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Rethinking the Price Formation Problem–Part 1: Participant Incentives under Uncertainty

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***Abstract*—Operators of organized wholesale electricity markets attempt to form prices in such a way that the private incentives of market participants are consistent with a socially optimal commitment and dispatch schedule. In the U.S. context, several competing price formation schemes have been proposed to address the non-convex production cost functions characteristic of most generation technologies. This paper considers how the design and analysis of price formation policies for non-convex markets are affected by the uncertainty inherent in electricity demand and supply. We argue that by excluding uncertainty, the analytical framework underlying existing policies mischaracterizes the incentives of market participants, leading to inefficient price formation and poor incentives for flexibility. We establish favorable theoretical properties of a new construct, *ex ante convex hull pricing*, and demonstrate the difference between this idealized benchmark and existing methods on a large-scale test system. Given increased operational uncertainty with a transition to wind and solar generation, distortions caused by poor incentives for flexibility are likely to grow without improved price formation in organized wholesale markets.**

***Index Terms*—Electricity market design, non-convexity, uncertainty, price formation, uplift**

The Commission's price formation efforts seek to ensure that market rules provide appropriate price signals, which compensate resources at prices that reflect the value of the service resources provide to the system and operational conditions and ensure resources accurately respond to dispatch instructions.

—Federal Energy Regulatory Commission statement on Energy Price Formation

I. Introduction

In recent years, many organized wholesale electricity markets in the U.S. have implemented enhanced pricing schemes intending to address the non-convex production cost functions characteristic of most generation technologies. Since it prevents the formation of uniform clearing prices, non-convexity can lead to a difference between the profit market participants would gain by maximizing their individual benefit and that they obtain by following the socially optimal schedule. The non-convex price formation problem has proven challenging, motivating a large number of competing proposals [1], modeling advances to enable computation of prices under these proposals [2], [3], and large-scale tests aimed at understanding the economic consequences of their adoption [4]–[8]. Efforts to address non-convexity have led to the introduction of Extended LMP in the Midcontinent Independent System Operator (MISO) [9] and Fast-Start Pricing in ISO New England (ISO-NE) [10], the New York Independent System Operator (NYISO) [11], PJM Interconnection (PJM) [12], and the Southwest Power Pool (SPP) [13].

Part 1 of this two-part paper demonstrates that existing policies fail, even in principle, to "ensure resources accurately respond to dispatch instructions." Part 2 provides evidence that existing policies may not "compensate resources at prices that reflect the value of the service resources provide to the system," in particular failing to provide adequate incentives for flexibility in operations. As such, the paper's primary contribution is in diagnosing inadequacies in existing policies. While the paper is diagnostic rather than constructive, our corresponding goal is to propose a new direction for research in this area by demonstrating a stochastic analysis of how price formation policies are affected by the uncertainty inherent in electricity demand and supply.

While difficult in its own right, the version of the non-convex price formation problem generally addressed in the literature understates the real-world challenge. Descending from [14] and [15], the main focus in the literature is the binary commitment variables included in deterministic day-ahead market models covering at least 24 hours of operations. Importantly, however, these seminal analyses occurred before the widespread adoption of financial trading in day-ahead markets. With lower uncertainty in supply and participation limited to physical resources, the day-ahead market could be analyzed as a standalone entity. Modern day-ahead markets include financial participants attempting to profit through arbitrage and are therefore linked to real-time markets. As a consequence, analyses of price formation policies that describe day-ahead markets without specifying how the policy leads to different outcomes in the real-time market are incomplete. Due to the back-propagation of expected real-time prices to the day-ahead market through financial participants, any effort to institute a new policy for pricing in day-ahead markets without also altering real-time price formation would be in vain [16], [17]. In U.S. markets, these real-time prices are calculated every five minutes using a simpler economic dispatch model that excludes binary variables, covers a much smaller time frame than the day-ahead market, and updates as new information is available throughout the day. In this setting, the challenge is not to identify a complete set of prices that accurately conveys the cost to serve load over a 24-hour operating period,

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but instead to identify a policy for setting spot prices in each individual five-minute period such that the generated sequence of prices preserves any relevant information about non-convexity and intertemporal constraints contributing to overall cost. Due to this complication, the relationship between price formation proposals with favorable properties described in the literature and those implemented in real-world markets is unclear.

Moving toward a more complete version of the price formation problem entails a consideration of uncertainty in addition to non-convexity. Price formation often depends on past decisions that were made without perfect information, so accounting for uncertainty has important implications for the interpretation of competing policies. Among uniform pricing schemes, convex hull pricing was proposed as a "best compromise" between a convex market equilibrium and what is obtainable in a market with non-convexities [15], [18], [19]. In deterministic simplifications, it has the property of minimizing lost opportunity costs, i.e., the difference between the profit market participants could have gained by maximizing their individual benefit and what they obtain by following a socially optimal schedule. Informally, these prices are often described as "ideal" because they are thought to align the incentives of market participants and the system operator to the extent possible given underlying non-convexity [20]. This description of aligned incentives, however, is an artifact of a deterministic model in which generators respond to a known market price. As shown in [21], convex hull pricing as traditionally defined can give poor incentives in the presence of uncertainty. In addition to non-convex production costs, thermal generators have intertemporal constraints linking decisions in present intervals with the future. An appropriate characterization of generator incentives therefore requires consideration of commitment decisions that are made under uncertainty of real-time conditions. The common definition of lost opportunity costs overestimates the incentives for inflexible generators to deviate from the system operator's commitment and dispatch instructions because it assumes that these past commitment decisions could have been made with perfect knowledge of real-time prices.

Just as the traditional *ex post convex hull pricing* may be considered ideal for generating prices in a day-ahead market without uncertainty, this paper defines a new pricing construct, *ex ante convex hull pricing* that may be considered ideal for generating spot prices. While this newly defined policy offers a clear exposition of the inadequacy of current approaches, it is not well-suited to real-world implementation. Accordingly, the paper's goal is to redefine the non-convex price formation problem rather than resolve it. In Part 2, we complement the theoretical results with a computational demonstration on an ISO-scale system with a large scenario set leveraging state-of-the-art stochastic programming methods to compare market outcomes of different schemes, including the traditional ex post locational marginal pricing (LMP), the ex post convex hull pricing (EP-CHP) sometimes described as an ideal in the literature, two variants of the fast-start pricing (FSP) implemented in several real-world markets, and the ex ante convex hull pricing (EA-CHP) defined here. We discuss the results in the context of ongoing debates about price formation in U.S. markets.

Two outcomes of the numerical tests are particularly relevant to these policy debates. First, with the growth of wind and solar, many systems have become concerned with ensuring sufficient flexible resources to manage variability and uncertainty. Consistent with [22], our tests suggest that current pricing policies could suppress volatility in prices and lead to weaker incentives for investment in flexible resources. It is well understood that suppressing the level of prices leads to a missing money problem and necessitates the introduction of supplemental revenue streams to achieve an efficient level of capacity [23]. Similarly, a failure to allow full-strength price volatility would imply a need for supplemental revenues to achieve an efficient level of flexibility in the resource mix. Second, currently accepted definitions of uplift include losses due to uncertainty as well as non-convexity. By misdiagnosing losses from uncertainty as instead arising due to non-convexity, uplift payments often described as "necessary" to providing good incentives can instead inappropriately subsidize inflexibility [21]. The resulting transfers from customers to generators are substantial. In PJM, for example, total energy uplift amounted to \$178.3M in 2021, of which two categories, balancing generator and lost opportunity cost credits, constituted \$157.8M [24]. Our analysis shows that the non-convexity-based justification typically provided for these categories of uplift is inadequate, and points to the type of uncertainty-based evidence that would constitute appropriate justification. Absent evidence along these lines, it may be questioned whether these transfers meet the regulatory standard of just, reasonable, and not unduly discriminatory.

We describe the price formation problem, define ex ante convex hull pricing in a general way, and describe its theoretical properties in Part 1 of this two-part paper. Part 2 develops a small example system to motivate the discussion, illustrate the key economic phenomena, and demonstrate the potential failure modes of seemingly plausible price formation policies that neglect the effect of uncertainty, then builds a large-scale example to provide a more complete demonstration of the properties of competing pricing schemes.

II. Models for Commitment, Dispatch, and Pricing

Operators of organized wholesale markets seek to identify commitment and dispatch schedules that maximize market surplus, as well as prices that support those optimal operational schedules. Our first task is therefore to identify a surplus-maximizing solution to the operational problem, for which we define a stochastic program. It bears mentioning that the stochastic program we define is a simplification of the true operational problem, which would be better characterized in the framework of sequential decision making under uncertainty. Given our focus on pricing rather than operations, our strategy is to describe a problem that is complex enough to reveal the economic properties of competing pricing policies, but not so complex as to obscure the economic analysis.

We then turn in the following two subsections to the question of what real-time price formation policy would best

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support that optimal commitment and dispatch. Focusing on the joint impact of uncertainty and non-convexity, we define a set of pricing models associated with the commitment and dispatch decisions. Here our representation of the system as a stochastic program becomes important, as it determines market schedules in each possible real-time scenario. Three ex post pricing models are defined that consider each real-time scenario independently. An ex ante pricing model is then defined that considers linkages between real-time scenarios that arise due to uncertainty and intertemporal constraints. We consider real-time scenarios from a discrete probability distribution that accurately represents all possible states of the world and the market operator and participants' beliefs. As will become clear, this assumption poses a serious challenge for implementation of the idealized ex ante convex hull pricing. In pursuit of practical implementations, future in-depth studies of competing approaches would benefit from more realistic simulations of rolling horizon decision models, the dynamic revelation of uncertainties, agents with asymmetric information and risk preferences, and decentralized methods to coordinate market-based pricing and scheduling decisions.

The section closes with a definition of forward market models that ensure consistency with the different real-time prices generated under each policy. We do not assume that the market operator explicitly employs stochastic programming for either operations or market clearing (cf. [25]–[28]). What is important for the purposes of our analysis is not the means by which a commitment and dispatch is identified, but the ability to describe a complete solution for all scenarios.

A. Stochastic Unit Commitment

The stochastic unit commitment problem (SUC) below minimizes the total production cost z^{SUC} considering dispatch decisions $x_{ns} \in \mathbb{R}$ and commitment $y_{ns} \in \mathbb{Z}$ for a set of generators $n \in \mathcal{N}$ and scenarios $s \in \mathcal{S}$. Here we consider a slight modification of the classic two-stage setup [29], [30], adding an additional intermediate stage to capture fast-start commitments, for a total of three stages. The generators are split into a subset of fast-start generators $\mathcal{F} \subset \mathcal{N}$ and slow-start generators $\hat{\mathcal{F}} = \mathcal{N} \setminus \mathcal{F}$. In the first stage, commitment decisions for all slow-start generators must be determined. Between the first and second stages, we gain knowledge about the uncertainties, allowing us to narrow the possible scenarios to a subset $\mathcal{S}_r \subset \mathcal{S}$, with $r \in \mathcal{R}$ indexing the subsets in the partition. In the second stage, commitment decisions for all fast-start generators are made. Lastly, in the third stage, uncertainties are revealed and dispatch decisions for all generators are made subject to the commitment decisions from prior epochs. The optimization problem is formulated below.

$$\min \quad z^{SUC} = \textstyle\sum_s \rho_s \left(c^\top x_s + d^\top y_s\right) \tag{1a}$$

$$\text{s.t.:} \quad \rho_s \left(A_0 x_s - b_{0s}\right) \geq 0 \qquad \forall s \in \mathcal{S} \tag{1b}$$

$$A_{ns} x_{ns} + B_{ns} y_{ns} \geq b_{ns} \qquad \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{1c}$$

$$y_{ns} \in \{0,1\} \qquad \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{1d}$$

$$y_{ns} = y'_{n0} \qquad \forall n \in \hat{\mathcal{F}}, \forall s \in \mathcal{S} \tag{1e}$$

$$y_{ns} = y'_{nr} \qquad \forall n \in \mathcal{F}, \forall r \in \mathcal{R}, \forall s \in \mathcal{S}_r. \tag{1f}$$

Note that the generator dispatch and commitment decisions are written more succinctly as the column vectors $x_s = [x_{1s}, \dots, x_{Ns}]$ and $y_s = [y_{1s}, \dots, y_{Ns}]$ where appropriate. Additional decision variables y'_{n0} and y'_{nr} define the first stage commitment decisions of slow-start resources and the second stage commitment decisions of fast-start resources, respectively. Constraint (1b) includes the system-wide constraints that define each scenario s. Constraints (1c) and (1d) include generator-specific feasibility and binary constraints, and constraints (1e) and (1f) define the nonanticipativity constraints of slow- and fast-start resources, respectively. The problem's parameters include scenario probability ρ_s, dispatch costs c, fixed start-up and no-load costs d, system-wide constraint matrix A_0, scenario s's system requirements b_{0s}, generator n's constraint matrices A_{ns} and B_{ns}, and generator n's constraint limits b_{ns}. Scenario-dependent generator constraints can be used to model economic offers by wind, solar, or other renewable resources that have uncertain maximum output, as well as possible contingency scenarios for traditional thermal generators. Solutions to (1) provide optimal commitment y^*_s and dispatch x^*_s decisions for each scenario $s \in \mathcal{S}$. When the s is dropped from the notation, it will be understood that x^* and y^* refer to the full solution to model (1). For convenience, z^* will refer to the optimal expected system cost identified by (1), and z^*_s will similarly to refer to the system cost in each scenario.

We define the following notation for the nonanticipativity constraints (1e) and (1f):

$$\mathcal{Y}_{ns} := \left\{ y_{ns} : \left\{ \begin{array}{ll} y_{ns} = y'_{n0}, & \forall n \in \hat{\mathcal{F}}, \forall s \in \mathcal{S}, \text{ or} \\ y_{ns} = y'_{nr}, & \forall n \in \mathcal{F}, \forall s \in \mathcal{S}_r, \forall r \in \mathcal{R} \end{array} \right\} \right\}.$$

To simplify notation of the generator-level constraints, we define feasibility sets utilizing these representations of the nonanticipativity constraints:

$$\mathcal{X}_{ns} := \{(x_{ns}, y_{ns}) : A_{ns} x_{ns} + B_{ns} y_{ns} \geq b_{ns}, y_{ns} \in \{0,1\}\} \quad \forall n \in \mathcal{N}, \forall s \in \mathcal{S};$$

$$\mathcal{X}_{n} := \{(x_{n}, y_{n}) : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns},\ y_{ns} \in \mathcal{Y}_{ns},\ \forall s \in \mathcal{S}\} \quad \forall n \in \mathcal{N},$$

where $x_n = [x_{n1}, \dots, x_{nS}]$ and $y_n = [y_{n1}, \dots, y_{nS}]$ will refer to generator n's dispatch and commitment decisions, respectively, across all scenarios $\mathcal{S} = \{1, \dots, S\}$.

In addition, we define the sets $\mathcal{X}^C_{ns}$ and $\mathcal{X}^C_{n}$ to be the convex hull relaxation of the associated set, i.e.:

$$\mathcal{X}^C_{ns} := \text{conv}(\mathcal{X}_{ns}); \qquad \mathcal{X}^C_{n} := \text{conv}(\mathcal{X}_{n}).$$

In general, a compact formulation for $\mathcal{X}^C_{ns}$ may be insufficient to ensure that a compact formulation for $\mathcal{X}^C_{n}$ is easily obtainable [31].

By formulating (1) generically, the proceeding analysis can be applied to many unit commitment-based (or more generally

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integer programming-based) market designs. In addition to system power balance, the constraint matrix A_0 may include constraints defining power flows in the transmission system and ancillary service definitions including spinning reserves, operating reserves and/or ramping products. Similarly, system requirements b_{0s} would typically define each scenario based on different levels of hourly inelastic energy demand and behind-the-meter solar output. It may also reflect scenario-specific transmission line limits in cases where dynamic line ratings are used. Also note that, while demand information would typically be given as fixed information in the b_{0s} parameters, the dispatch decisions x_{ns} have no explicit sign restriction and could therefore be used to model controllable or price-responsive loads in addition to economic offers from renewable resources and traditional thermal generators.

B. Ex Post Pricing

Prices are calculated after dispatch decisions are decided. Therefore, the following pricing models consider the scenario s to be fixed and do not consider the probabilities ρ_s. This section will first formulate a few of the standard pricing models that are found in the electricity pricing literature. The dual variables of each pricing model constraint are shown in brackets to the right of each model.

The traditional method to calculate LMPs is formally presented in [14] and consists of fixing all binary variables to their optimal value in the solution to (1). It can be written as follows.

$$\min_{x_s,y_s} \quad z_s^{LMP} = c^\top x_s + d^\top y_s \tag{2a}$$
$$\text{s.t.:} \quad A_0 x_s \geq b_{0s} \qquad [\lambda_s^{LMP}] \tag{2b}$$
$$A_{ns}x_{ns} + B_{ns}y_{ns} \geq b_{ns} \qquad [\sigma_{ns}^{LMP}] \quad \forall n \in \mathcal{N} \tag{2c}$$
$$y_{ns} = y_{ns}^* \qquad [\delta_{ns}^{LMP}] \quad \forall n \in \mathcal{N}. \tag{2d}$$

The pricing model (2) has two advantageous properties since it is a linear program and obtains the same optimal objective function value as model (1) when weighted across all scenarios. Similarly to the optimal dispatch and commitment variables x^* and y^*, for LMP and subsequent pricing schemes, the notation λ^{LMP} indicates the vector of prices in all scenarios, i.e., $\lambda^{LMP} = [\lambda_s^{LMP}]_{s\in\mathcal{S}}$.

The model's prices λ^{LMP} are consistent with the socially optimal production quantities x_s^* in each scenario s, given that the generators follow the socially optimal commitment solution y_s^* [14]. Since generators may be otherwise unwilling to follow the commitment schedule y_s^*, revenues based on this traditional method of calculating LMPs are typically supplemented by make-whole payments equal to the difference between the generator's as-offered costs and market revenues, if positive. It is argued that large make-whole payments tend to dilute the pricing signals from model (2) [19], which has motivated attempts to find new pricing policies.

The ex post convex hull pricing (EP-CHP) model, proposed by [15], is based on minimizing a broad category of side payments called uplift payments. The pricing model can be formulated as below.

$$\min_{x_s,y_s} \quad z_s^{EP} = c^\top x_s + d^\top y_s \tag{3a}$$
$$\text{s.t.:} \quad A_0 x_s \geq b_{0s}, \qquad [\lambda_s^{EP}] \tag{3b}$$
$$(x_{ns}, y_{ns}) \in \mathcal{X}_{ns}^C \qquad [\sigma_{ns}^{EP}] \quad \forall n \in \mathcal{N}. \tag{3c}$$

It will be useful to compare the objective function values of each pricing model. Since constraint (3c) relaxes the constraints (2c) and (2d), it is clear that the EP-CHP model is a relaxation of the LMP model and therefore $z_s^{LMP} \geq z_s^{EP}$. The subsequent pricing schemes will also refer to the convex hull constraint set $\mathcal{X}_{ns}^C$.

Arguments in favor of the EP-CHP model are often based on improved generator dispatch incentives, since the perceived need for uplift payments is minimized [15], [19]. In contrast to make-whole payments, uplift is defined by the difference between the maximum operating profit that a generator can earn by changing its production schedule and the actual operating profit it earns by following the production schedule provided by the ISO. If it is assumed that generators can produce a zero quantity and therefore earn at least zero operating profit, then uplift payments are always greater than make-whole payments, and make-whole payments can be considered a component of uplift.

The MISO, PJM, ISO-NE, SPP, and NYISO markets have implemented Extended LMP and fast-start pricing (FSP) models that have some similarities to the original convex hull pricing proposal by [15] but only allow certain types of generators under certain conditions to be relaxed [32]. Although some specifics differ between each market's implementation pricing models (see [9]–[12]), the main aspects are captured in the model formulation below.

$$\min_{x_s,y_s} \quad z_s^{F1} = c^\top x_s + d^\top y_s \tag{4a}$$
$$\text{s.t.:} \quad A_0 x_s \geq b_{0s}, \qquad [\lambda_s^{F1}] \tag{4b}$$
$$(x_{ns}, y_{ns}) \in \mathcal{X}_{ns}^C \qquad [\sigma_{ns}^{F1}] \quad \forall n \in \mathcal{N} \tag{4c}$$
$$y_{ns} = y_{ns}^*, \qquad [\delta_{ns}^{\hat{F}1}] \quad \forall n \in \hat{\mathcal{F}} \tag{4d}$$
$$y_{ns} \leq y_{ns}^*, \qquad [\delta_{ns}^{F1}] \quad \forall n \in \mathcal{F}. \tag{4e}$$

Rather than relaxing the commitment constraints of all generators like the EP-CHP pricing model, the fast-start pricing model above (FSP-I) only relaxes the commitment variables of fast-start resources and only if those resources are committed in the schedule calculated by the ISO. Arguments in favor of adopting FSP models often make similar, though vague, appeals to improved price signals and lower uplift or make-whole payments as previously discussed for the EP-CHP model. However, the model (4) differs from (3) since far fewer commitments will typically be relaxed, comparing (2d) with (4d) and (4e). Therefore model (4) may be seen as only a small adjustment to the traditional LMP model (2).

An additional FSP model called FSP-II modifies FSP-I to relax all fast-start resources instead of only those that are committed by the ISO. This pricing model is formulated below.

$$\min_{x_s, y_s} \quad z_s^{F2} = c^\top x_s + d^\top y_s \tag{5a}$$
$$\text{s.t.:} \quad A_0 x_s \geq b_{0s}, \quad [\lambda_s^{F2}] \tag{5b}$$
$$(x_{ns}, y_{ns}) \in \mathcal{X}_{ns}^C \quad [\sigma_{ns}^{F2}] \quad \forall n \in \mathcal{N} \tag{5c}$$
$$y_{ns} = y_{ns}^*, \quad [\delta_{ns}^{\hat{F}2}] \quad \forall n \in \hat{\mathcal{F}} \tag{5d}$$
$$y_{ns} \leq 1 \quad [\delta_{ns}^{F2}] \quad \forall n \in \mathcal{F}. \tag{5e}$$

Due to the binary variable restrictions in each formulation, it can be seen that (3) is a relaxation of (5), (5) is a relaxation of (4), and (4) is a relaxation of (2). Therefore, the pricing model objective functions can be arranged in the order $z_s^{LMP} \geq z_s^{F1} \geq z_s^{F2} \geq z_s^{EP}$ for each scenario s.

Like EP-CHP and in contrast to FSP-I, prices from FSP-II may reflect the production costs of fast-start resources that are not dispatched by the ISO. However, this aspect of FSP-II and EP-CHP may plausibly be justified by the same appeals to improved price signals that motivate other modifications to the traditional LMP model (2).

C. Ex Ante Pricing

A potential shortcoming of ex post pricing methods is that they may not provide incentives for generators to make efficient commitment decisions ex ante. For example, inefficient generators, i.e., those that are not committed in the optimal solution of model (1), may have incentives to self-commit ex ante if there is an expectation of high ex post prices. Conversely, efficient generators may fail to make necessary ex ante arrangements, such as purchasing fuel contracts, if the expectation of ex post prices is too low [33]. This paper therefore proposes a new pricing model called ex ante convex hull pricing (EA-CHP) which attempts to provide the best possible ex ante incentives.

In contrast to the ex post pricing policies, under which the spot price can be calculated from a deterministic model using data from only the realized scenario, EA-CHP computes a vector of prices covering all possible future states of the world and allows the prices in any state to be partially dependent upon conditions that might occur in other states. The appropriate price can then be selected from this vector after uncertainty is realized. This construction raises an important practical question regarding how to choose prices when the scenario realized in real time is not included in the lookahead stochastic model. Since we are primarily interested in comparing the economic properties of EA-CHP against the ex post policies, for this analysis we assume that the stochastic unit commitment problem includes all possible future scenarios.

The EA-CHP model is formulated analogously to the EP-CHP model for a deterministic unit commitment model, but is based on the stochastic unit commitment model (1), as shown below.

$$\min_{x,y} \quad z^{EA} = \textstyle\sum_s \rho_s \left(c^\top x_s + d^\top y_s\right) \tag{6a}$$
$$\text{s.t.:} \quad \rho_s \left(A_0 x_s - b_{0s}\right) \geq 0, \quad [\lambda_s^{EA}] \quad \forall s \in \mathcal{S} \tag{6b}$$
$$(x_n, y_n) \in \mathcal{X}_n^C \quad \forall n \in \mathcal{N}. \tag{6c}$$

By construction, model (6) is a convex relaxation of (1). While the convex hull representation of an individual generator's schedule, $\mathcal{X}_{ns}^C$, may have a known compact formulation [34], [35], it does not necessarily generate a compact description of $\mathcal{X}_n^C$. Accordingly, in the computational experiments we use a relaxation of (6) which considers the convex hull of every generator per scenario:

$$\min \ \hat{z}^{EA} = \textstyle\sum_s \rho_s \left(c^\top x_s + d^\top y_s\right) \tag{7a}$$
$$\text{s.t.:} \ \rho_s \left(A_0 x_s - b_{0s}\right) \geq 0 \quad [\hat{\lambda}_s^{EA}] \ \forall s \in \mathcal{S} \tag{7b}$$
$$(x_{ns}, y_{ns}) \in \mathcal{X}_{ns}^C \quad \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{7c}$$
$$y_{ns} = y'_{n0} \quad \forall n \in \hat{\mathcal{F}}, \forall s \in \mathcal{S} \tag{7d}$$
$$y_{ns} = y'_{nr} \quad \forall n \in \mathcal{F}, \forall s \in \mathcal{S}_r, \forall r \in \mathcal{R}. \tag{7e}$$

To compare the objective values z^{EA} and $\hat{z}^{EA}$ with the other price models, first note that $z^{SUC} \geq z^{EA} \geq \hat{z}^{EA}$ since the EA-CHP model (6) is a relaxation of (1), and (7) is in turn a relaxation of (6). Since the objective function in (2) is equal to (1) in each scenario, it is also clear that $z^{SUC} = \mathbb{E}_s[z_s^{LMP}]$. Additionally, the commitment and dispatch decisions of (3) are unrestricted by any nonanticipativity constraints such as (7d) and (7e), so the objective's expectation is $\mathbb{E}_s[z_s^{EP}] \leq \hat{z}^{EA}$. As a result, objective values are ordered by $z^{SUC} = \mathbb{E}_s[z_s^{LMP}] \geq z^{EA} \geq \hat{z}^{EA} \geq \mathbb{E}_s[z_s^{EP}]$.

The above descriptions of the stochastic unit commitment problem and pricing models are kept brief for easy reference. Although the models include simplifying assumptions such as the discrete probability distribution ($\rho_s, s \in \mathcal{S}$), the formulations are sufficient to illustrate shortcomings of standard pricing methods and to describe how the proposed EA-CHP pricing policy's properties overcome these shortcomings.

D. Forward Markets

Here we construct models for forward markets corresponding to the first stage as well as each node of the second stage, labeling the first a day-ahead market and the second an intraday market. We emphasize that the forward market models are distinct from the stochastic program with which optimal commitment and dispatch schedules are identified. The forward markets are deterministic unit commitment models that avoid the need for the system operator to specify scenarios and probabilities. The key modeling requirement of these markets is that they do not introduce arbitrage opportunities between stages. In each forward market, we assume that virtual bidders participate in a perfectly competitive, risk-neutral manner, such that prices converge to the expected (or conditionally expected) spot price under the chosen pricing policy. As such, the presence or absence of forward markets does not affect remuneration in expectation but can affect scenario-specific outcomes. We note the contrast between this assumption and previous examinations of stochastic market clearing, where the absence of virtual bidders can lead to inconsistencies between day-ahead and expected real-time prices [26]. We do not enforce any constraint on the physical supply quantity cleared in the day-ahead market, e.g., by setting it equal to expected demand. Since the price at the expected demand is

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not in general equal to the expected price given uncertainty, a mismatch between the cleared physical supply and expected demand can be interpreted as the net position taken by virtual bidders.

Quantities awarded in the day-ahead market assuming pricing scheme $PS \in \{LMP, EP, F1, F2, EA\}$ are calculated by solving

$$\begin{aligned} \min_{x^{DAM}, y^{DAM}} \quad & z^{DAM} = c^\top x^{DAM} + d^\top y^{DAM} \\ & \quad - \overline{\lambda}^{PS} \left(A_0 x^{DAM} - \overline{b}_0 \right) && \text{(8a)} \\ \text{s.t.:} \quad & A_n x_n^{DAM} + B_n y_n^{DAM} \geq \overline{b}_n \qquad \forall n \in \mathcal{N} && \text{(8b)} \\ & y_n^{DAM} \in \{0,1\} \qquad \forall n \in \mathcal{N}. && \text{(8c)} \end{aligned}$$

Here $\overline{\lambda}^{PS} = \mathbb{E}[\lambda_s^{PS}]$, $\overline{b}_0 = \mathbb{E}[b_{0s}]$, and $\overline{b}_n = \mathbb{E}[b_{ns}]$. As a result, the forward market is a deterministic unit commitment problem with the power balance constraint relaxed at a penalty corresponding to the expected real-time price. The constraints in Eq. (8b) ensure that generators are awarded a feasible schedule in the day-ahead market.

Similarly, positions in the intraday market in scenario r using pricing scheme PS are calculated by solving

$$\begin{aligned} \min_{x_r^{IDM}, y_r^{IDM}} \quad & z_r^{IDM} = c^\top x_r^{IDM} + d^\top y_r^{IDM} \\ & \quad - \overline{\lambda}_r^{PS} \left(A_0 x_r^{IDM} - \overline{b}_0^r \right) && \text{(9a)} \\ \text{s.t.:} \quad & A_n x_{nr}^{IDM} + B_n y_{nr}^{IDM} \geq \overline{b}_n^r \qquad \forall n \in \mathcal{N} && \text{(9b)} \\ & y_{nr}^{IDM} = y_{n0}^{\prime *} \qquad \forall n \in \hat{\mathcal{F}} && \text{(9c)} \\ & y_{nr}^{IDM} \in \{0,1\} \qquad \forall n \in \mathcal{F}. && \text{(9d)} \end{aligned}$$

Here $\overline{\lambda}_r^{PS} = \mathbb{E}[\lambda_s^{PS} | s \in \mathcal{S}_r]$, $\overline{b}_0 = \mathbb{E}[b_{0s} | s \in \mathcal{S}_r]$, and $\overline{b}_n = \mathbb{E}[b_{ns} | s \in \mathcal{S}_r]$.

Because they depend on the expected (or conditionally expected) price, commitments and quantities in the forward markets are contingent on the policy chosen for real-time price formation. Previous work has demonstrated that in principle, the presence of virtual bidders can push the solution of deterministic market clearing models toward a higher-quality solution of the underlying stochastic problem [17], [21], [36]–[39]. Our formulation makes it clear that this property depends on the policy chosen for real-time price formation. In Part 2 of the paper, we show an example in which the back-propagation of prices generated under some policies would induce poor commitment decisions in the day-ahead market. For our analysis of the financial outcomes in Part 2, we assume that if the forward markets yield suboptimal decisions, they will be overruled by a reliability unit commitment process performed by system operators that restores the optimal commitment but does not grant any committed generators a financial position. Along these lines, in the context of model (9), slow-start generators in the intraday market are fixed to their optimal commitment in the stochastic unit commitment, rather than the position awarded in the day-ahead market.

While we include their definition here to make the deterministic nature of the forward markets clear, the remainder of the analysis in Part 1 focuses on spot prices. We return to the topic of forward markets in Part 2 of the paper.

III. Properties of Pricing Policies

The following subsections describe the properties of the pricing policies in more detail. First, the expected value of perfect information (EVPI) is introduced for the SUC problem (1), which can be viewed as either the expected reduction in production costs with perfect forecast accuracy, or as the expected regret of ex ante decisions after the ex post scenario is realized. Next, a Lagrangian dual formulation of (1) is presented. This Lagrangian formulation develops an ex ante uplift definition that can be compared with the ex post uplift definition provided by [15]. It is then proven that the EA-CHP pricing model minimizes ex ante uplift, an analogous condition to classical economic equilibrium for problems with integer variables [15], [40]. Lastly, this section provides a dual interpretation of the EVPI called the expected nonanticipativity opportunity cost (ENOC) that is equal to the difference between the ex post and ex ante uplifts. The analysis in this section largely follows standard Lagrangian relaxation procedures for integer programs (e.g., see Section 11.4 in [41] and Chapter 10 in [42]). The analysis demonstrates the following propositions:

- EVPI becomes positive when ex ante decisions are required to minimize expected operations costs.
- Lagrangian duality can be applied to integer-constrained stochastic programs in a similar manner to integer-constrained deterministic programs.
- Ex post CHP maximizes the Lagrangian dual of the deterministic SCUC problem and minimizes *ex post* uplift.
- Similarly, ex ante CHP maximizes the Lagrangian dual of the stochastic SCUC problem and minimizes *ex ante* uplift.
- The difference between total ex ante uplift and ex post uplift is a quantity called ENOC that is analogous to a dual formulation of EVPI.

In summary, the ex ante uplift minimization condition yields a pricing policy that minimizes incentives to deviate from the centralized dispatch schedule given profit maximizing behavior of the market participants, the nonconvex operating characteristics of their resources, and the need to make planning decisions in advance under certainty. By analogy to EVPI, this ex ante pricing policy becomes important if nonanticipativity prevents market participants from maximizing profit with perfect foresight of the real time prices.

A. Expected Value of Perfect Information

As described above, EVPI is the expected reduction in production costs if the future scenarios could be forecasted with perfect accuracy. To calculate EVPI, the nonanticipativity constraints in (1) are dropped so that an ex post optimal solution can be determined for each scenario. The ex post optimal unit commitment problem is formulated as

$$\min_{x,y} \quad z^{EPO} = \textstyle\sum_s \rho_s \left(c^\top x_s + d^\top y_s\right) \tag{10a}$$

$$\text{s.t.:} \quad \rho_s \left(A_0 x_s - b_{0s}\right) \geq 0 \qquad \forall s \in \mathcal{S} \tag{10b}$$

$$A_{ns} x_{ns} + B_{ns} y_{ns} \geq b_{ns} \qquad \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{10c}$$

$$y_{ns} \in \{0,1\} \qquad \forall n \in \mathcal{N}, \forall s \in \mathcal{S}. \tag{10d}$$

Since (10) is a relaxation of (1), the total expected production cost is $z^{EPO} \leq z^{SUC}$, and the inequality is strict if a perfectly accurate forecast would necessarily change decisions that are made ex ante. EVPI is simply $EVPI = z^{SUC} - z^{EPO}$, or more explicitly,

$$\begin{aligned} &EVPI := \\ &\min_{x,y} \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s\right) : (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N} \right\} \\ &- \min_{x,y} \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s\right) : \right. \\ &\qquad \left. (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \right\}. \end{aligned} \tag{11}$$

Recall that $\mathcal{X}_n$ includes the set of nonanticipativity constraints (1e) and (1f). Because the two optimizations only differ due to the presence of nonanticipativity constraints, any difference in objective values can be solely attributed to the need to make decisions in advance.

B. Lagrangian Duality

To present the UC problem's economic properties, the Lagrangian relaxation of (1) is defined as

$$\begin{aligned} L(\lambda) := \min_{x,y} &\left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top (A_0 x_s - b_{0s})\right) : \right. \\ &\qquad \left. (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N} \right\}. \end{aligned} \tag{12}$$

Note that the price vector λ is a set of real time price vectors $[\lambda_s]_{s \in \mathcal{S}}$. For $\lambda \geq 0$, problem (12) is a relaxation of (1) for the following reasons: all feasible solutions to (1) are also feasible in (12), any feasible solution to (1) will have as objective function value in (12) that is no more than the objective value in (1), and there may be feasible solutions to (12) that are not feasible in (1).

Next, let the Lagrangian dual of (1) be defined as

$$L^* := \max_{\lambda \geq 0} L(\lambda). \tag{13}$$

Similarly, a Lagrangian relaxation can be defined for each scenario in the ex post optimal UC problem (10) as

$$\begin{aligned} \tilde{L}_s(\lambda_s) := \min_{x_s, y_s} &\left\{ c^\top x_s + d^\top y_s - \lambda_s^\top (A_0 x_s - b_{0s}) : \right. \\ &\qquad \left. (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\}, \end{aligned} \tag{14}$$

and a Lagrangian dual as

$$\tilde{L}_s^* := \max_{\lambda_s \geq 0} \tilde{L}_s(\lambda_s). \tag{15}$$

Both Lagrangian relaxations (12) and (14) are equivalent to profit maximization for all market participants, the former considering nonanticipativity across scenarios and the latter considering a single scenario.

C. Uplift and Lost Opportunity Costs

As described by [43], uplift consists of two components called lost opportunity cost and product revenue shortfall. Lost opportunity cost is the sum of foregone profits due to following the market operator's commitment and dispatch instructions instead of individually profit-maximizing commitment and dispatch schedules at the given market prices. Product revenue shortfall is collected to maintain revenue sufficiency (i.e.,the ability to collect and pay for all market positions, see [44]). The rest of this section will be used to show that deterministic analyses of uplift fail to identify how nonanticipativity can impact uplift calculations. Including this consideration results in identifying a new subcomponent of uplift called the expected nonanticipativity opportunity cost.

The standard ex post uplift is defined as follows:

$$\begin{aligned} U_s^P(\lambda_s) := \max_{x_s, y_s} &\left\{ \left(A_0^\top \lambda_s - c\right)^\top x_s - d^\top y_s : \right. \\ &\qquad \left. (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} \\ &- \left(\left(A_0^\top \lambda_s - c\right)^\top x_s^* - d^\top y_s^* \right) \\ &+ \lambda_s^\top \left(A_0 x_s^* - b_{0s}\right). \end{aligned} \tag{16}$$

In contrast to the deterministic analysis applied in [15], Eq. (16) indexes the optimal solution (x^*, y^*) by a scenario s considered in the stochastic problem (1). To reduce the potential for poor incentives, [15] suggests that market operators set prices to minimize uplifts that are then paid respectively to each market participant. Such prices can be defined as the vector $\lambda_s \geq 0$ that minimizes $U_s^P(\lambda_s)$ and can be determined by solving the Lagrangian dual (15):

$$\begin{aligned} &\min_{\lambda_s \geq 0} U_s^P(\lambda_s) \\ &= \min_{\lambda_s \geq 0} \left\{ \max_{x_s, y_s} \left\{ \left(A_0^\top \lambda_s - c\right)^\top x_s - d^\top y_s : \right. \right. \\ &\qquad \left. \left. (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} - \lambda_s^\top b_{0s} + z_s^* \right\} \\ &= \min_{\lambda_s \geq 0} \max_{x_s, y_s} \left\{ -\left(c^\top x_s + d^\top y_s - \lambda_s^\top \left(A_0 x_s - b_{0s}\right)\right) : \right. \\ &\qquad \left. (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} + z_s^* \\ &= z_s^* - \max_{\lambda_s \geq 0} \min_{x_s, y_s} \left\{ c^\top x_s + d^\top y_s - \lambda_s^\top \left(A_0 x_s - b_{0s}\right) : \right. \\ &\qquad \left. (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} \\ &= z_s^* - \tilde{L}_s^*. \end{aligned}$$

That is, the minimum ex post uplift is exactly equal to the duality gap between the optimal schedule cost z_s^* and Lagrangian cost $\tilde{L}_s^*$ in each scenario s.

An alternative approach would be to apply nonanticipativity constraints on the commitment and dispatch decisions in each potential scenario, resulting in the following ex ante uplift definition:

$$\begin{aligned} U^A(\lambda) := \max_{x,y}\Bigg\{ & \sum_s \rho_s \left(\left(A_0^\top \lambda_s - c\right)^\top x_s - d^\top y_s\right) : \\ & (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N}\Bigg\} \\ & - \sum_s \rho_s \left(\lambda_s^\top b_{0s} - c^\top x_s^* - d^\top y_s^*\right). \end{aligned} \tag{17}$$

In contrast to the ex post uplift, the ex ante uplift defined above requires that the profit maximizing schedules obey the same nonanticipativity constraints that are satisfied in the set of socially optimal schedules, $\{(x_s^*, y_s^*), \forall s \in \mathcal{S}\}$, from the solution to (1).

Following the same steps as before, minimizing ex ante uplift is equivalent to solving the SUC problem's Lagrangian dual (13).

$$\min_{\lambda \geq 0} U^A(\lambda) = z^{SUC} - L^*$$

Assuming that the ex ante optimal commitment solution is chosen, then $z^{SUC} = \sum_s \rho_s z_s^*$ and definitions (16) and (17) imply that the expected ex post uplift must be greater than or equal to the ex ante uplift, i.e., $\mathbb{E}[U_s^P(\lambda_s)] \geq U^A(\lambda)$. Accordingly, the ex post uplift definition may include some uplift that is not included in the ex ante definition. This difference will be called the expected nonanticipativity opportunity cost (ENOC), $U^N(\lambda)$, and is defined as follows:

$$\begin{aligned} U^N(\lambda) &:= \sum_s \rho_s U_s^P(\lambda_s) - U^A(\lambda) \\ &= \sum_s \rho_s \Bigg(\max_{x,y}\Bigg\{\left(A_0^\top \lambda_s - c\right)^\top x_s - d^\top y_s : \\ &\qquad (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N}\Bigg\}\Bigg) \\ &\quad - \max_{x,y}\Bigg\{\sum_s \rho_s \left(\left(A_0^\top \lambda_s - c\right)^\top x_s - d^\top y_s\right) : \\ &\qquad (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N}\Bigg\}. \end{aligned} \tag{18}$$

Notice that (18) can be rewritten as the difference of two minimization problems:

$$\begin{aligned} U^N(\lambda) = \min_{x,y}\Bigg\{ & \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top A_0 x_s\right) : \\ & (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N}\Bigg\} \\ - \min_{x,y}\Bigg\{ & \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top A_0 x_s\right) : \\ & (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N}\Bigg\} \\ + & \sum_s \left(\lambda_s b_{0s} - \lambda_s b_{0s}\right). \end{aligned}$$

Further,

$$\begin{aligned} U^N(\lambda) = \min_{x,y}\Bigg\{ & \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top \left(A_0 x_s - b_{0s}\right)\right) : \\ & (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N}\Bigg\} \\ - \min_{x,y}\Bigg\{ & \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top \left(A_0 x_s - b_{0s}\right)\right) : \\ & (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N}\Bigg\} \\ = L(\lambda) - & \sum_s \rho_s \tilde{L}_s(\lambda_s). \end{aligned}$$

Whereas EVPI was previously defined as the cost difference between (1) and (10), ENOC is defined above as the difference of the Lagrangian duals of the same two optimization problems. In other words, ENOC is a dual formulation of EVPI that depends on the the real time pricing policy vectors $\lambda = [\lambda_s]_{s \in \mathcal{S}}$ rather than commitment and dispatch solutions. It can be deduced from the ENOC definition (18) that ENOC is a component of the lost opportunity cost component of uplift described in (16). ENOC therefore represents the component of ex post uplift that would inappropriately compensate resources given that the realized market prices could not have been known in advance with perfect foresight.

Remark III.1. *The derived expressions for ex ante uplift* (17) *and ENOC* (18) *are defined in the context of the three-stage stochastic unit commitment model* (1)*, which is a simplification of the real-world operational problem. We note, however, that the existence of a gap between ex ante and ex post uplift only requires that nonanticipativity constraints will limit the ability of market participants to maximize their operating profits in all possible scenarios. While more sophisticated stochastic models of real-world operations would be able to generate better estimates of the magnitude of the gap, our focus is instead on a clear exposition of the directional consequences.*

D. Primal Convex Hull Equivalence

The ex post CHP model (3) and ex ante CHP model (6) were previously described in terms of minimizing a specific set of side-payments called uplift. Those statements can be made more precise now that the ex post and ex ante uplift definitions, (16) and (17), have been provided. Uplift payments are intended to remove incentives for participants to deviate from the socially optimal schedule calculated by the market operator since they would ensure that all participants receive their maximum profit, and they would be paid on condition that the participant follows the market operator's dispatch schedule to a reasonable degree of accuracy.

Supposing that the ex ante CHP model (6) produces a set of prices λ^{EA} that solve the Lagrangian dual problem (13), then it would be clear that λ^{EA} minimizes the expected ex ante uplift in equation (17). Accordingly, the following proofs show that the prices calculated by solving (6) in fact also

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solve the Lagrangian dual problem (13). Let the ex ante CHP model's Lagrangian relaxation be defined as follows:

$$L^{EA}(\lambda) = \min_{x,y} \Bigg\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top (A_0 x_s - b_{0s}) \right) : (x_n, y_n) \in \mathcal{X}_n^C, \forall n \in \mathcal{N} \Bigg\}.$$

Lemma III.1. *Suppose $L^{EA*} = \max_{\lambda \geq 0} L^{EA}(\lambda)$ is the Lagrangian dual solution for problem* (6). *Then $L^{EA*} = L^*$.*

Proof. Proof of Lemma III.1: For given λ, the feasible region of $L(\lambda)$ only differs from the feasible region of $L^{EA}(\lambda)$ in that the variables (x, y) are optimized over the possibly non-convex set $\mathcal{X}_n$ for each $n \in \mathcal{N}$ rather than the convex hull relaxation $\mathcal{X}_n^C$. Clearly, $L(\lambda) \geq L^{EA}(\lambda)$. Since the optimal points $(x_n^*, y_n^*) \in \mathcal{X}_n^C$ for L^{EA} will be extreme, the relaxed solution will be such that $(x_n^*, y_n^*) \in \mathcal{X}_n$ for every $n \in \mathcal{N}$. Therefore $L^{EA}(\lambda) = L(\lambda)$ for every λ, so $L^{EA*} = L^*$. □

Theorem III.1. *Suppose λ^{EA} is the optimal dual variable of constraint* (6b). *Then $z^{EA} = L^{EA*} = L^*$, and λ^{EA} solves $L^* = L(\lambda^{EA})$.*

Proof. Proof of Theorem III.1: For the first part of the theorem, note that (6) is a convex linear program. Therefore, strong duality implies that $z^{EA} = L^{EA*}$. The second equality $L^{EA*} = L^*$ is implied by Lemma III.1.

The second part of the proof will show, first, that $L^{EA}(\lambda^{EA}) \leq z^{EA}$, and second, that $L^{EA}(\lambda^{EA}) \geq z^{EA}$. As a result, $L^{EA}(\lambda^{EA}) = L^*$, so we can conclude that λ^{EA} solves the Lagrangian dual problem.

For the upper bound, let (x^{EA}, y^{EA}) be the optimal primal solution to (6) and λ^{EA} the optimal dual variable to constraint (6b). Then we have the following upper bound based on the minimization in $L(\lambda^{EA})$ and complementary slackness in the primal and dual solutions of (6):

$$\begin{aligned} L(\lambda^{EA}) &\leq \sum_s \rho_s \Big(c^\top x_s^{EA} + d^\top y_s^{EA} \\ &\quad - (\lambda^{EA})^\top \left(A_0^\top x_s^{EA} - b_{0s} \right) \Big) \\ &= \sum_s \rho_s \left(c^\top x_s^{EA} + d^\top y_s^{EA} \right) \\ &= z^{EA}. \end{aligned}$$

For the lower bound, it must be true that $L(\lambda^{EA}) \geq L^{EA}(\lambda^{EA})$ since $L^{EA}(\lambda)$ is a relaxation of minimization problem $L(\lambda)$. Further, since $L^{EA}(\lambda)$ is a convex optimization problem, strong duality implies that $L^{EA}(\lambda^{EA}) = z^{EA}$. Applying the first part of the proof, we have shown that $L(\lambda^{EA}) = L^*$. Therefore, λ^{EA} is a solution to the Lagrangian dual problem. □

Corollary III.1. *Suppose $\hat{L}^{EA}(\lambda)$ is the Lagrangian relaxation of* (7) *and $\hat{\lambda}^{EA}$ is the optimal dual variable of constraint* (7b). *Then $\hat{z}^{EA} = \hat{L}^{EA}(\hat{\lambda}^{EA}) \leq L^*$.*

Proof. Proof: The proof is immediate from the fact that (7) is a relaxation of (6). □

Remark III.2. *There may be considerable difficulty in solving either the exact EA-CHP model* (6) *or the Lagrangian problem L^{EA*}. If the relaxed EA-CHP model* (7) *is solved instead, there may be uncertainty whether the resulting prices provide similar economic properties as the theoretically ideal prices. Even if the approximation is unable to reproduce the theoretically ideal price vectors, it is unlikely that the approximation introduces significantly different incentives so long as ex ante uplift is nearly minimized. From Corollary III.1, the increase in ex ante uplift due to approximation is $z^{EA} - \hat{z}^{EA} \geq 0$, which will be close to zero if* (7) *is a close approximation of* (6). *In addition, recalling the result of Corollary 1 in [4], there is an upper bound on the redistribution of market surplus that results from sub-optimal scheduling decisions, provided that the pricing model is a convex relaxation of a mixed-integer scheduling problem. Those conditions are satisfied by the approximated ex ante pricing model* (6) *or by any other reasonably tight relaxation of the stochastic scheduling problem, which implies a limited amount of economic distortions in the approximated model.*

IV. Conclusion

This two-part paper examines the impact of uncertainty on the design and analysis of policies for price formation in non-convex markets. Part 1 develops a theoretical framework to distinguish the effects of uncertainty and non-convexity on price formation. In Part 2 of the paper, we complement the theoretical development in Part 1 with two examples demonstrating the differences between the idealized benchmark of *ex ante convex hull pricing* and existing policies. The examples illustrate the potential consequences of a failure to make this distinction, including poor incentives for commitment in short-term operations and poor incentives for flexibility in long-term investments.

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V. Biography Section

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