Fractional Turbulence Modeling

$$\left\langle r^2 \right\rangle = \overline{C}\,\varepsilon \Delta t^3, \qquad \left\langle (\Delta u)^2 \right\rangle \propto \Delta t.$$

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- Kolmogorov turbulence in the inertial range corresponds to normal diffusion for the velocity increments and super-diffusion for the displacement increments.

$$\frac{\partial P}{\partial t} + \gamma(-\Delta)^{1/3} P - \upsilon \Delta P = 0; \quad \gamma = (\overline{C}\,\varepsilon/2)^{1/3}$$

- **Intermittency** for finite Reynold number turbulence:
- inertial interactions + molecular viscosity

$$P(k,t) = \exp(-\gamma |k|^{2/3} - \upsilon |k|^2)t$$

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \cdot \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho}\nabla \overline{p} + \upsilon \Delta \overline{u}_i - \frac{\partial}{\partial x_j}\left\langle \widetilde{u}_i \widetilde{u}_j \right\rangle, \qquad \frac{\partial}{\partial x_j}\left\langle \widetilde{u}_i \widetilde{u}_j \right\rangle = \gamma(-\Delta)^{1/3}\overline{u}_i$$

- **Reynolds equation** model with fractional derivative; fractional Laplacian reflects self-similarity & guarantees positive definiteness of energy dissipation