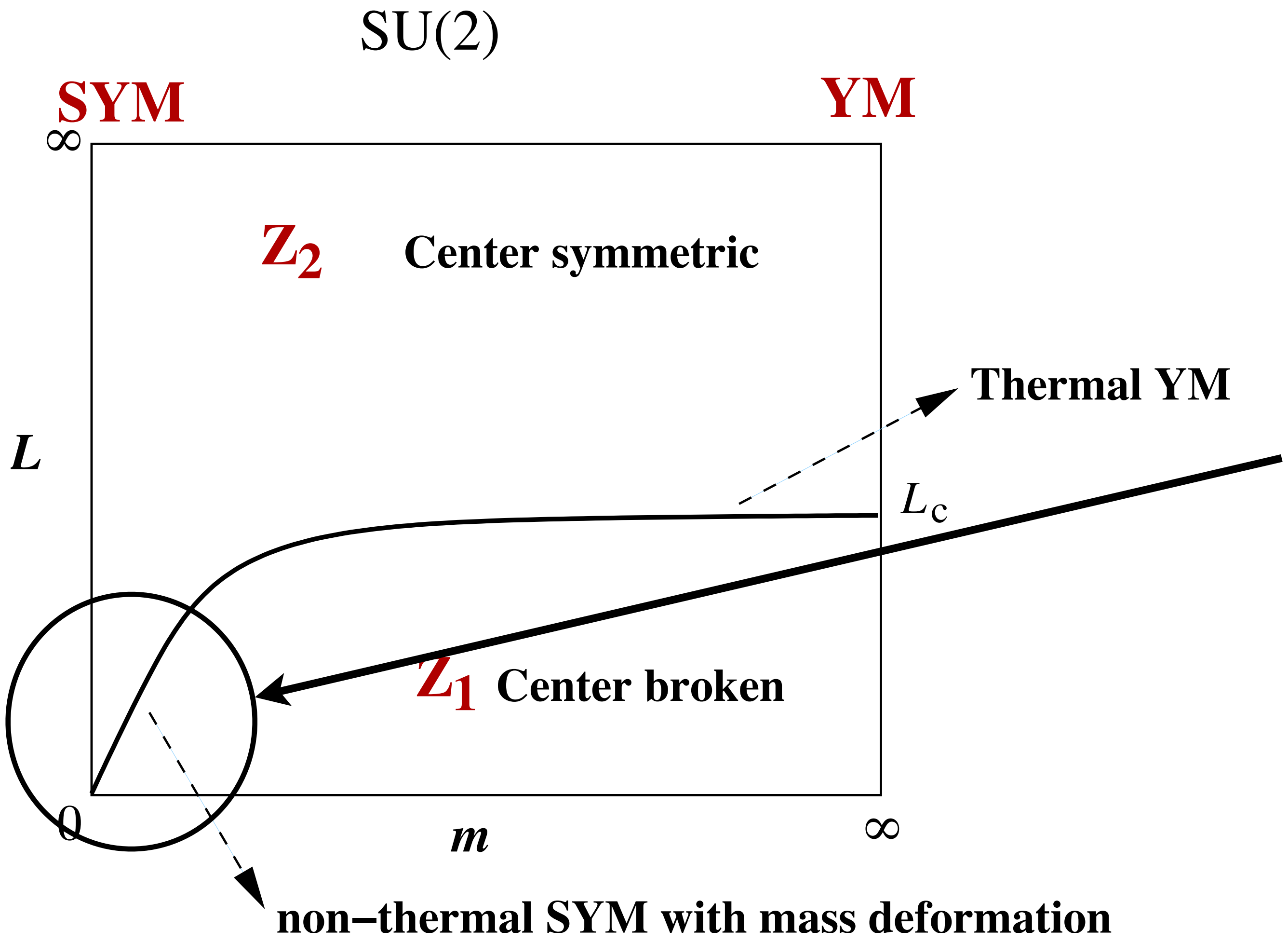
In the small-(m,L) corner, transition is semiclassically calculable with rather rich physics...

Pure SYM on $\mathbb{R}^3 \times S^1_{\mathsf{L}}$ with periodic (supersymmetric) b.c. for gaugino.

iv.) at small L, the mass-deformed SYM theory has an interesting phase structure - depending on the order of limits as m, L --> 0, there is a center-symmetry breaking phase transition



...expected to connect smoothly to the large-m thermal-YM deconfinement.



In the semiclassical regime, center-symmetry breaking occurs due to competition between contributions to the potential for Polyakov loop due to various topological objects and the perturbative V(eff):

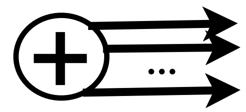
relevant bosonic fields: A_4 (gauge field in compact direction) and A_i (3d gauge field) in the unbroken U(1) of SU(2), equivalent to:

σ - 3d dual to A_i = "dual photon" (potential for magnetic charge)

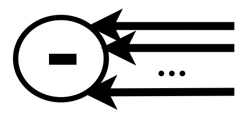
ϕ - deviation of A_4 from center symmetric value

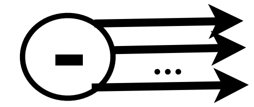
monopole-instantons - M,M*,KK,KK*

$$e^{-S_0} e^{+i\sigma-\phi} \lambda\lambda$$

M: 

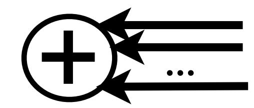
$$e^{-S_0} e^{-i\sigma-\phi} \overline{\lambda\lambda}$$

M*: 

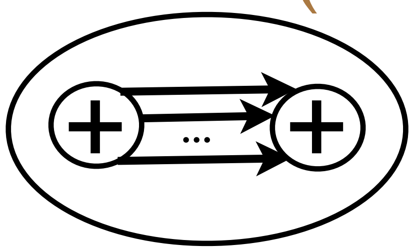
KK: 

$$e^{-S_0} e^{-i\sigma+\phi} \lambda\lambda$$

$$e^{-S_0} e^{+i\sigma+\phi} \overline{\lambda\lambda}$$

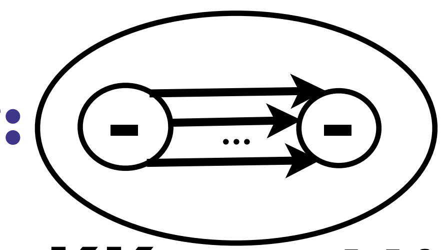
KK*: 

type-I bions (M-KK* "molecules") "magnetic bions" - confinement!

B: 

M **KK***

$$e^{-2S_0} e^{+i2\sigma}$$

B*: 

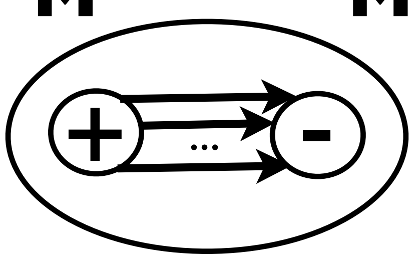
KK **M***

$$e^{-2S_0} e^{-i2\sigma}$$

type-II bions (M-M*,KK-KK* "molecules") "neutral bions"

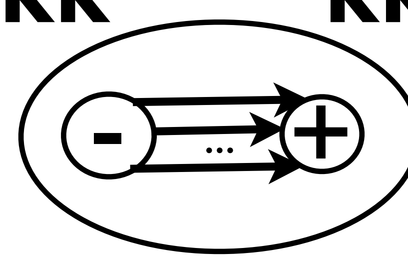
in pure-SYM: center-stabilizing

M **M***

N: 

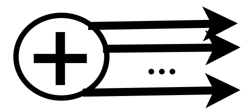
$$e^{-2S_0} e^{-2\phi}$$

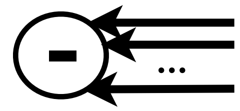
KK **KK***

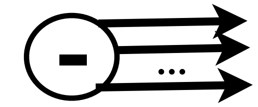
N*: 

$$e^{-2S_0} e^{+2\phi}$$

monopole-instantons - M,M*,KK,KK*

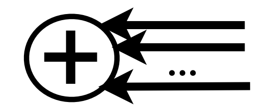
$e^{-S_0} e^{+i\sigma - \phi} \lambda\lambda$ **M:** 

$e^{-S_0} e^{-i\sigma - \phi} \overline{\lambda\lambda}$ **M*:** 

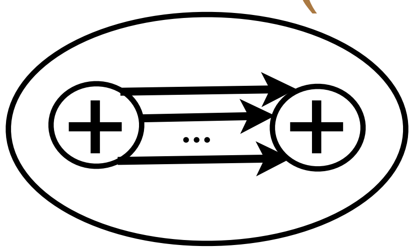
KK: 

$e^{-S_0} e^{-i\sigma + \phi} \lambda\lambda$

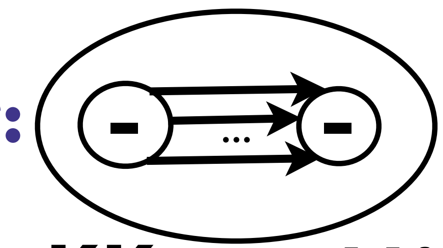
$e^{-S_0} e^{+i\sigma + \phi} \overline{\lambda\lambda}$

KK*: 

type-I bions (M-KK* "molecules") "magnetic bions" - confinement!

B:  $e^{-2S_0} e^{+i2\sigma}$

M KK*

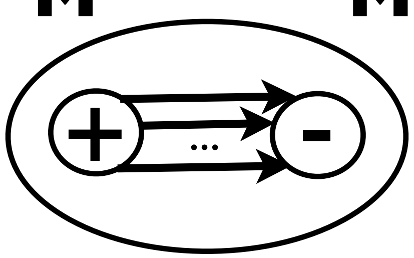
B*:  $e^{-2S_0} e^{-i2\sigma}$

KK M*

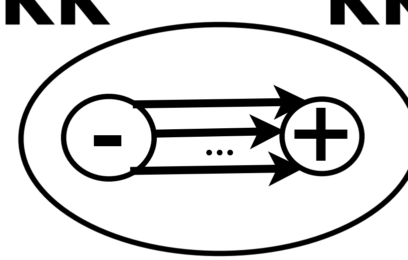
type-II bions (M-M*,KK-KK* "molecules") "neutral bions"

in pure-SYM: center-stabilizing

M M*

N:  $e^{-2S_0} e^{-2\phi}$

KK KK*

N*:  $e^{-2S_0} e^{+2\phi}$

$$\frac{1}{L^3} e^{-2S_0} \left(\cosh 2\phi - \cos 2\sigma\right)$$

The neutral "center-stabilizing bion" molecules' contribution can be computed using supersymmetry, V = $|W'|^2$, with W from monopole-instantons, or via the Bogomolnyi-Zinn-Justin (BZJ) prescription [late 1970s, also Balitsky, Yung mid-1980s; Yung ~1990]. BZJ allows one to identify molecules also in non-SUSY Yang-Mills theory... check: minus sign via BZJ = SUSY

$$\frac{1}{L^3} e^{-\frac{8\pi^2}{g^2(L)}} (\cosh 2\phi - \cos 2\sigma)$$

at small-L the SYM vacuum is Z_Nc symmetric
Nc monopole-instanton amplitudes are same, respect Z_Nc

center-stabilizing
"bions" - II and I

now turn on small gaugino mass "m":

$$\frac{1}{L^3} e^{-\frac{8\pi^2}{g^2(L)}} (\cosh 2\phi - \cos 2\sigma) + \frac{m}{L^2} e^{-\frac{4\pi^2}{g^2(L)}} (\cosh \phi \cos \sigma) - \frac{m^2}{L} \phi^2$$

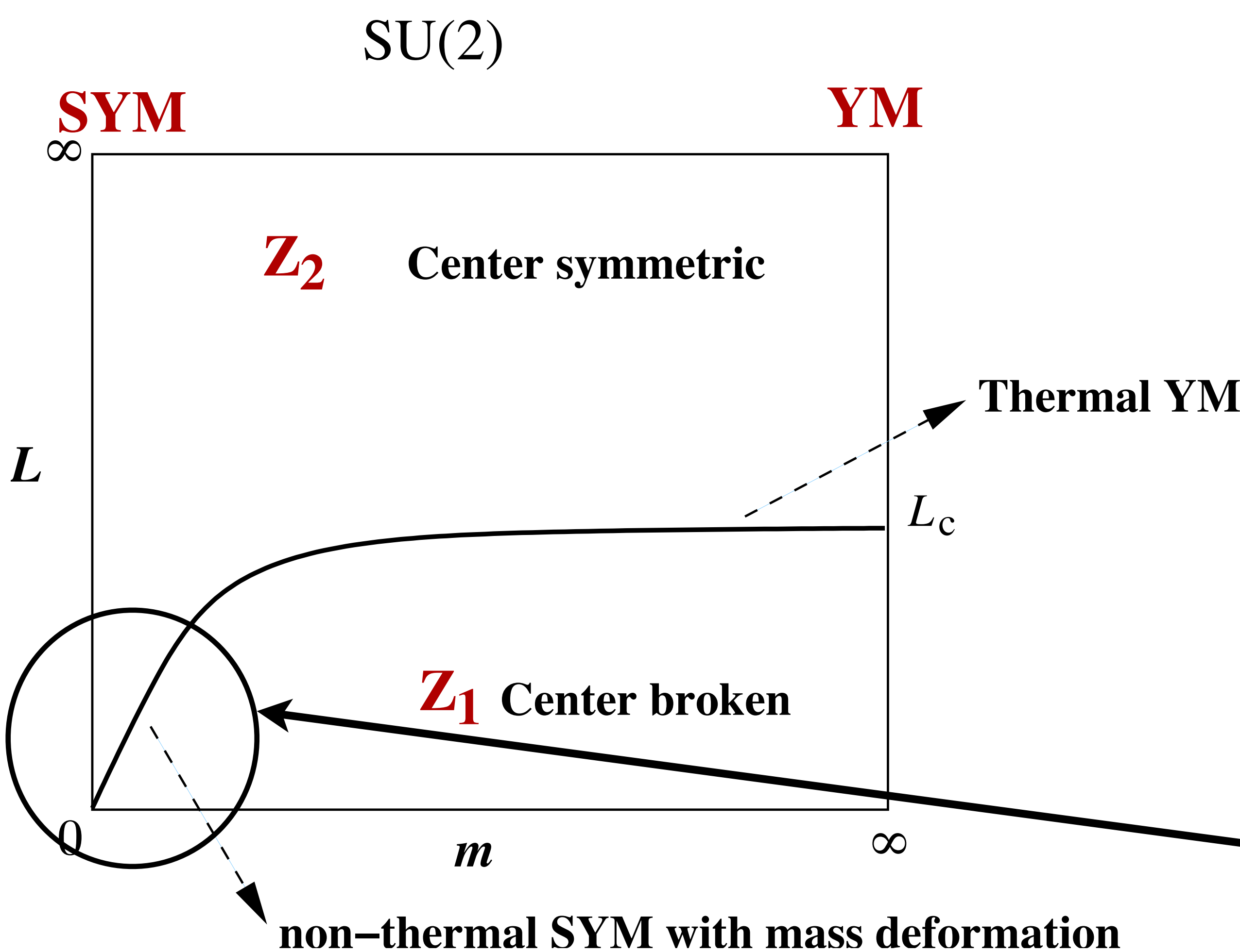
center-stabilizing
"bions" - II and I

center-breaking (sigma=Pi is min)
"monopole-instantons"

center-breaking
GPY potential shown before, expanded for small phi

$\frac{m_{soft}}{L^2 \Lambda^3}$ dimensionless parameter controlling the transition

now turn on small gaugino mass "m":



In the semiclassical regime, center-symmetry breaking occurs due to competition, as L (1/T) is varied, between contributions to the potential for Polyakov loop due to various topological objects and the perturbative V(eff):

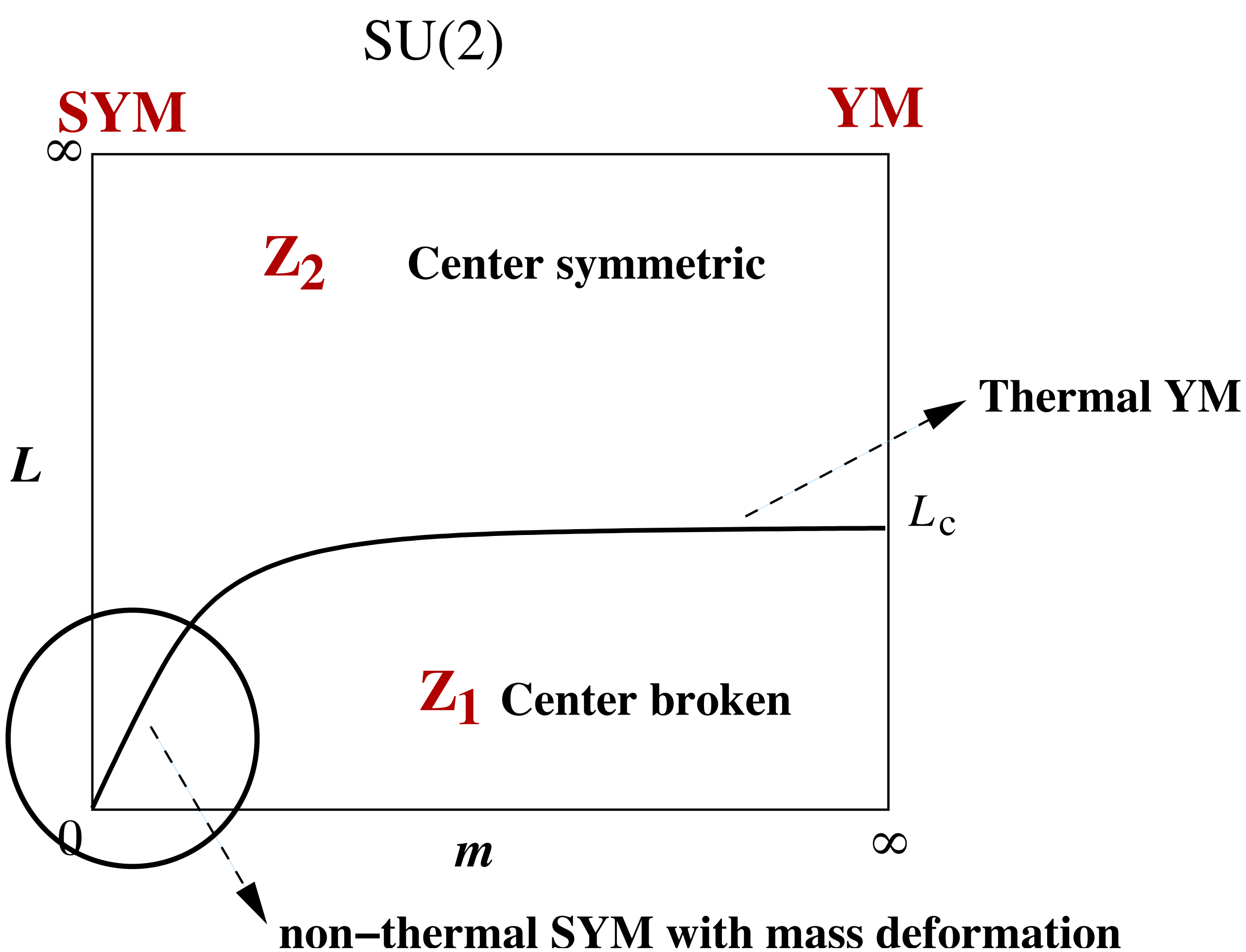
$$\frac{1}{L^3} e^{-\frac{8\pi^2}{g^2(L)}} \left(\cosh 2\phi - \cos 2\sigma\right) + \frac{m}{L^2} e^{-\frac{4\pi^2}{g^2(L)}} \left(\cosh \phi \cos \sigma\right) - \frac{m^2}{L} \phi^2$$

center-stabilizing "bions" - II and I

center-breaking (sigma=Pi is min) "monopole-instantons"

center-breaking GPY potential shown before, expanded for small phi

$\frac{m_{soft}}{L^2 \Lambda^3}$ dimensionless parameter controlling the transition



Our main result:

Center-breaking *quantum* phase transition, second order for SU(2), with causes that are well understood and under theoretical control - "fight" between topological molecules and perturbative contribution to holonomy potential - appears continuously connected to thermal deconfinement transition.

same topological excitations can be used to model pure YM deconfinement: Shuryak, Sulejmanpasic 1305.0796

$$\frac{1}{L^3} e^{-\frac{8\pi^2}{g^2(L)}} (\cosh 2\phi - \cos 2\sigma) + \frac{m}{L^2} e^{-\frac{4\pi^2}{g^2(L)}} (\cosh \phi \cos \sigma) - \frac{m^2}{L} \phi^2$$

center-stabilizing
"bions" - II and I

center-breaking (sigma=Pi is min)
"monopole-instantons"

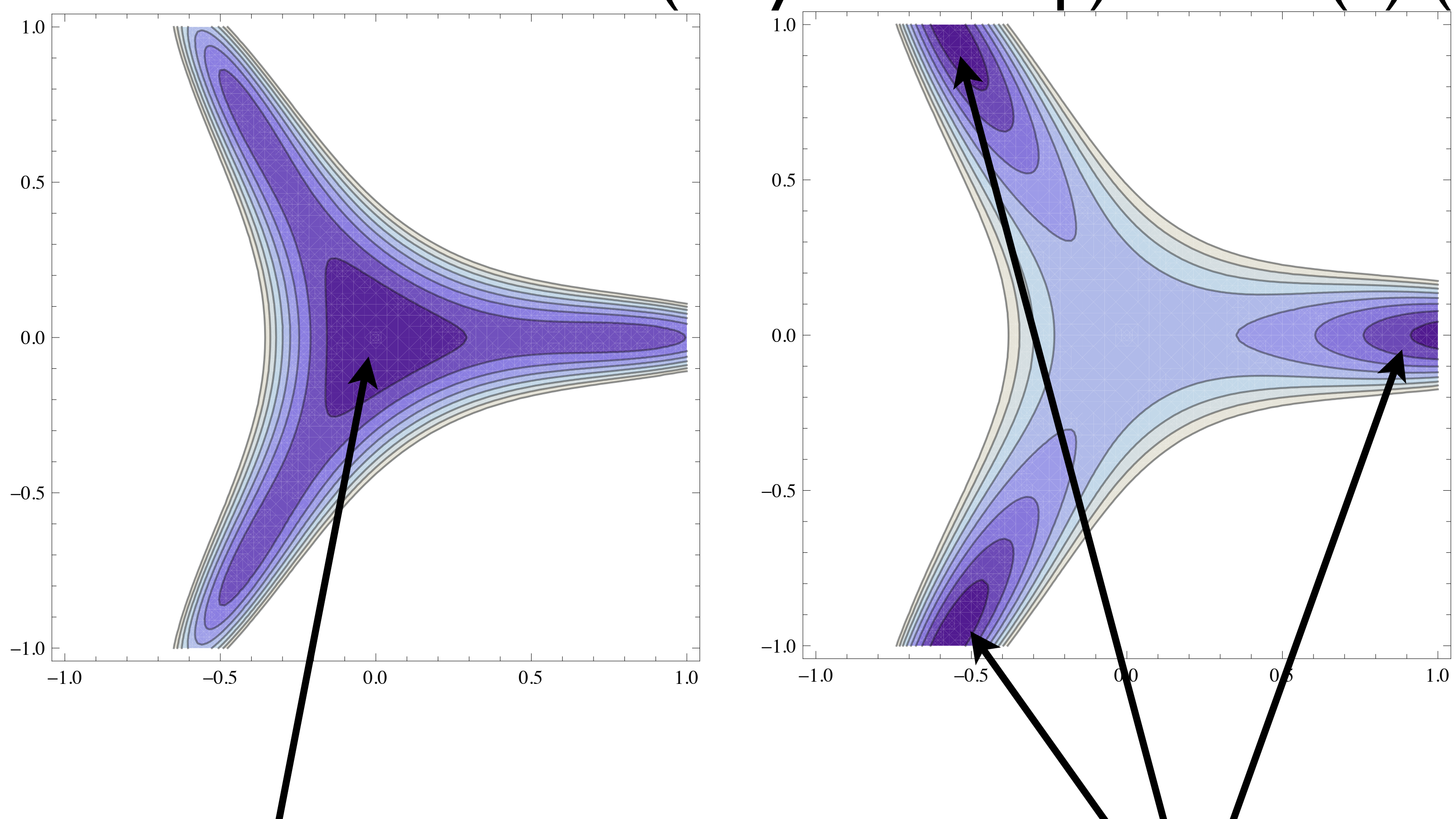
center-breaking
GPY potential shown before, expanded for small phi

$\frac{m_{soft}}{L^2 \Lambda^3}$ dimensionless parameter controlling the transition

Similar story holds for other gauge groups. [EP, Schaefer, Unsal, 1212.1238]

For SU(N_c>2), 1st order, as known from the lattice; e.g.:

contours of constant V(Polyakov loop) for SU(3) (Z_3 center):



L>Lc center-symmetric L<Lc center-broken

Apart from correct order of deconfinement transition, the theta-angle dependence of T_c, recently studied on the lattice [D'Elia, Negro 1205.0538] is also correct. Theta dependence of T_c occurs because monopole-instantons carry topological charge, physics: "topological interference"... T_c(theta) first seen by Unsal in `deformed' QCD (2012)

(for theta-dependence at T>0 above and below T_c, see Zhitnitsky(w/ Parnachev/Thomas 2000/9))

[theta-dependence for SU(N_c): Mohamed Anber 1302.2641]

Extensions...

Theories without center symmetry: pure G_2 YM ... or QCD?

I. pure G_2 (s)YM small-L: semiclassical result vs. lattice

[EP, Schaefer, Unsal 1212.1238]

both show discontinuous change of Polyakov loop, without symmetry breaking

just below transition   just above transition

$\langle \mathrm{tr}\Omega \rangle = -0.15\ \frac{g^2}{4\pi}$ $\langle \mathrm{tr}\Omega \rangle = 3.21\ \frac{g^2}{4\pi}$

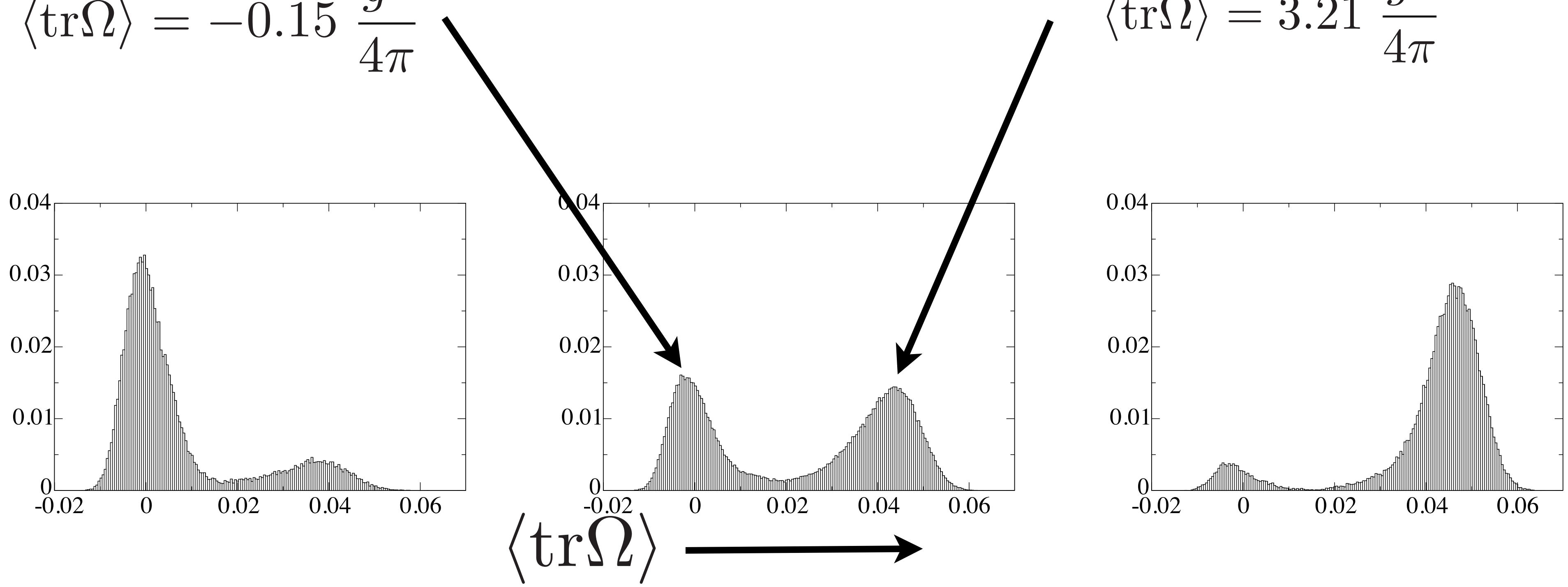


Figure 4: Polyakov loop probability distributions in the region of the deconfinement

lattice study of G_2 [Pepe, Wiese 2006; Cossu et al. 2007]

Extensions...

Theories without center symmetry: pure G_2 YM ... or QCD?

II. towards QCD?

[EP, Sulejmanpasic...in progress] 1307.xxxx

- take $SU(N_c)$ SQCD with N_f fundamental flavors on $R^3 \times S^1$ of size L
- take vector supermultiplet periodic and N_f flavors antiperiodic (w/"real masses")
- turn on gaugino mass, scalar mass induced by "gaugino mediation"
- limit of infinite gaugino mass

= thermal T=1/L QCD with N_f flavors of fundamental fermions

what does this theory "do"? is it calculable at small L? is it center symmetric?

- quarks do not respect center - on $R^3 \times S^1$ seen by the fact that different monopoles-instantons have different fundamental zero modes

  [Nye-Singer index (2000), Unsal, EP (2008)]

- zero quark mass SQCD on $R^3 \times S^1$ not calculable at $N_f>0$...

  ... various - often strongly coupled - dual descriptions (incl. "Aharony dualities", etc...)

- but finite-M calculable:

Extensions...

Theories without center symmetry: pure G_2 YM ... or QCD?

II. towards QCD?

[EP, Sulejmanpasic...in progress] 1307.xxxx

- finite quark mass calculable: M and KK contribute to superpotential

- one-loop fermionic and bosonic nonzero-mode determinants around monopole-instantons do not cancel, but instead related to "index function" (see Unsal, EP '08)
  - fermion-boson density of continuum states do not match (e.g., Kaul/E.Weinberg 1970s) -

  the ratio of one-loop dets is thus exactly calculable in M and KK backgrounds

- thus, the relation between monopole superfield Y (W=Y + 1/Y) and holonomy "deformed" by quarks; holonomy vev shifted away from center symmetry:

$$\langle \mathrm{tr}\, \Omega \rangle \sim N_f \frac{g^2}{(2\pi)^{3/2}} \frac{e^{-ML}}{\sqrt{ML}}$$

- leading term at large M; SUSY limit, at small L - cf. exp(-M/T) [correlator - string breaking behavior]

- at least up to quark masses > dual photon mass topological excitations same, but "deformed" - precise range & details of "deformation" can be found numerically

Extensions...

Theories without center symmetry: pure G_2 YM ... or QCD?

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- with nonzero small SUSY breaking, there is a calculable transition from the small Polyakov loop regime to one where it is O(1), similar to G_2 - but smooth, for SU(2) at least - as seen on lattice, always done with finite quark mass.

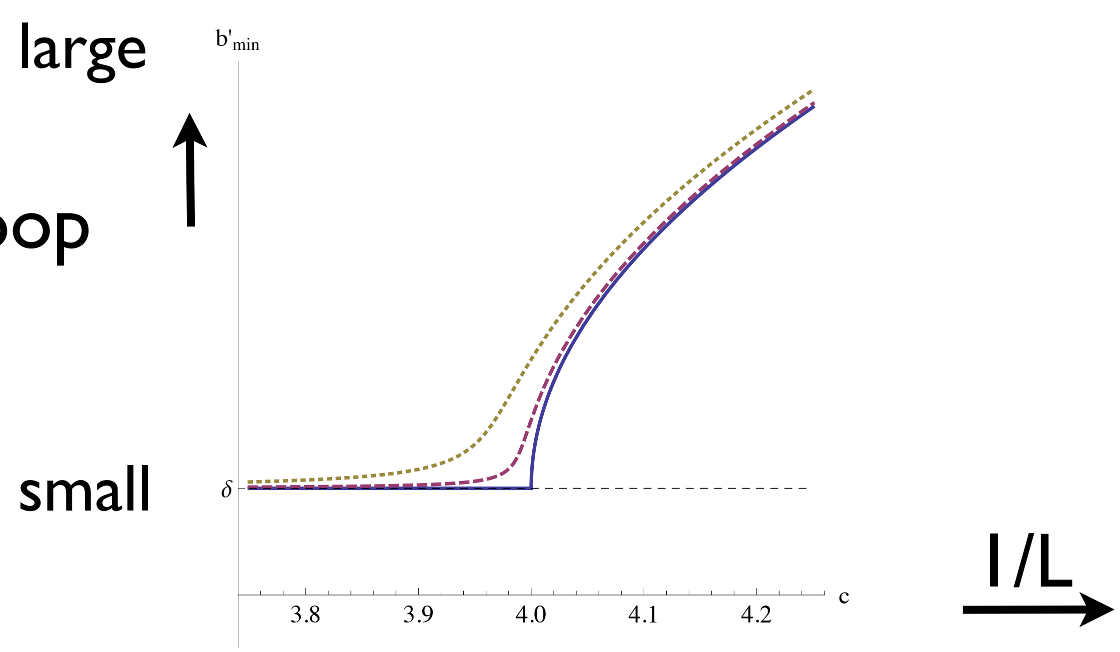

Figure 1. The minimum of b', proportional to the Polyakov loop trace, $\mathrm{tr}\,\Omega \approx \frac{g^2}{4\pi} b'$, as a function of the

[in correlator, see string breaking behavior]

- for small physical quark mass, however, calculable semiclassical picture breaks down - always before quarks are to contribute to the long-range instanton-monopole binding into bions

SUMMARY

Main message:

SYM with soft masses on a (non-)thermal S^1 provides a theory laboratory allowing study of deconfinement transition, at infinite V, in a controlled setting: a quantum phase transition appears continuously related to the thermal deconfinement one.

In particular:

It appears, from the examples we studied, that, quite generally, deconfinement occurs due to a competition between center-stabilizing topological molecules ("neutral bions") and center-breaking monopole-instanton and perturbative contributions.

The various topological objects' contributions can be computed using SUSY, or via the BZJ prescription. The latter helps identify them in non-SUSY theories - but no semiclassical limit where they dominate exists there . However, monopole-instanton-liquidmodels of deconfinement can be constructed, studied, and compared with lattice data...

Shuryak, Sulejmanpasic - "excluded volume", instead of BZJ...

Ways to go? *- some concrete things and some throwaway questions...*

Short(ish) term:
To understand other calculable cases with "quarks", e.g. with "baryon chemical potential" (= imaginary Wilson line for U(1)_Baryon)...

Our results lead to/support the conjectured phase diagram. The entire m-L plane in pure SYM can be studied on the lattice with current technology and future effort.

In particular, "topological" (with Qtop=0) "molecules" in pure YM - via defect localization of probe Dirac eigenmodes on the lattice - not a dilute gas, likely
[e.g., Bruckmann, Kovacs, Schierenberg 2011]

Relation to various bions to R^4 center vortices/monopoles
- Abelian projection vs. Poisson duality? As in Seiberg-Witten? [EP, Unsal 2011]
Can one make precise?

Is there a relation (precisely what?) between the streamline and BZJ prescription in SYM/SQCD?
- Yung's 1990 (heroic, in my view) calculation in R^4 SQCD, now for M-M*, KK-KK*, etc.?

Do the Seiberg/Aharony type dualities shed any light on deconfinement?