LETTERS

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Simulations of turbulent thermal convection

L. Sirovich, S. Balachandar, and M. R. Maxey
Center for Fluid Mechanics, Turbulence, and Computation, Brown University, Providence, Rhode Island 02912

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A direct numerical simulation of thermal convection between horizontal plane boundaries has been performed, at a Rayleigh number $\mathrm{Ra} = 9800\ \mathrm{Ra}_c$, where Ra_c is the critical Rayleigh number for the onset of convection ($\mathrm{Pr} = 0.72$). The flow is found to be fully turbulent and analysis of the probability distributions for temperature fluctuations indicates that this is within the "hard turbulence" regime, as defined by the Chicago group. Good agreement is shown to exist between their experiments and the present simulation.

Thermal convection has been the subject of active research as a result of its fundamental role in many applications. The complex nature of such coherent structures as thermal plumes and convective rolls, makes the problem of turbulent convection at high Rayleigh numbers (Ra) particularly interesting. The experimental investigations of turbulent convection by Deardorff and Willis[1] and by Krishnamurti[2] are among the most detailed. Several studies[3–5] using direct numerical simulations have been carried out for Ra up to 3.8×10^5.

Recent experiments by Heslot *et al.*[6] and by Castaing *et al.*[7] (the Chicago group) at very high Ra has prompted new interest. They observed that at sufficiently high Ra the turbulent flow underwent a transition in its structure from *soft turbulence to hard turbulence*. In particular, the probability distribution of temperature fluctuations in the core region shows a switch from a Gaussian to an approximately exponential distribution. Another significant change is in the relation of the Nusselt number to the Ra. In this Letter we present a first report on a direct numerical simulation of thermal convection at high Ra, which supports the existence of a hard turbulence range.

The experiments of the Chicago group were carried out in a vertical circular cylinder of unit aspect ratio. Heat flux boundary conditions were applied at the top and bottom and the side walls were insulated. Viscous no-slip velocity conditions applied on all the boundaries. By contrast the simulation we report on was carried out with specified temperatures and slip boundary conditions at square horizontal bounding planes and aspect ratio $2\sqrt{2}$. The fluid was taken to be periodic in both horizontal directions.[8] This permits a Fourier decomposition in all three directions.

Therefore the two cases are different in regard to boundary conditions and geometry. To relate the two phenomena we assume that the relevant active parameter is the ratio, $r = \mathrm{Ra}/\mathrm{Ra}_c$, where Ra_c is the critical Ra number for instability of the conduction regime. If the Ra at which hard turbulence first appears is denoted by Ra_h, then it is found[7] that $r_h = \mathrm{Ra}_h/\mathrm{Ra}_c = 4\times 10^7/5.8\times 10^3 = 6.9\times 10^3$. For our case $r = 9.8\times 10^3$ and therefore lies in the hard turbulence regime. Power law scalings for a variety of physical quantities were also found.[7] For example, the Nusselt number is given by $\mathrm{Nu}\sim Cr^{\beta}$, where $\beta\approx 0.282$ for hard turbulence,[6,7] and is given by the classical value[9] of $\beta = \frac{1}{3}$ for soft turbulence $r < r_h$. Since the coefficient C may be a strong function of geometry and boundary conditions, we will not comment on this aspect of the results. Work by Yakhot further supports these results. Yakhot has analyzed the transition from soft to hard turbulence in the framework of renormalization group (RNG),[10] and more recently presented a theoretical argument accounting for the structural changes that occur in the transition,[11] see also the recent discussion by She.[12]

We first comment on the resolution of the numerical simulation. If H denotes the plate spacing and δ the thermal sublayer, then the Nusselt number is given by $\mathrm{Nu} = H/2\delta$. For the present simulation $\mathrm{Nu} = 23.7$. The computation was performed on a $(96)^3$ lattice and at least two grid spacings occur in the thermal sublayer. Since this is a linear zone, two spacings were deemed more than adequate. Our simulation yielded a Kolmogorov length microscale of 0.008, based on $\eta^* = (\nu^3/\epsilon)^{1/4}/H$, where ϵ is the average value of dissipation. A conservative condition[13] to meet the requirements of spatial resolution in thermal convection is that $\pi\eta^* H/h > 1$, where h is an average grid spacing. In our case, this ratio has a value of 2.4. The ratio of the Kolmogorov time scale $(\nu/\epsilon)^{1/2}$ to the time step Δt used in the simulation is 67. Thus flow scales are adequately resolved spatially and temporally. Further confirmation came from inspection of the energy content of the Fourier modes, which at least show three to four orders of decay in the energy levels and indicate small-

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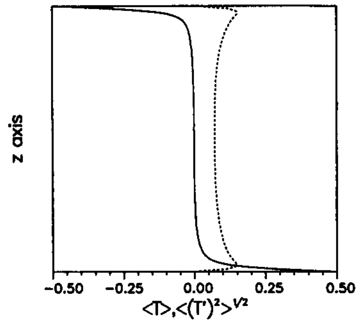

FIG. 1. Profiles of mean and fluctuating rms values: (a) mean temperature $\langle T \rangle$ (solid line), rms fluctuation $\langle T'^2 \rangle^{1/2}$ (broken line).

scale isotropy in the horizontal velocities. In Fig. 1 the mean temperature profile and rms temperature fluctuation are plotted versus layer height. The slight vertical temperature reversal observed[8] at lower Ra is absent, which is consistent with experiments.[1] From the figure we infer that there are roughly five grid spacings within the full thermal boundary layer region (90% of the core temperature) resolving well the rapid changes in the temperature profiles.

Flatness factor, skewness factor, Peclet number, Taylor microscale, and microscale Reynolds number based on vertical velocity gradients, evaluated at the midplane, are listed in Table I. Also given are those corresponding to the lower Rayleigh number of 70 Ra_c reported earlier.[8] Predictably, all the above parameters, except the Taylor microscale, increase with increasing Rayleigh number. The values for derivative skewness and flatness factors are consistent with those found in experiments.[14] Such higher-order statistics give further support to the soundness of our numerical resolution. The horizontal dissipation length scale is $0.8H$ (based on $\langle u^2 \rangle^{3/2}/\epsilon$).

Based on our results, we now describe a scenario, for the heat transfer mechanism, consistent with the experimental results of Castaing *et al.* First we consider Fig. 2, which shows quadrant plots of vertical velocity w against temperature fluctuation T', using an approach similar to that of Adrian.[15] These are based on all the grid points in three horizontal planes at one instant of time. (Note that $\langle w \rangle$ and $\langle T' \rangle$ are zero.) The structure of these *sluglike* plots changes considerably from close to the bottom to the center of the cell. The I and the III quadrants represent an upward positive heat flux and the II and the IV quadrants negative values. Close to the cell bottom the distribution of points in the I and III quadrants are symmetrically placed and the heat transport by upward moving, hot fluid is as probable as the heat transport by downward moving cold fluid. Slightly away from the bottom this symmetry is broken and Fig. 2(b) shows that there are a large number of small negative T' excursions countered by a small number of large positive T' fluctuations. The probability distribution of T' is skewed toward positive values, with a maximum at some T' less than zero and a long positive tail. At the center of the cell Fig. 2(c) the symmetry is restored. The T' fluctuations are

TABLE I. Taylor microscale, microscale Reynolds number, and other statistics evaluated at the midplane, $z = H/2$. Data are presented for both the present simulation and for the earlier[8] simulation at Ra = 70 Ra_c.

	Present	Ra = 70 Ra_c
$\lambda_w = \left[\frac{\langle w^2 \rangle}{\langle (\partial w/\partial z)^2 \rangle}\right]^{1/2}$	0.26	0.49
$\mathrm{Re}_{\lambda_w} = \langle w^2 \rangle^{1/2} \lambda_w / \nu$	117.0	38.1
$\frac{\langle (\partial w/\partial z) \rangle^3}{\langle (\partial w/\partial z)^2 \rangle^{3/2}}$	0.67	0.65
$\frac{\langle (\partial w/\partial z)^4 \rangle}{\langle (\partial w/\partial z)^2 \rangle^2}$	5.22	4.85

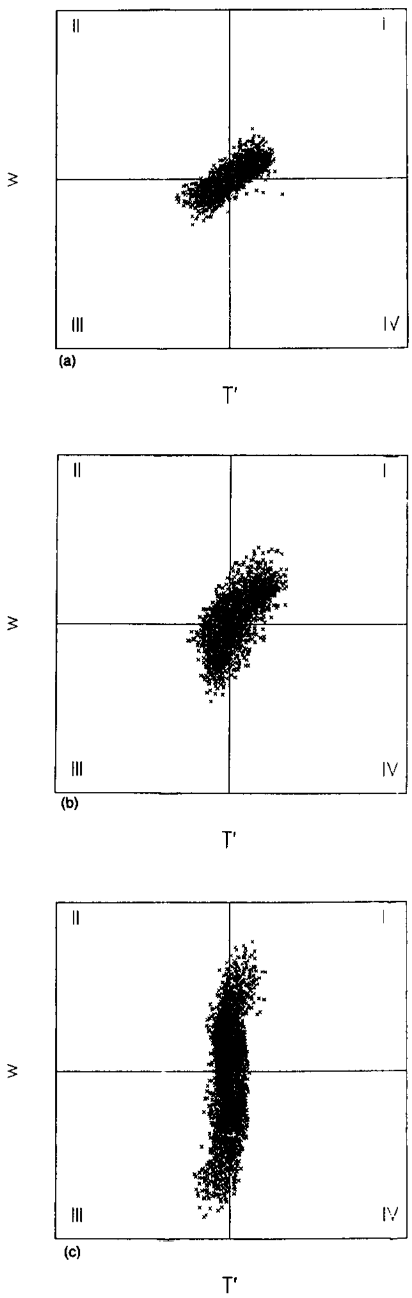

FIG. 2. Quadrant plots of fluctuating temperature T' and vertical velocity w from all grid points in a horizontal plane at one instant: (a) $z/H = \frac{2}{96}$; (b) $z/H = \frac{6}{96}$; (c) $z/H = \frac{48}{96}$.

smaller and the vertical velocity fluctuations are larger and likely have a bimodal distribution.

Further analysis of the data shows that the temperature fluctuations have a positive skewness from the region above the lower conductive sublayer up to the midplane. At the midplane the correlation coefficient for w and T' is only 0.55, compared to a value of 0.71 found at 575 Ra_c[4] and 0.76 found at 70 Ra_c.[8] These observations and further analysis of the quadrant data suggest three basic regions. In particular, they indicate the existence of narrowly confined, strong upward moving regions of warm fluid surrounded by larger areas of weak, downward moving cool fluid. These upward moving regions of warm fluid are found to dominate the heat transfer in the lower half of the layer.

Within the wall sublayer conduction is dominant and at the edge of the sublayer, the conductive heat transfer equals the convective heat transfer. Also at the edge $\langle T'^2 \rangle$ reaches a maximum. An isothermal surface at the edge of sublayer will be irregular, with a balance of positive and negative patches corresponding to the approximate balance of both positive and negative temperature fluctuations. Above this sublayer is a second region, about six mesh points thick in our simulation, where a predominance of hot tongues of fluid develops in otherwise cooler ambient surroundings. Analysis of the points shown in Fig. 2(b) shows that in this region the number of points, and hence horizontal area, where $T' > 0$ (quadrants I and IV) reaches a minimum while the number of points where $T' < 0$ (quadrants II and III) reaches a maximum. The heat transfer is dominated by these narrowly confined, rising tongues of warm fluid. At the edge of this *mixing region*,[7] the heat conduction is zero. Beyond this is a third region, which is the central mixed zone, where the mean temperature is uniform. At the midplane $\langle T'^2 \rangle$ has a minimum. As the level of T' decreases the level of w increases to maintain the relatively constant heat flux. The fraction of horizontal area where $T' > 0$ increases toward the midplane indicating a broadening of the warm tongues or plumes, and possibly a pinching off of the tongues into discrete thermals.

The most striking feature of the hard turbulence regime is found in the probability distribution for temperature fluctuations. In Fig. 3 we show this distribution at three levels in the cell. At the edge of the thermal sublayer this shows the expected tendency to negative fluctuations. At the intermediate stage, the distribution is still skewed toward negative fluctuations but a peak is clearly present. Finally at the midplane a symmetric, exponential-like distribution appears. The experiments reported by Castaing *et al.*[7] fall close to the last curve, however, since their measurements were taken at an off-center location their distribution is asymmetric. Recent results[16] measured at the center of the cell show a symmetric form. (Exponential probability distributions have been observed in past turbulence investigations, e.g., Sreenivasan *et al.*[17]) If the distribution shown in Fig. 3 is fitted by back to back exponentials

$$p(T/\Delta_c) \propto \exp(-\alpha|T/\Delta_c|), \qquad (1)$$

then we obtain $\alpha \approx 1.25$. Our estimate of the Chicago group value is $\alpha \approx 1.2$. Yakhot,[11] using a theoretical model, has obtained a preliminary value of $\alpha \approx 1.1$.

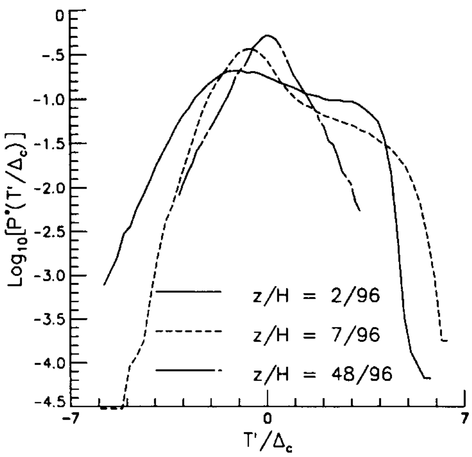

FIG. 3. Probability distribution for local normalized temperature fluctuations at $z/H = \frac{2}{96}$, $z/H = \frac{6}{96}$, and $z/H = \frac{48}{96}$. Here, $\Delta_c = 0.084\,\Delta T$, is the box averaged value of T'_{rms}.

Aside from the difference in geometries, the most essential contrast in the two studies lies in the role played by what the Chicago group refer to as the wind. They find a symmetry breaking in their experiment, which results in a sustained circulation about a relatively fixed horizontal axis. This they assert is responsible for the *breaking* off of the plumes at the boundaries and accounts for the observed preferred frequency of oscillation in the cell.

We find no such sustained *wind* in our simulation. But an analysis of the data by means of the method of empirical eigenfunctions[18,19] shows that roughly 25% of the energy of the motion is contained in a cellular mode. The rolling cellular motion described by this mode *jitters* horizontally, reverses direction, and rotates by 90° at seemingly random time. In our normalization the time scale of this rolling motion is roughly 3×10^{-3}. Our estimate for the formation and breaking of plumes is roughly 5×10^{-4}. (Present records are not long enough to give more than these rough figures.)

In conclusion, the present simulation captures thermal convection in the hard turbulence regime. The numerical results suggest the existence of three distinct domains, a thin thermal sublayer close to the boundaries, an intermediate region of hot (and cold) plumes, and a central turbulent zone. A possible scenario for the evolution of the thermal plumes may be as follows: the hot and cold fluids from their respective sublayers accumulate at random localized spots forming hot and cold plumes extending like fingers from the sublayer surface. These plumes grow and move around in time. At some stage the prevailing circulation pinches these off from the sublayer. They then accelerate up into the central zone, where they mix with the surrounding fluid by turbulent mixing. Though this picture is well supported by the Chicago experiments further work needs to be done in order to verify and to understand the dynamics of these coherent structures and their importance in the overall heat transfer mechanism.

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