# Optimal driving of autonomous vehicle platoons on arterial streets to reduce fuel consumption

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#### Abstract

Traffic signals, while serving an important function to coordinate vehicle movements through intersections, also cause frequent stops and delays, particularly when they are not properly timed. Such stops and delays contribute to significant amount of fuel consumption and greenhouse gas emissions. The recent development of connected and automated vehicle (CAV) technology provides new opportunities to enable better control of vehicles and intersections, that in turn reduces fuel consumption and emissions. In this paper, we propose a trajectory optimization method, PTO-GFC, to reduce the total fuel consumption of a CAV platoon through a signalized intersection. In this method, we first apply platoon-trajectory-optimization (PTO) to obtain the optimal trajectories of the platoon vehicles. In PTO, all CAVs in one platoon are considered as a whole, that is, all other CAVs follow the trajectory of the leading one with a time delay and minimum safety gap, which is enabled by vehicle to vehicle communication. Then, we apply gap-feedback-control (GFC) to control the vehicles with different speeds and headways merging into the optimal trajectories. We compare the PTO-GFC method with the other two methods, in which the leading vehicle adopts the optimal trajectory (LTO) or drive with maximum speed (AT), respectively, and the other vehicles follow the leading vehicle with a simplified Gipps' car-following model. Furthermore, we extend the controls into multiple platoons by considering the interactions between the two platoons. The numerical results demonstrate that PTO-GFC has better performance than LTO and AT, particularly when CAVs have enough space and time to smooth their trajectories. The reduction of travel time and fuel consumption shows the great potential of CAV technology in reducing congestion and negative environmental impact of automobile transportation.

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reduce fuel consumption

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Traffic signals, while serving an important function to coordinate vehicle movements through intersections, also cause frequent stops and delays, particularly when they are not properly timed. Such stops and delays contribute to significant amount of fuel consumption and greenhouse gas emissions. The recent development of connected and automated vehicle (CAV) technology provides new opportunities to enable better control of vehicles and intersections, that in turn reduces fuel consumption and emissions. In this paper, we propose a trajectory optimization method, PTO-GFC, to reduce the total fuel consumption of a CAV platoon through a signalized intersection. In this method, we first apply platoon-trajectory-optimization (PTO) to obtain the optimal trajectories of the platoon vehicles. In PTO, all CAVs in one platoon are considered as a whole, that is, all other CAVs follow the trajectory of the leading one with a time delay and minimum safety gap, which is enabled by vehicle to vehicle communication. Then, we apply gap-feedback-control (GFC) to control the vehicles with different speeds and headways merging into the optimal trajectories. We compare the PTO-GFC method with the other two methods, in which the leading vehicle adopts the optimal trajectory (LTO) or drive with maximum speed (AT), respectively, and the other vehicles follow the leading vehicle with a simplified Gipps' car-following model. Furthermore, we extend the controls into multiple platoons by considering the interactions between the two platoons. The numerical results demonstrate that PTO-GFC has better performance than LTO and AT, particularly when CAVs have enough space and time to smooth their trajectories. The reduction of travel time and fuel consumption shows the great potential of CAV technology in reducing congestion and negative environmental impact of automobile transportation.

#### 1. Introduction

Transportation is a major consumer of non-renewable energy. In 2018, the U.S. transportation sector alone consumed over 143 billion gallons of motor fuel, and it is predicted that the fuel consumption in transportation in the U.S. will remain at a high level in the foreseeable future (EIA, 2019). Furthermore, the world consumption of transportation fuel is forecast to increase significantly with a steady increase in vehicle ownership as incomes in developing countries rise (Sperling and Gordon, 2010). There has been a practice of the so-called eco-driving among environmentally conscious drivers, which tries to avoid hard accelerations and decelerations based on real-time driving conditions, particularly on urban streets with numerous traffic lights (af Wåhlberg, 2007; Delhomme et al., 2013; Pampel et al., 2018). This practice was shown

to reduce personal fuel consumption, but without the advance knowledge of traffic signal status, the practice is based on ad hoc rules and furthermore, its impact on other drivers, and hence at a system level, is not certain. Fortunately, the rapidly evolving connected and autonomous vehicle (CAV) technology can overcome these limitations of eco-driving through better communication and greater vehicle control, and hence provides a powerful tool to reduce both fuel consumption and greenhouse gas emissions more effectively (Mahmassani, 2016; Taiebat et al., 2018; Wang et al., 2018).

In the transportation system, intersections play a crucial role in assigning and controlling traffic flow. In many cases, traffic streams on arterial roads are controlled by traffic signals at intersections. Vehicles must stop at signals on red, which increases their fuel consumption, emission levels and travel time due to acceleration/deceleration maneuvers and idling required at traffic signals. In this paper, we first use platoontrajectory-optimization (PTO), to optimize CAVs moving through a signalized intersection as so to minimize the total fuel consumption of the platoon. In this method, we assume the CAV platoon knows the traffic light's schedule before entering the approach of the intersection, and consider all CAVs in one platoon as a whole. If not all CAVs in the platoon can pass through the intersection within one green light window, the platoon will split into several subplatoons and cross the intersection within successive traffic signal cycles. In one subplatoon, the trajectory of the leading CAV is copied by the other ones with minimum reaction time delay and minimum safety gap, enabled by V2V communication. As a result, we transform the problem that controls and optimizes multiple CAVs in one platoon into a problem that controls and optimizes the leading CAV in each subplatoon. To make the control operational, we apply gapfeedback-control (GFC) to generate modified trajectories to merge into the optimized trajectories and maintains them. We denoted the extended control method as PTO-GFC.

Besides, we also study the other two methods based on a simplified Gipps' carfollowing model (Gipps, 1981), i.e., leading-trajectory-optimization (LTO) and aggressive driving (AT). In LTO method, we suppose the leading vehicle is a CAV, and the others are human-driven vehicles. The strategy of the leading CAV is to minimize its fuel consumption with optimal control and pass the signalized intersection without considering the following vehicles. The human-driven vehicles travel across the intersection with a simplified Gipps' car-following model and stop before the intersection when the red light is on. In AT method, we suppose all vehicles in one platoon are human-driven. The leading vehicle travels with maximum speed and stops before the intersection until the green light is on. As similar as LTO, the other vehicles follow their preceding vehicles with a simplified Gipps' car-following model in AT. Furthermore, we apply the PTO-GFC method to control multiple platoons across a signalized intersection in consideration of the intersections between two platoons. A virtual trajectory generated based on the last CAV of the platoon in front is taken as a constraint of the back platoon to ensure safety. The results of case studies and sensitivity analysis demonstrate PTO-GFC outperforms LTO and AT in reducing both fuel consumption and travel time when the CAVs have enough space and traffic throughput to smooth their trajectories.

The rest of this paper is organized as follows. Section 2 reviews related literature.

Section 3 presents the results of optimizing one vehicle with optimal control. In Section 4, the frameworks of PTO-GFC and the other two methods, LTO and AT, are described. Case studies and sensitivity analysis are conducted to compare the performance of the three methods. In Section 5, we extend the three methods into multiple platoons. As similar as Section 4, we conduct case studies and sensitivity analysis in the multiple-platoon level. Section 6 concludes the paper and discusses some further research directions.

#### 2. Literature review

The basic idea to reduce fuel consumption and emission when vehicles pass through the signalized intersections is to avoid sharp acceleration/deceleration and idling as much as possible (Rakha and Kamalanathsharma, 2011; Wang et al., 2018). With the development of wireless communication technology such as vehicle-tovehicle (V2V) and vehicle-to-infrastructure (V2I) communications, it is becoming possible to better deploy and deliver eco-driving speed advisory messages to drivers or automated vehicles (AVs). These proposed speed advisory strategies such as Green Light Optimized Speed Advisory (GLOSA) (Mandava et al., 2009; Katsaros et al., 2011; Alsabaan et al., 2013; He et al., 2015; Wan et al., 2016) and Advisory-Speed Limit (ASL) (Liu et al., 2012; Yang and Jin, 2014; Ubiergo and Jin, 2016; Jiang et al., 2017; Yao et al., 2018) can smooth the vehicles' trajectories and save fuel. The speed advisory strategies are hierarchically constrained by the compliance ratio of the drivers and the ratio of the vehicles that equipped with the communication equipment. In general, they have better performance when more vehicles are equipped with communication equipment. Besides fuel consumption, many studies also take multiple factors including travel time, safety, comfort level and emissions into consideration (Dresner and Stone, 2008; Zhou et al., 2017; Ma et al., 2017; Zhou et al., 2019). For example, Zhou et al. (2017) and Ma et al. (2017) propose a parsimonious shooting heuristic algorithm to construct vehicle trajectories on a signalized highway segment with multiple factors included in the objective functions. Jin et al. (2016) developed a power-based longitudinal control algorithm for a connected eco-driving system in order to reduce fuel consumption and emissions under a variety of traffic conditions, including road grade, upcoming traffic signal status, and the preceding vehicle's state. In addition to optimize vehicles' trajectories under fixed signal timing, some studies also optimize traffic signal timing and vehicle's movements simultaneously as so to improve the traffic efficiency and reduce fuel consumption (Jung et al., 2016; Xu et al., 2018; Feng et al., 2018; Yu et al., 2018; Pourmehrab et al., 2019; Wang et al., 2019).

The above studies mainly focus on solving the problem of trajectory smoothing across a signalized intersection at the individual or multiple vehicle-level. According to Lioris et al. (2017), the potential mobility benefits of platooning with connected vehicle technology can double throughput in urban roads. Even though there are lots of literature about platooning in the highway transportation systems, especially for heavy-duty vehicles (Tsugawa et al., 2016), only limited studies focus on reducing platoon energy consumption from eco-driving speed advisory or control at signalized intersections. Chen et al. (2015) developed a speed control algorithm to optimize the

acceleration-deceleration profile for a platoon rather than only one vehicle. The optimization objective of the algorithm is to avoid drivers from idling and to let them clear the signalized intersection during the green light as often as possible. Wei et al. (2017) present a set of integer programming and dynamic programming models for scheduling longitudinal trajectories based on a space-time lattice. By adjusting the lead vehicle's speed and platoon-level reaction time at each time step, their framework can control the complete set of trajectories in a platoon efficiently. Stebbins et al. (2017) propose a trajectory optimization method by optimizing for the delay over the entire trajectory instead of suggesting an individual speed. Moreover, they extend the framework to platoon-level, in which other vehicles follow the leading vehicle with a car-following model. He and Wu (2018) developed an optimal control model to provide eco-driving advisory for the mixed-traffic platoon with electric vehicles and traditional gasoline vehicles. Two eco-driving advisory strategies, i.e., acceleration-based advisory strategy for automated leading vehicle and stepwise speed advisory strategy for the human-driven leading vehicle, were proposed in order to obtain the platoon energyoptimal speed trajectory. In both strategies, the following vehicles in the platoon follow their preceding vehicles automatically. Zhao et al. (2018) proposed a real-time cooperative eco-driving strategy for a platoon of vehicles with mixed automated vehicles and human-driven vehicles approaching a signalized intersection. In the strategy, the automated vehicles are assumed to be leaders whose trajectories are generated by a model predictive control (MPC) method, and the human-driven vehicles follow the leaders with car-following model. They found that the cooperation between AVs and human-driven vehicles can smooth the trajectories of the human-driven vehicles and reduce fuel consumption. Liu et al. (2019) proposed a CAV platoon trajectory planning approach to pass through a signalized intersection. In this method, they optimize throughput first, and then to maximize comfort while minimizing travel delay and fuel consumption. The vehicles in the platoon that cannot pass the signalized intersection in the first green time window should smoothly decrease and wait before the intersection until the traffic light is on green.

In the above mentioned platoon trajectory planning works, it is usually to optimize the leading vehicle and assume the other vehicles follow their preceding vehicles based on car-following models (Chen et al., 2015; Wei et al., 2017; Stebbins et al., 2017; He and Wu, 2018; Zhao et al., 2018). However, controlling multiple vehicles in one platoon is a hard problem. Even though the control objective in the above references is to optimize the performance of the platoon, their optimization did not apply to all the vehicles in one platoon together, which may fail to realize the full potential of CAV technology. In this paper, we develop a vehicle trajectory control framework for CAV platoons to reduce fuel consumption. To take advantage of V2V and V2I communication in a CAV traffic environment, traffic signal timing status is transmitted to the leading CAV vehicle before it enters the intersection, and the platoon leaves the intersection at free-flow speed (or the speed limit of the road), which serves at the final state condition for our formulated optimal trajectory control problem. In the method, we transform the problem that optimizes and controls the multiple vehicles in one platoon into a problem that optimizes and controls the leading vehicle of each

subplatoon with trajectory copying in order to minimize the total fuel consumption of a platoon approaching and passing through a signalized intersection. Moreover, we also apply a gap feedback control framework to deal with platoon vehicles with different speeds and headways. By applying the gap feedback control framework, the platoon vehicles with different speeds and headways can merge into the optimal trajectories and pass through the signalized intersection effectively. With optimization and control, the vehicles can form a tight platoon and pass through the signalized intersection with free-flow speed. Our approach first develops the optimal control policy for a single CAV, then extends it to a vehicle platoon, and finally designs a mechanism to control multiple platoons traversing a signalized intersection considering the interactions between platoons.

#### 3. Optimal control of one CAV

First, let us optimize the trajectory of one CAV with optimal control from location  $s_1$  to location  $s_f$  ( $s_f > s_1$ ) without traffic signal. Suppose, at time  $t_0$ , one CAV with maximum speed  $v_0$  travel at location  $s_1$ , and the vehicle must arrive at location  $s_f$  at the maximum speed of  $v_0$ . In this situation, we need to optimize one trajectory to minimize fuel consumption for the vehicle traveling from location  $s_1$  to location  $s_f$  with speed limit. The control space for the CAV is  $s_f - s_1$ . The framework for solving this problem can be presented as follows.

(1) System Model: For a single vehicle, state vector  $\mathbf{x}(t)$  is defined as,

$$\mathbf{x}(t) \triangleq [x_1(t) \ x_2(t)]^T = [s(t) \ v(t)]^T$$

where s(t) is the distance from  $s_1$ , and v(t) is the speed of the vehicle. Those two variables denote the state of the vehicle. The control vector only contains one variable, i.e., the acceleration rate, which is defined as,

$$\mathbf{u}(t) \triangleq [a(t)]^T$$

Therefore, the dynamics of the system can be described with differential equations,

$$\dot{\mathbf{x}}(t) \triangleq \begin{pmatrix} \dot{x}_1(t) = v(t) \\ \dot{x}_2(t) = a(t) \end{pmatrix}$$

(2) Optimal Control Problem Formulation: The problem of controlling the CAV is formulated to minimize the fuel consumption as follows,

$$J = \int_{t_0}^{t_1} c(v(t), a(t)) dt,$$

where  $t_0$  and  $t_1$  are the corresponding time points at locations  $s_1$  and  $s_f$ , respectively; c(v(t), a(t)) is an instantaneous fuel consumption model presented at the Conference of Australian Institutes of Transportation Research (CAITR) (Akcelik and Besley, 2003; Liu et al., 2012)

The above optimal control problem is challenging to find analytical solutions (Liu et al., 2012). Instead, the numerical Gauss pseudospectral method (GPM) is used to discretize a continuous optimal control problem into a mixed integer nonlinear program (MINLP) and obtain the optimal solution. The technique is an orthogonal collocation method where the collocation points are the Legendre-Gauss (LG) points (Benson,

2005). Here, we employ the General Algebraic Modeling System (GAMS) to obtain the optimal control solution (Andrei and Andrei, 2013).

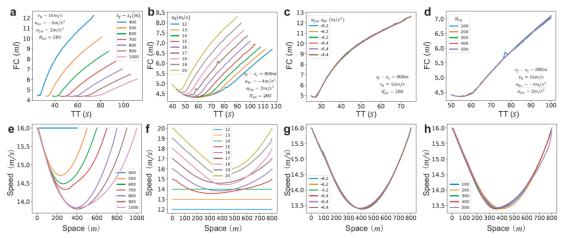


Figure 1: Optimal control outcomes of one CAV. (a-d) The relationship between optimal fuel consumption per 100 meters and travel time; (e-h) The relationship between travel speed and space for optimal trajectories with lowest fuel consumption.

Figure 1 presents the optimal results with different travel distance, maximum speed, deceleration/acceleration constraint, and LG points. Figure 1(a)-(d) show the relationship between optimal fuel consumption (FC) and travel time (TT). Figure 1(e)-(h) show the optimal trajectories with lowest fuel consumption. At a given maximum speed, as shown in Figure 1 (a, e), it is better for a vehicle to keep a constant speed when traveling through a short distance, e.g., 400m. However, as the increase of the travel distance, the corresponding travel time with the lowest fuel consumption is a little longer than the shortest travel time with constant maximum speed. In this case, the optimal fuel consumption decreases firstly and then increases over travel time. The CAV traveling with lowest fuel consumption needs to decelerate firstly, and then gradually accelerate to maximum speed. It is a bit counterintuitive at first because it is generally believed that keeping a constant velocity would consume less fuel in contrast to a trajectory with speed variations. When the acceleration  $a \ge 0$ , even though it has a high impact on the fuel consumption in the third term, vehicle speed v(t) dominates in both the second and third terms. The implication is that the effects of the lower speed could offset the impact of high acceleration rate on fuel consumption. Hence, as shown in Figure 1 (b, f), when the maximum speed is relatively low, e.g., 14m/s, keeping traveling in a constant speed is in favor of reducing fuel consumption at a given travel distance. Besides, we find the deceleration/acceleration constraint and the number of LG points do not have a significant influence on the performance of optimal control. Therefore, in the following sections, we set the maximum brake deceleration as  $a_{br} =$  $-4m/s^2$ , maximum acceleration as  $a_{fw} = 2m/s^2$  and the number of LG points as  $N_{LG}=200.$ 

#### 4. Platoon optimization

#### 4.1 The framework of PTO-GFC method

Based on the optimal control framework for one vehicle described in the above section, we propose the PTO method to optimize one platoon across a signalized intersection by considering all CAVs in the platoon as a whole. The components of the PTO method are described as follows.

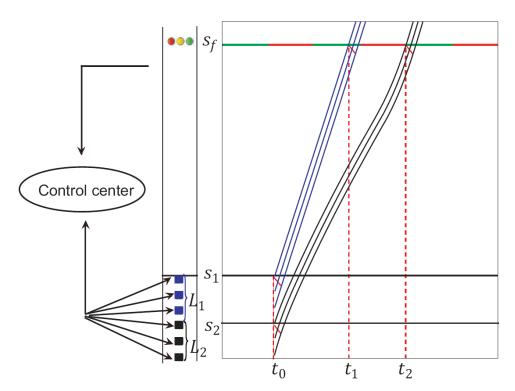


Figure 2: Illustration of control framework of PTO method.

**Road:** We only consider one single lane leading to a signalized intersection. The leading CAV in one platoon enters location  $s_1$  and arrives location  $s_f$  with maximum speed  $v_0$ . The traffic signal is installed at location  $s_f$ .

**Traffic Signal:** The traffic signal we consider here is a fixed signal timing including a sufficient length of G and an effective red time of R. Thus, the cycle length of the traffic signal is C := G + R.

**Platoon**: The number of CAVs in one platoon is N. The initial time of the leading CAV arriving at location  $s_1$  is  $t_0$ . The minimum reaction time of CAV is  $\tau_0$  and the minimum gap between the two vehicles is d. Suppose the initial speed and headway of the following CAV follow the uniform distributions, i.e.,  $v(t_0) \sim v_0 - \alpha * U(0,1)$  and  $\tau(t_0) \sim \tau_0 + \beta * U(0,1)$ , where  $v_0 \geq \alpha \geq 0$ ,  $\beta \geq 0$  and U(0,1) is the standard uniform distribution. According to the initial condition of all CAVs, we can obtain the length of the platoon L.

**Platoon splitting:** If not all CAVs in one platoon can pass through the intersection within one green time window, then the platoon will split into several subplatoons and cross the intersection within successive green time windows.

**Trajectory copying:** The basic idea of PTO method is that all vehicles in one subplatoon can copy the trajectory of the leading vehicle with minimum reaction time

delay and minimum gap delay.

Through platoon splitting and trajectory copying, we transform the problem that optimizes and controls multiple vehicles in one platoon to optimize and control the leading CAVs of the subplatoons. Figure 2 illustrates the operations of controlling one platoon across a signalized intersection with PTO. The platoon is composed of six CAVs. Yet, they cannot pass the intersection within one green light window. Therefore, to minimize the total fuel consumption of all vehicles, the platoon splits into two subplatoons at time  $t_0$ . The locations of the leading vehicle in Subplatoon 1 (blue trajectories) and Subplatoon 2 (black trajectories) are  $s_1$  and  $s_2$ , respectively. Then, the control space for Subplatoon 1 and Subplatoon 2 are  $s_f - s_1$  and  $s_f - s_2$ , respectively.

The total fuel consumption of one platoon with PTO method across a signalized intersection can be formulated as,

$$J_{p} = \sum_{m=1}^{M} N_{m} J_{m},$$

Where  $N_m$  ( $N_M > 0$ ) and  $J_m$  are the number of CAVs and the fuel consumption of one CAV in Subplatoon m, respectively. Substituting  $J_m$  with fuel consumption model, we can obtain,

$$J_{p} = \sum_{m=1}^{M} N_{m} \int_{t_{0}}^{t_{m}} c(v(t), a(t)) dt,$$

Where  $t_0$  and  $t_m$  are the starting time and ending time of optimizing the leading CAV in Subplatoon m, respectively.

The platoon optimization of passing a signalized intersection is to minimize  $J_p$  with all CAVs traveling across the intersection in green light windows. The constraints of guaranteeing all CAVs crossing the intersection with maximum speed  $v_0$  within successive green light windows can be described as,

$$\begin{cases} t_m \backslash C \leq G \\ (t_m + L_m/v_0) \backslash C \leq G \\ \lfloor t_m/C \rfloor = \lfloor (t_m + L_m/v_0)/C \rfloor \\ \lfloor t_m/C \rfloor = \lfloor t_m/C \rfloor + 1 \\ 0 \leq v \leq v_0 \\ a_{br} \leq a \leq a_{fw} \\ v(t_m) = v_0 \\ s(t_m) = s_f \end{cases}$$

The first three equations can guarantee all CAVs in Subplatoon m crossing the intersection in one green light window; the fourth equation can ensure all subplatoons pass through the intersection at successive traffic signal cycles; the fifth and sixth equations are the restricted conditions for speed and acceleration of all CAVs, the last two equations can ensure the leading CAV of each subplatoon pass through the intersection with maximum speed  $v_0$ .

Up to now, we have proposed a general framework to make the CAV platoon

traveling with the least fuel consumption. Although the framework serves as a starting point for optimal platoon control, it assumes that the platoon is already formed before the optimization process. To supplement PTO control, we introduce a gap feedback control (GFC) to form the initial platoon and make it to track the optimal platoon trajectory.

$$a(t) = a^*(t) + K_v(v^*(t) - v(t)) + K_s(s^*(t) - s(t)),$$

where  $K_v$ ,  $K_s$  are the two positive parameters that determines convergence speed of the GFC;  $a^*(t)$ ,  $v^*(t)$ , and  $s^*(t)$  are the acceleration, speed and location in the tracked optimal trajectory. The transfer function for the GFC is given by H(s) = 1. Therefore, the acceleration of the modified trajectory will equal to the acceleration of the optimal trajectory through long enough adjustments by using the GFC. The constraints for acceleration and speed also be applied to make sure the feedback acceleration is reasonable. In combination of PTO and GFC, we have the framework PTO-GFC, which can deal with a platoon of CAVs that have different initial speeds and headways.

#### 4.2 Two other methods for comparison

We compare our trajectory optimization framework PTO-GFC with two other methods that adopt a simplified Gipps' car-following model, namely leading-trajectory-optimization (LTO) and aggressive-trajectory (AT). In the LTO method, we assume the leading vehicle in a platoon is a CAV, and optimize its trajectory based on the optimal control framework. The other vehicles in the platoon are human-driven ones and follow their preceding vehicles with the simplified Gipps' car-following model (Gipps, 1981; Treiber and Kesting, 2012; Ubiergo and Jin, 2016). If the following vehicles arrive at the signalized intersection in red, they need to stop until the green light is on. For the AT method, we assume there is no CAV in the platoon, and the leading vehicle travel from  $s_1$  to  $s_f$  with maximum speed  $v_0$ . If the leading vehicle arrives at the signalized intersection in red, it is forced to wait until the green light is on; otherwise, it travels through the intersection with maximum speed. The other vehicles in the platoon follow their preceding vehicle with the simplified Gipps' car-following model and stop if the red light is on.

#### 4.3 Case study

In this section, we conduct one case study to illustrate the performance of our proposed platoon optimization method PTO-GFC and the other two methods, LTO and AT. The parameters in the case study are set as follows: enter location  $s_1 = 0$ , traffic signal location  $s_f = 800m$ , maximum speed  $v_0 = 16m/s$ , CAV minimum reaction time  $\tau_0 = 1s$ . Moreover, even though it is generally assumed that the CAVs have shorter reaction time than human drivers, here we assume the human drivers have the same reaction time as the CAVs, i.e.,  $\tau_c = \tau_0$ , in order to highlight the effectiveness of the PTO-GFC method in reducing fuel consumption and travel time. In the case study, we suppose the platoon vehicles travel with the maximum speed and minimum headway at  $t_0$ , i.e.,  $\alpha = 0$  and  $\beta = 0$ . The number of vehicles in the platoon is N = 1

12. The cycle length of traffic signal is C = 12 and R = G. Figure 3 displays the trajectories of the platoon vehicles under different initial time  $t_0$  (the time that the leading vehicle arriving at location  $s_1$ ). The data in the Figure 3 are the unit fuel consumption and travel time per vehicle per 100 meters. We calculate the unit fuel consumption and travel time with counting in the length of the platoon and acceleration process of the human-driven vehicles when they are blocked by the red light in order to compare the performance of PTO-GFC, LTO and AT. For simplicity, the acceleration process that we count in fuel consumption is set as 200 meters. Figure 3 (a-c) show the results of PTO-GFC, LTO and AT when the initial time  $t_0 = 30s$ . In this case, the vehicles in one platoon cannot pass the signalized intersection within one green time window. When the platoon vehicles are optimized with PTO-GFC, they can split into two subplatoons and pass the intersection in two successive green time windows. However, when applying LTO and AT, some vehicles are blocked by the red light and have to wait until the light is green. Moreover, in LTO, the leading vehicle travel with minimum fuel consumption, leading more following vehicles are blocked by the red light. Therefore, in the case, in contrast to AT, the platoon controlled by LTO consumes more fuel and takes longer time to travel past the intersection. Figure 3 (d-f) show the results of PTO-GFC, LTO and AT when the initial time  $t_0 = 45s$ . When the platoon is optimized by the PTO-GFC method, all vehicles can pass through the signalized intersection with maximum speed. Moreover, when the leading vehicle is optimized to cross the signalized intersection with maximum speed, the following vehicles also can pass through the intersection without stopping. While the PTO-GFC and LTO both can make the platoon pass through the intersection without stopping, PTO-GFC method can reduce about 3.13% fuel consumption in contrast to LTO. When platoon vehicles travel without optimization and control, all vehicles are blocked by the red light. The idling waiting time and sharp deceleration/acceleration process not only waste lots of time but also increase fuel consumption. Therefore, in comparison to AT, PTO-GFC and LTO can save about 27.7% and 25.4% of fuel, respectively.

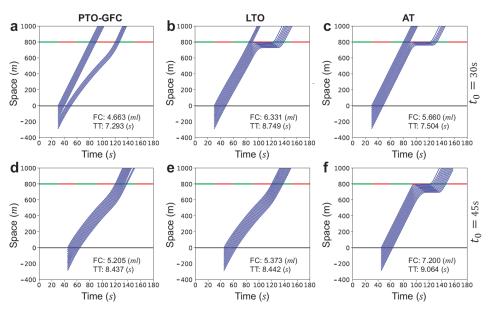


Figure 3: Trajectories of one platoon entering at different times. (a,d) Trajectories with

PTO, (b,e) trajectories with LTO and (c,f) trajectories with AT. Unit fuel consumption (FC) and travel time (TT) are labeled in each figure.

## 4.4 Sensitivity analysis

From the previous case study, we find the PTO-GFC method is beneficial for reducing fuel consumption and travel time. In this section, we analyze how the values of the critical parameters influence the performance of PTO-GFC, LTO and AT methods. Figure 4 presents the results of sensitivity analysis about different control space for the leading CAV in one platoon, maximum speed, the length of traffic signal cycle, and the number of vehicles in one platoon.

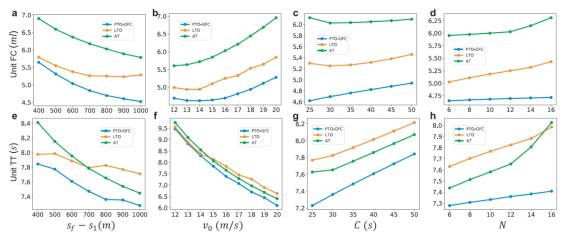


Figure 4: Sensitivity analysis of one platoon across a signalized intersection with different parameters. (a-d) Unit fuel consumption and (e-h) travel time.

As shown in Figure 4 (a), the unit fuel consumption in the three methods all decrease with the increase of control space. The gap in fuel consumption between PTO-GFC and the other two methods also increases with the increase of control space. Figure 4 (b) shows the relationship between unit fuel consumption and maximum speeds. Overall, the fuel consumption in the three methods all increases with the increase of maximum speed. Figure 4 (c) shows the sensitivity of the length of the traffic signal cycle on fuel consumption. The fuel consumption of the PTO-GFC method increases with the increase of the cycle length. It is because the fuel consumption with optimal control first decreases and then increases with travel time (see Figure 1). When the traffic signal has a significantly long cycle, the PTO-GFC method, by requiring the CAV platoon to arrive at the start of the green interval, does not take full advantage of the long green time window. Figure 4 (d) shows that the fuel consumption increases slightly with large platoon size, and the number of vehicles in one platoon does not have a significant influence on the fuel consumption of PTO-GFC. Figure 4 (e-h) show the unit travel time under different conditions. Even though we only take fuel consumption as the optimization objective, the PTO-GFC method is also beneficial to reduce travel time compared with LTO and AT methods. In summary, our PTO-GFC method considerably outperforms the LTO method in reducing fuel consumption and increasing traffic throughput in the situation with longer control distance, lower maximum speed and shorter traffic signal cycle. Moreover, even though only the leading vehicle is CAV in LTO method, it can improve the performance in fuel consumption and travel time in comparison with AT, which is consistent with both theoretical and experimental found in the literature results (Stern et al., 2018; Ubiergo and Jin, 2016; Yao et al., 2018).

In the above analysis, we have presented the impact of several factors on the performance of PTO-GFC, LTO and AT. However, in reality, the vehicles in the platoon may have different speeds and spacing before joining the platoon. In this case, we apply the GFC feedback control to the vehicles to form the initial platoon and make it track the optimal platoon trajectory. Next, we analyze the performance of the three methods under different initial speeds and headways. For simplicity, we set  $K_v = 0.8$  and  $K_s =$ 0.4 to deal with different initial conditions. Figure 5 shows the percentage increase in fuel consumption and travel time in comparison to the base condition with  $\alpha = 0$  and  $\beta = 0$ . Overall, the PTO-GFC method outperforms LTO and AT under different initial conditions no matter in reducing fuel consumption or travel time. As shown in Figure 5 (a), when  $\beta = 0$ , the platoon vehicles have the same headway at initial time  $t_0$ . In this case, the length of the platoon decreases with the increase of  $\alpha$ . Even though the platoon has a tight initial state, e.g.,  $\alpha = 4$ , the PTO-GFC method can reduce fuel consumption in comparison to the other two methods. Moreover, when the initial state of the platoon is relatively disperse, as shown in Figure 5 (b-c), the PTO-GFC method also can significantly reduce fuel consumption. Except for reducing fuel consumption, as shown in Figure 5 (d-f), the PTO-GFC method also can reduce travel time under different  $\alpha$  and  $\beta$ . All in all, the PTO-GCF method has good performance in reducing fuel consumption and travel time under different initial conditions.

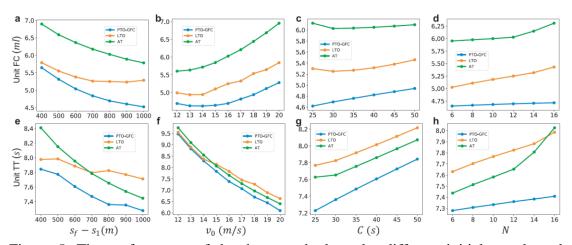


Figure 5: The performance of the three methods under different initial speeds and headways.

## 5. Optimization of multiple platoons

#### 5.1 The constraint between two platoons

The previous results are for one platoon, and no interactions between multiple platoons is considered. Therefore, in this section, we extend the PTO-GFC method to multiple platoons. The probability of the leading vehicle in platoon k arriving location

 $s_1$  based on the time of the last vehicle in platoon k-1 arriving location  $s_1$  is described as.

$$p(t_{k,0}) = \lambda e^{-\lambda[(t_{k,0} - t_{k-1,N}) - \tau_p]},$$

Where  $\lambda$  is the average event rate,  $\tau_p$  is the minimum time gap between the leading vehicle in platoon k arriving  $s_1$  and the last vehicle in platoon k-1 arriving  $s_1$ , and  $t_{k-1,N}$  denotes the time of the leading vehicle in platoon k and the last vehicle in platoon k-1 arriving location  $s_1$ , respectively.

The previous results are for one platoon, and no interactions between multiple platoons is considered. Therefore, in this section, we extend the PTO-GFC method to multiple platoons. The probability of the leading vehicle in platoon k arriving location  $s_1$  based on the time of the last vehicle in platoon k-1 arriving location  $s_1$  is described as,

$$s_{k-1}^{\text{virtual}}(t) \ge s_{k,1}(t),$$

where  $s_{k-1}^{\text{virtual}}(t)$  and  $s_{k,1}(t)$  denote the locations of virtual vehicle in platoon k-1 and the leading vehicle in platoon k at time t, respectively.

#### 5.2 sensitivity analysis

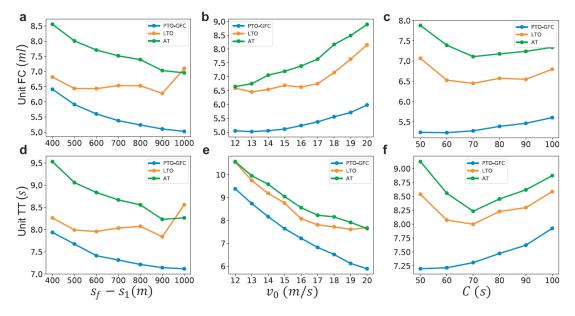


Figure 6: Sensitivity analysis of multiple platoons across a signalized intersection with different parameters.

From the above case study, we find the PTO-GFC method can reduce more than 30% fuel consumption and 20% travel time than the other two methods. In this section, we analyze the impact of key parameters on the performance of PTO-GFC, LTO and AT methods. Figure 6 shows the unit fuel consumption and travel time under different control space, maximum speed, and the length of traffic signal cycle. As shown in Figure 6 (a), there is a negative relationship between the unit fuel consumption and

control space in the PTO-GFC method. Moreover, as similar as one platoon, it is obvious that the increase of  $v_0$  and C will contribute more to the unit fuel consumption in the PTO-GFC method (see Figure 6 (b-c)). In addition to reduce fuel consumption, as shown in Figure 6 (d-f), in comparison to LTO and AT, the PTO-GFC method can reduce travel time under different scenarios.

Table 1: The percentage increase in fuel consumption and travel time under different  $\tau_p$  with applying CAITR and VT-micro fuel consumption models.

FC model	+ (a)	FC (ml)	TT(s)
	$t_{p}(s)$	PTO-	PTO-
		GFC/LTO/AT	GFC/LTO/AT
CAITR	15	5.28/9.28/10.24	7.30/11.04/12.07
	20	5.23/7.89/8.78	7.19/9.44/10.19
	25	5.23/6.53/7.39	7.21/8.07/8.56
	30	5.26/6.00/6.81	7.30/7.70/8.05
	35	5.29/6.05/6.80	7.34/7.75/8.05
VT-micro	15	10.37/13.07/13.36	7.26/8.62/9.02
	20	10.27/11.91/12.14	7.17/7.69/7.96
	25	10.33/11.52/11.76	7.20/7.39/7.69
	30	10.47/11.62/11.77	7.39/7.62/7.82
	35	10.47/11.58/11.79	7.36/7.50/7.77

Besides the parameters analyzed in Figure 6, we analyze the performance of the three methods in dealing with multiple platoons under different  $\tau_p$  (arrival rate of platoons). In addition to the above-mentioned fuel consumption model CAITR, we also test the performance of the PTO-GFC method with another fuel consumption model VT-micro (Ahn et al., 2002; Rakha et al., 2011). The parameters in the VT-micro model can be found in Zegeye et al. (2013). The VT-micro model is a nonlinear regression model, and the fuel consumption will sharply increase when the deceleration is too large. Therefore, we set the maximum brake deceleration as  $a_{br} = -2m/s^2$ when applying the VT-micro model. The other parameters are the same as the case study mentioned above. Table 1 shows the unit fuel consumption and travel time under different  $\tau_p$ . We find that the PTO-GFC method can tolerate relatively higher arrival rate of platoons. However, for LTO and AT, the unit fuel consumption and travel time significantly increase with the increase of  $\tau_p$ . For example, when  $\tau_p = 15s$ , under CAITR fuel consumption model, the PTO-GFC method can save 43.1% and 48.4% of fuel consumption in comparison to LTO and AT, respectively. Moreover, because congestion occurs under high arrival rate of platoons, the PTO-GFC method also can reduce 33.9% and 39.5% of travel time in contrast to LTO and AT, respectively. However, with the increase of  $\tau_p$ , the unit fuel consumption in PTO-GFC is approaching LTO. When  $\tau_p = 35s$ , the PTO-GFC method only save about 12.6% of fuel consumption and 5.3% of travel time in contrast to LTO. This tells us that PTO-GFC type of control is more effective under heavy than light traffic demand. Furthermore, the underlying patterns in fuel consumption and travel time are the same

as the CAITR model when we apply the VT-micro model, indicating the robustness of the PTO-GFC method when facing different instantaneous fuel consumption models.

#### 6. Conclusions and discussion

In this paper, we propose a platoon-based trajectory optimization method, i.e., PTO-GFC, to reduce fuel consumption of vehicles passing through a signalized intersection. In the PTO-GFC method, we transform the problem of optimizing and controlling multiple vehicles in one platoon into a problem of optimizing and controlling the leading vehicle in each subplatoon. A gap feedback control (GFC) makes sure the platoon vehicles with different initial speeds and headways merge into the optimal trajectories. The method can smooth the trajectories of vehicles, eliminate full stops, economize fuel consumption, and ease traffic congestion. Moreover, we compare the PTO-GFC method with the other two methods, LTO and AT. In LTO, only the leading vehicle is a CAV with optimized trajectory, and the other vehicles follow their preceding vehicles with Gipps' car-following model. In AT, we simulate the condition that all vehicles are human-driven and no optimization is applied.

Through a series of case studies and sensitivity analysis, we verify that our PTO-GFC method has advantages in economizing fuel consumption and reducing travel time over the other two methods. We find there are positive relationships between fuel consumption and the length of the traffic signal cycle and maximum speed when applying the PTO-GFC method. Because when those factors have large values, it is equivalent to reducing the space used for trajectory optimization. This indicates that the PTO-GFC method is best suited to undersaturated traffic conditions with shorter or moderately long cycles. Moreover, because the platoons controlled by PTO-GFC can pass the signalized intersections tightly with maximum speed, in comparison to LTO and AT, the PTO-GFC method has better performance in reducing fuel consumption and travel time under heavy than light traffic demand.

Several research directions can be pursued to extend this research, which includes, but is not limited to (1) to develop a PTO-GFC method for electric vehicles (EV), (2) to extend the PTO-GFC method for a network of traffic intersections, and (3) to extend the PTO-GFC method with actuated control traffic signals.

#### 7. Acknowledgement

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