Rethinking the Price Formation Problem–Part 2: Rewarding Flexibility and Managing Price Risk

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***Abstract*—Part 1 of this two-part paper describes the impact that uncertainty has on the design and analysis of price formation policies in the non-convex auctions conducted by U.S. wholesale electricity market operators. Using first a toy model and then a large-scale test system, Part 2 demonstrates the difference in prices under the idealized benchmark of *ex ante convex hull pricing* defined in Part 1 versus existing methods, in particular documenting the potential for suppression of volatility and therefore under-compensation of flexibility by existing methods. The examples highlight that inefficient spot price formation can induce inefficient forward commitments of generators, necessitating out-of-market intervention to restore a reliable and efficient operating plan. Given the potential side effects of existing policies for investment and operation, we suggest two elements in a reoriented approach to the price formation problem: first ensuring that prices exhibit full-strength volatility, and second ensuring that risk-averse market participants have sufficient ability to manage this volatility.**

***Index Terms*—Electricity market design, price formation, risk trading, virtual bidding**

I. Introduction

THE analysis in Part 1 of this two-part paper highlights that lost opportunity costs for market participants can arise from both uncertainty and non-convexity. Attempts to address non-convexity focus on the property of individual rationality: faced with negative profits, resources would alter their offers or choose not to participate in the market, leading to lower overall efficiency. Losses due to uncertainty, on the other hand, are an inherent feature of competitive markets, and there is no efficiency-based argument to making participants whole when they occur. Accordingly, addressing lost opportunity costs in an efficient way requires a clear distinction between the two sources. Moreover, since prices formed in spot markets serve as the basis for trades used to manage financial risk in forward markets, efforts to address non-convexity will necessarily affect efforts to address uncertainty.

Part 2 complements the theoretical development in Part 1, using a toy model and then a large-scale test to demonstrate the features of different pricing policies relative to the idealized benchmark of *ex ante convex hull pricing*. The numerical studies suggest that policies currently in use suppress volatility relative to the ideal, leading to poor incentives for investment and operation. A potential counterargument to restoring full-strength price volatility and a higher probability of incurring losses is that it could prompt risk-averse market participants to alter their offers, leading to lower efficiency overall. Along these lines, Part 2 also assesses the role of financial trading, including day-ahead markets, in reducing risk and aligning the incentives of participants in short-term markets.

In addition to helping manage financial risk, trading in the day-ahead market can improve the physical performance of power systems by pushing the solution of the deterministic market clearing model toward that of the true underlying stochastic problem [1]–[4]. In practice, many complications can interfere with this salubrious property [5]. In an extreme example, the root cause analysis of the August 2020 outages in California argues that virtual bidding contributed to the need for rolling blackouts [6]. This paper highlights that the effect of forward contracts on physical system performance depends on the efficiency of the spot prices on which they are based. In this context, while suppressing volatility of spot prices relative to the ideal may have risk reduction benefits, a better approach for losses due to uncertainty may be ensuring that market participants have greater ability to trade risk [7], [8]. To manage increasing variability and uncertainty due to the growth of wind and solar, some have suggested the introduction of intraday markets [9]–[12]. Our results suggest that intraday markets could help reduce the perceived need for uplift payments to address misaligned incentives. However, along the lines of [13], fixed-quantity swaps alone do not give an efficient way to manage risk associated with the positive correlation between price and dispatch quantity for most near-marginal generators. To address this correlation, supplementing day-ahead markets with option-like instruments may be more effective than introducing more frequent intraday markets.

II. Toy Example

To elaborate on the incentives of market participants, the potential failure of pricing policies that neglect uncertainty, and the role of forward markets, here we adapt an example from [14] and construct a system with a single uncertain demand, single hour-long time period, and single node. The set of scenarios $\mathcal{S} = \{1, 2, \ldots, 100\}$ contains 100 possible outcomes for demand b, with $b_s = (99.5 + s)$ MW $\forall s \in \mathcal{S}$. The system is served by 101 thermal generators. Generator 0 has capacity 100 MW, has a marginal production cost of \$50/MWh, and has no minimum operating level, start-up cost, or no-load cost. Generators $n = 1, 2, \ldots, 100$ are each block loaded units of size 1 MW with zero energy cost and a start-up cost of $n + 50$, such that Generator 1 costs \$51 to commit

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and operate for the hour while Generator 100 costs \$150. All generators with even indices are slow-start generators, while those with odd indices have fast-start capability. Lastly, the system can also engage a demand-side resource at a cost of \$500/MWh in the event that the committed generators are insufficient to meet demand. In stochastic programming terms, the presence of this resource guarantees complete recourse.

At the time of first stage decisions, the probability $\rho_s = 0.01$ for each scenario $s \in \mathcal{S}$. The scenarios are partitioned into 20 sets of cardinality 5; i.e., with $\mathcal{R} = \{1, \ldots, 20\}$, $\mathcal{S}_r = \{5(r-1)+1, 5(r-1)+2, \ldots, 5(r-1)+5\}\ \forall r \in \mathcal{R}$. The conditional probability of scenario $s \in \mathcal{S}$ is 0.20 if $s \in \mathcal{S}_r$ and zero otherwise.

A. Optimal solution

If the demand scenario s were known in advance, it can be seen that the optimal solution would be to commit exactly s of the block-loaded units. For instance, $s = 40$ would give a total demand of 139.5, optimally served by 99.5 MW from generator 0 and 40 MW from generators 1 through 40. Instead, the stochastic problem requires the system operator to weigh the commitment cost of the block-loaded generators against the probability that a more expensive recourse action will be required in the next epoch. Table I shows the optimal commitment solution for the example system, i.e., the solution to model (1) from Part 1, which depends on the subset of demand scenarios $\mathcal{S}_r$ identified before the commitment of fast-start units. The system commits 38 slow-start units in stage one. In the event of low demand, i.e, $r \in \{1, \ldots, 7\}$, no fast-start units are committed. For $r \in \{8, \ldots, 12\}$, the operator commits enough fast-start units to cover demand in all scenarios that remain possible. For $r \in \{13, \ldots, 17\}$, it is optimal to engage the \$500/MWh demand-side resource 20 percent of the time rather than commit an additional fast-start unit. Lastly, in the highest demand scenarios with $r \in \{18, 19, 20\}$, the system commits all available fast-start units but the demand-side resource is frequently needed since many slow-start units have been held offline.

TABLE I
OPTIMAL COMMITMENT SOLUTION FOR EXAMPLE SYSTEM FOR EACH DEMAND RANGE

Range	Max Demand	Block-loaded Units Committed	Probability of Shortfall
1	104.5	38	0.0
2	109.5	38	0.0
3	114.5	38	0.0
4	119.5	38	0.0
5	124.5	38	0.0
6	129.5	38	0.0
7	134.5	38	0.0
8	139.5	40	0.0
9	144.5	45	0.0
10	149.5	50	0.0
11	154.5	55	0.0
12	159.5	60	0.0
13	164.5	64	0.2
14	169.5	69	0.2
15	174.5	74	0.2
16	179.5	79	0.2
17	184.5	84	0.2
18	189.5	88	0.4
19	194.5	88	1.0
20	199.5	88	1.0

B. Pricing

We now consider how to establish a pricing policy that supports this optimal solution. Building on the discussion in Part 1, we make several aspects of this challenge more concrete here. For reference, we summarize the five pricing schemes defined in Part 1 as follows:

- **Locational Marginal Pricing (LMP):** Binary commitment variables are fixed to their optimal values, with only marginal costs able to contribute to energy prices.
- **Ex Post Convex Hull Pricing (EP-CHP):** Generator feasible regions are relaxed to their convex hulls and a deterministic solution for relaxed commitment and dispatch is found using the realizations of random variables.
- **Fast-Start Pricing I (FSP-I):** The feasible regions of fast-start generators committed by the ISO are relaxed to their convex hulls while other binary variables are fixed to their optimal values, and a deterministic solution for relaxed commitment and dispatch is found using the realizations of random variables.
- **Fast-Start Pricing II (FSP-II):** The feasible regions of all fast-start generators are relaxed to their convex hulls while other binary variables are fixed to their optimal values, and a deterministic solution for relaxed commitment and dispatch is found using the realizations of random variables.
- **Ex Ante Convex Hull Pricing (EA-CHP):** Generator feasible regions are relaxed to their convex hulls per scenario and a stochastic solution for relaxed commitment and dispatch is found before learning the realizations of random variables.

Table II shows the average price across all 100 scenarios under the five pricing schemes. The system is constructed such that the marginal unit in every scenario is either generator 0, with a marginal cost of \$50/MWh, or the demand-side resource, with a marginal cost of \$500/MWh. Examination of Table I shows that the demand-side resource is engaged in 17 of the 100 scenarios (one scenario each when $r \in \{13, \ldots, 17\}$, two scenarios when $r = 18$, and five scenarios each when $r \in \{19, 20\}$). Under the traditional LMP pricing policy, this means that the price set after the dispatch is determined will be \$500/MWh in 17 percent of scenarios and \$50/MWh in the remaining 83 percent, for an average of \$126.50/MWh.

TABLE II
EXPECTED SPOT PRICE UNDER DIFFERENT PRICING SCHEMES (\$/MWH)

LMP	EP-CHP	FSP-I	FSP-II	EA-CHP
126.50	100.50	147.85	129.00	127.90

The price under EP-CHP is substantially lower. Suppose we knew in advance that $s = 40$ and were able to optimally

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commit 40 block-loaded units. In this instance, generator 0 would still be the price-setting resource under traditional LMP, leading to a price of \$50/MWh and losses for generators 1 through 40. The EP-CHP scheme attempts to minimize conflicts between the system operator and individual market participants by instead setting a price of \$90/MWh, i.e., the total cost of the most expensive unit that would have been committed had demand been known in advance. Given this price, generators 0 through 40 would be content with their non-negative profit, while generators 41 through 100 would still prefer not to operate. Applying this logic across all scenarios, an EP-CHP policy would lead to generator s setting the price in each scenario $s \in \mathcal{S}$, giving an average price of \$100.50.

1) Forward Contracting and Ex Post Lost Opportunity Costs: In the optimal solution, 38 slow-start units are committed in the first stage, the most expensive of which is generator 76 at a total cost of \$126. With the LMP pricing policy, generator 76 incurs a loss of \$76 in the 83 scenarios with a price of \$50/MWh. In the other 17 scenarios, the uncommitted slow-start generator 78 does not operate even though it could hypothetically earn a profit of $\$500 - \$128 = \$372$. In both cases, owners of the generator may complain that the commitment and dispatch schedule directed by the system operator led to lower profits than would have been obtained under a different schedule.

As described in Part 1 of the paper, suppose we identify the first stage of the model as a day-ahead market that includes virtual bidders driving the day-ahead price to the expected real-time price. Under LMP, this expected price is \$126.50/MWh, while EP-CHP gives \$100.50/MWh. Under LMP or EA-CHP, forward contracts awarded in the day-ahead market alleviate the lost opportunity costs problem noted above: generator 76 locks in a profit of \$0.50, allowing it to avoid losses in scenarios with a price of \$50/MWh.

2) Backpropagation and Operational Efficiency: Given the no-arbitrage condition leading to day-ahead prices that match expected real-time prices, it is worth highlighting the interaction between the chosen policy for real-time price formation and the operational efficiency of the system. With an expected price under EP-CHP of \$100.50/MWh, generator 76 would not clear in the day-ahead market despite being included in the optimal solution to the stochastic unit commitment. Instead, it would be supplanted by virtual suppliers bidding closer to \$100.50. As a result, implementation of EP-CHP would require that system operators supplement or override the results of the day-ahead market, e.g., through a residual unit commitment process, in order to restore a near-optimal solution. Without a day-ahead market position, generator 76 would be fully exposed to real-time prices and would expect to incur an operating loss of \$25.50. The FSP-I scheme exhibits the opposite issue. With an expected price of \$147.85/MWh, many slow-start units would be awarded a contract in the day-ahead market despite not being in the efficient solution to the stochastic unit commitment problem.

For purposes of the analysis in this paper, we set aside the consequences of pricing policies for reliability and operational efficiency, instead focusing on the financial outcomes. In other words, we assume that operators have the ability to restore an efficient commitment and dispatch solution. However, we highlight the need for further analysis on the operational ramifications of different pricing policies, since 1) operator interventions to override market outcomes, which introduce the need for other penalties or uplift payments, are typically seen as undesirable and 2) such interventions are inherently limited in their scope to the resources under centralized control, leading to challenges if some resources are self-scheduled.

3) Ex Ante Lost Opportunity Costs: The discussion so far suggests that attempts to address incentive issues through EP-CHP may in fact be counterproductive: for generator 76, instituting EP-CHP results in poor incentives at the time a commitment decision must be made. In effect, the pricing policy misdiagnoses losses arising due to uncertainty as instead resulting from non-convexity. This observation, however, does not imply that retaining LMP removes the potential for misaligned incentives.

Table III shows the cost of the most expensive fast-start unit committed for each $r \in \mathcal{R}$, as well as the conditional mean of the price under each pricing policy. Consider the case of $r = 10$, in which 12 fast-start units are committed. Since the demand-side resource is never engaged, under LMP the price will be \$50/MWh. Since the 12 fast-start units all have a start-up cost above $\$50$, they are guaranteed to lose money despite being included in the optimal commitment. In other words, the fast-start generators would prefer not to be committed in the second stage given the conditional distribution of prices in the third stage. Under EA-CHP, the conditional mean is determined by the most expensive committed fast-start unit, which has a total cost of \$73.

4) Uplift and Incomplete Markets: The presence of a day-ahead market substantially changes the lost opportunity costs calculation for slow-start units. In U.S. markets, however, market operators do not provide an opportunity to update financial positions between the day-ahead and real-time markets. This limitation can be contrasted with European markets, where continuous trading is available until closer to real time [9]. In the second stage of the example problem, consider the case of $r = 16$, which leads to the commitment of 41 fast-start units and a 20 percent chance of engaging the demand-side resource. In this case, the most expensive fast-start unit is generator 81, with a cost of \$131. The conditional mean of the price under LMP is \$140/MWh, i.e., high enough to make commitment of the generator profitable in expectation. Without a mechanism to lock in the new expected price of \$140/MWh, however, generator 81 will incur losses 80 percent of the time.

To alleviate problems with potentially misaligned incentives, market operators in the U.S. use uplift payments to supplement compensation from the uniform prices seen by all market participants. These side payments can be seen as part of the overall pricing policy. While the theoretical results in Part 1 pertain to all lost opportunity costs, no system actually pays all lost opportunity costs as uplift because such a policy would lead to indefensibly high compensation in practice [15]. Instead, most systems authorize smaller categories of uplift payments. The most important of these are make-whole payments, which guarantee non-negative profit for generators that are included in the efficient commitment solution, and

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TABLE III
TOTAL COST OF MOST EXPENSIVE COMMITTED FAST-START UNIT COMPARED TO CONDITIONAL MEAN OF PRICE IN EACH DEMAND RANGE

Range (r)	Highest FS Cost ($)	LMP ($/MWh)	EP-CHP ($/MWh)	FSP-I ($/MWh)	FSP-II ($/MWh)	EA-CHP ($/MWh)
1	0	50.00	53.00	50.00	50.00	50.00
2	0	50.00	58.00	50.00	50.00	50.00
3	0	50.00	63.00	50.00	50.00	50.00
4	0	50.00	68.00	50.00	50.00	50.00
5	0	50.00	73.00	50.00	50.00	50.00
6	0	50.00	78.00	50.00	50.00	50.00
7	0	50.00	83.00	50.00	50.00	50.00
8	53	50.00	88.00	50.80	50.80	53.00
9	63	50.00	93.00	59.00	59.00	63.00
10	73	50.00	98.00	69.00	69.00	73.00
11	83	50.00	103.00	79.00	79.00	83.00
12	93	50.00	108.00	89.00	89.00	93.00
13	101	140.00	113.00	178.40	99.00	103.00
14	111	140.00	118.00	186.40	109.00	113.00
15	121	140.00	123.00	194.40	119.00	123.00
16	131	140.00	128.00	202.40	129.00	133.00
17	141	140.00	133.00	210.40	139.00	141.00
18	149	230.00	138.00	288.20	288.20	230.00
19	149	500.00	143.00	500.00	500.00	500.00
20	149	500.00	148.00	500.00	500.00	500.00

we focus on these potential make-whole payments in the discussion in Part 2.

In the case of $r = 16$ discussed above, under current rules generator 81 could be entitled to a make-whole payment of $81 to cover its losses in the 4 out of 5 scenarios that result in an LMP of $50/MWh. While the justification for these side payments relies on the need to satisfy individual rationality constraints, the example highlights the potential issue with assessing profitability ex post. Given $r = 16$, at the time of commitment generator 81 is profitable in expectation, earning $9 on average. In this case, providing make-whole payments whenever the price is $50/MWh socializes the losses that occur 80 percent of the time and privatizes the gains that occur when the price is $500/MWh. A more efficient route to resolving the incentive issues associated with these potential losses could be to attach a financial position to commitments occurring in the second stage, moving toward more complete markets in risk. A forward market executed at the time of the second stage would allow generators to update their financial positions based on the conditionally expected price. In the case of $r = 16$, generator 81 could sell its power in this intraday market at a price of $140/MWh, enabling it to avoid losses in the scenarios with a real-time price below $131/MWh.

C. Scenario Profits

With prices and quantities in forward markets defined, we can calculate the profit earned by generators in each scenario under each pricing policy and trading regime. With the superscript 1 indicating a single settlement, i.e., no forward trades, profit for generator n under policy PS is entirely dependent on the spot price in the given scenario and the production according to the efficient schedule:

$$\pi_{ns}^1 = (\lambda_s^{PS})^\top x_{ns}^* - \left(c^\top x_{ns}^* + d^\top y_{ns}^*\right).$$

With superscript 2 indicating a two-settlement system with a day-ahead market in addition to the spot market, profit is calculated as

$$\pi_{ns}^2 = (\overline{\lambda}^{PS})^\top x_{ns}^{DAM} + (\lambda_s^{PS})^\top (x_{ns}^* - x_n^{DAM}) - \left(c^\top x_{ns}^* + d^\top y_{ns}^*\right).$$

With the two-settlement system, sales in the real-time market are calculated with reference to the forward position awarded in the day-ahead market. With superscript 3 indicating an additional settlement in an intraday market, profit is calculated as

$$\pi_{ns}^3 = (\overline{\lambda}^{PS})^\top x_{ns}^{DAM} + (\overline{\lambda}_r^{PS})^\top (x_{ns}^{IDM} - x_n^{DAM}) + (\lambda_s^{PS})^\top (x_{ns}^* - x_n^{IDM}) - \left(c^\top x_{ns}^* + d^\top y_{ns}^*\right).$$

The expected prices in Table II are an indication of the superior ex ante incentives offered by EA-CHP, and to a lesser extent LMP and FSP-II, in the example system. Here we turn the focus to ex post results, in particular the potential for losses in individual scenarios. Table IV shows the number of generator scenarios which result in a negative profit under each pricing policy and trading regime. Given 100 scenarios and 101 generators, the total count of generator scenarios is 10,100. With a price of $50/MWh frequently set by generator 0, LMP and EA-CHP lead to frequent occurrence of negative profit scenarios. However, the introduction of a day-ahead market substantially reduces this number, and in the case of EA-CHP the addition of an intraday market brings the number

TABLE IV
GENERATOR SCENARIOS WITH NEGATIVE PROFIT

Settlements	LMP	EP-CHP	FSP-I	FSP-II	EA-CHP
One	4,324	1,523	2,250	2,271	4,106
Two	35	953	247	35	159
Three	50	984	250	29	0

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of negative profit generator scenarios to zero. Forward trading brings limited benefit in the case of EP-CHP, because the underlying prices are too low to result in a profit for many committed generators.

Table V shows the make-whole payments that would be authorized on average if generators were guaranteed non-negative profit in each scenario. Table V exhibits the degree to which make-whole payments may be driven more by the variance of prices created by a pricing policy, rather than the expected value. Despite having a lower expected price than other policies, EP-CHP generates fewer make-whole payments in the single settlement regime by lifting prices above \$50/MWh in low-demand scenarios, reducing its variance. With two settlements, both LMP and FSP-II lead to lower make-whole payments on average than EA-CHP. We return to the topic of price volatility, corresponding incentives for flexibility, and its relationship to market completeness in the larger test system.

TABLE V
AVERAGE MAKE-WHOLE PAYMENTS TO ALL GENERATORS, WITH LOSSES COMPUTED BY SCENARIO INCLUSIVE OF FINANCIAL TRADES

Settlements	LMP	EP-CHP	FSP-I	FSP-II	EA-CHP
One	\$1,623	\$377	\$754	\$756	\$1,545
Two	\$146	\$272	\$828	\$46	\$164
Three	\$172	\$272	\$843	\$46	\$0

III. LARGE-SCALE TEST SYSTEM

A. Case Study

To demonstrate the difference between pricing policies on a more realistic example, we selected a known-to-be-challenging day (2013-05-11) from [16], which considers 100 hypothetical wind scenarios drawn from real-world data from the Bonneville Power Administration over the WECC-240 system. To stress the system further, load was increased 10% from its given value, resulting in a mean wind penetration rate of 26% for this day over 100 wind scenarios, with a maximum hourly variability (at hour 20) of 79% of load, and a maximum net-load at hour 15 in scenario 38. Load is modeled deterministically, and wind is considered a zero marginal cost resource which is fully curtailable. Finally, to create a slightly less flexible system, we down-selected from 50 fast-start resources to 27 fast-start resources out of a total of 85 thermal units.

As is common in power systems operations, it may be difficult to distinguish between different policies on “typical” days. In our selected case study, FSP-I and FSP-II resulted in identical prices across all scenarios and time periods. It should be noted that these identical prices were obtained from the optimal solution of a modestly sized test system. Suboptimal commitments are common in real-world systems, in which case the FSP-II policy’s results would be relatively stable while those of FSP-I could change substantially [17]. In this section, we merge the results into a single policy labeled FSP. Further convergence in policies can occur. In tests that used the original set of 50 fast-start resources instead of our selected 27, for example, FSP yielded the same prices as EP-CHP. Here we present an instance with some separation so as to distinguish between the different pricing policies.

B. Computational Setup

All computations were done on a MacBook Pro (16-inch, 2019) with an 8-core 2.4GHz Intel Core i9 processor with 64 GB of DDR4 memory. All optimization problems were solved using Gurobi 9.0.2 [18]. Stochastic unit commitment problems were formulated using `EGRET` [19], [20] and `mpi-sppy` [21], [22]. All stochastic unit commitment problems were solved using the “extensive form” with 0% optimality gap to ensure an optimal commitment schedule is obtained (within numerical tolerances). Suboptimal commitments are a practical reality in large markets and can have a large effect on pricing results [17]. Accordingly, the LMP and FSP-I results could change substantially if the first or second stage commitments are not optimal, whereas results would be relatively stable due to suboptimality in either stage for EA-CHP or the second stage for FSP-II. Pricing problems were solved using the “extensive form” for EA-CHP, FSP, and LMP. To ensure convex hull prices (or variants thereof) were obtained, the row-generation procedure introduced in [23] was adapted to iteratively refine the convex hull relaxations of individual generators within stochastic pricing problems. The deterministic-equivalent formulation of the problem, i.e., the load-balance and generator constraints, was also taken from [23].

The 100-scenario stochastic unit commitment used in this case study, with 85 thermal generators and a power-balance constraint over 24 time periods, was readily solvable by Gurobi within reasonable wall-clock times of less than 1 hour. With a larger test system or a transmission network further decomposition schemes would need to be considered; however, approaches such as lazy-constraint generation for transmission constraints and scenario decomposition techniques such as progressive hedging [24] are readily applicable to the pricing problems demonstrated here.

C. Average Prices

Figure 1 shows the average price in each hour under the four pricing schemes. In the low-demand hours of the early morning and late evening, all the pricing schemes result in similar prices, with EP-CHP tracking slightly above the other three. Relative to LMP and FSP, EP-CHP results in slightly elevated prices on average through the middle of the day. EA-CHP also result in higher prices than LMP overall, but price spikes are concentrated in hours 14 and 15. Because the set of fast-start resources is relatively small, LMP and FSP return similar results. In other instances with a larger number of fast-start resources, FSP could instead mirror EP-CHP.

Figure 2 shows the distribution of load-weighted average prices arising in the 100 scenarios, corresponding to the total revenue earned by generators over the course of the day. It can be seen that, while average compensation is similar under the four pricing schemes, EA-CHP leads to a wider spread between scenarios.

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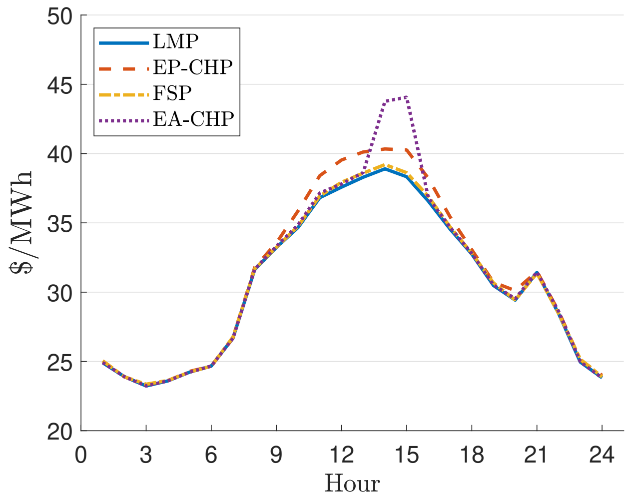

Fig. 1. Average Price by Hour

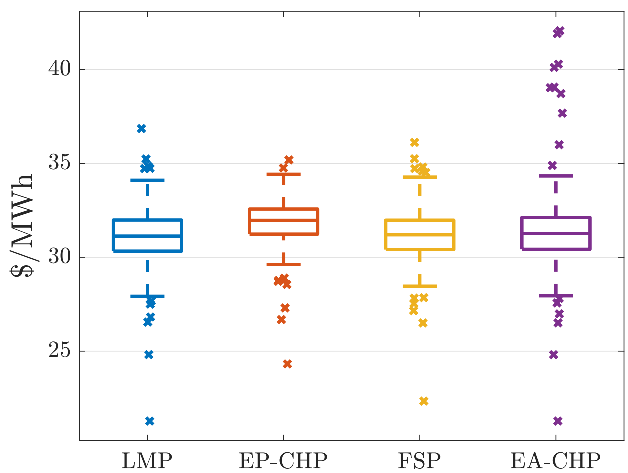

Fig. 2. Distribution of Weighted Average Prices

D. Scenario Prices and Incentives for Flexibility

The spread of daily compensation observed in Figure 2 suggests that under EA-CHP, generators may have more potential to monetize flexible attributes (e.g., being able to defer a commitment decision until closer to real time). Differing incentives for flexibility also arise in real-time operations. Figure 3 shows the paths that prices take in each scenario under each pricing scheme. As reflected by the averages in Figure 1, it can be seen that EA-CHP leads to more significant price spikes in the afternoon of some scenarios, corresponding to the hours and scenarios that drive the need for commitment of the highest-priced generators.

Since price volatility is an important signal for flexibility in operations, these pricing patterns suggest that EA-CHP may provide stronger incentives for long-run investment than LMP, EP-CHP, and FSP [25]. As a metric for price volatility, we compute the value

$$\frac{1}{100}\sum_{s\in\mathcal{S}}\sum_{t\in 2\ldots 24}|\lambda_{st}^{PS}-\lambda_{s(t-1)}^{PS}|,$$

where λ_{st}^{PS} indicates the price in scenario s and hour t under pricing scheme PS. Table VI reports the value of this price volatility metric under each pricing scheme.

TABLE VI
AVERAGE OF ABSOLUTE HOURLY PRICE DIFFERENCES

LMP ($/MWh)	EP-CHP ($/MWh)	FSP ($/MWh)	EA-CHP ($/MWh)
44.68	49.34	46.07	65.38

E. Scenario Profits and Forward Markets

The distribution of profits faced by generators at the time of commitment informs their willingness to follow a socially optimal schedule. The results in this subsection consider the ability of the pricing policies to support the socially optimal schedule, as well as the effect of risk trading in ensuring that incentives are aligned. Table VII shows the number of generators with negative expected profit under each pricing scheme before accounting for any side payments. We note that these values would be zero in a convex setting. Consistent with the theoretical development in Part 1, EA-CHP exhibits superior performance on this metric, with expected losses limited to a single generator incurring a loss amounting to 0.0002% of total expected operating cost. Because the EA-CHP prices are approximated, the near-zero expected losses provide considerable confidence that the approximation is very close for our test problems (see Remark II.2 in Part 1).

TABLE VII
NUMBER OF GENERATORS (OUT OF 85) WITH NEGATIVE EXPECTED PROFIT AND TOTAL NEGATIVE EXPECTED PROFIT AS A PERCENTAGE OF EXPECTED OPERATING COST.

	LMP	EP-CHP	FSP	EA-CHP
Number of generators	6	2	4	1
Relative expected losses	0.1877%	0.0433%	0.1517%	0.0002%

Suppose the market satisfies the assumptions of complete risk trading [7], [8], with the added assumption of at least one risk-neutral market participant. In this setting, the risk-adjusted probability attached by each market participant to each scenario $s \in \mathcal{S}$ would be 1%, and an Arrow–Debreu security for each scenario would be priced at \$0.01, returning \$1.00 in scenario s and \$0.00 otherwise [26]. Accordingly, by selling securities for each scenario in a quantity equal to its profit in that scenario, a generator could lock in its expected profit. Given this ability to trade, the expected losses in Table VII correspond to the make-whole payments that would be required under each scheme. As such, the need for make-whole payments would almost be eliminated under EA-CHP, while the other policies leave greater need for side payments.

Now suppose that markets in risk are incomplete. In this case, market participants may not use the same probability measure, due either to differing underlying assessments or to risk aversion. The current two-settlement system used in U.S. electricity markets, for example, does not meet the

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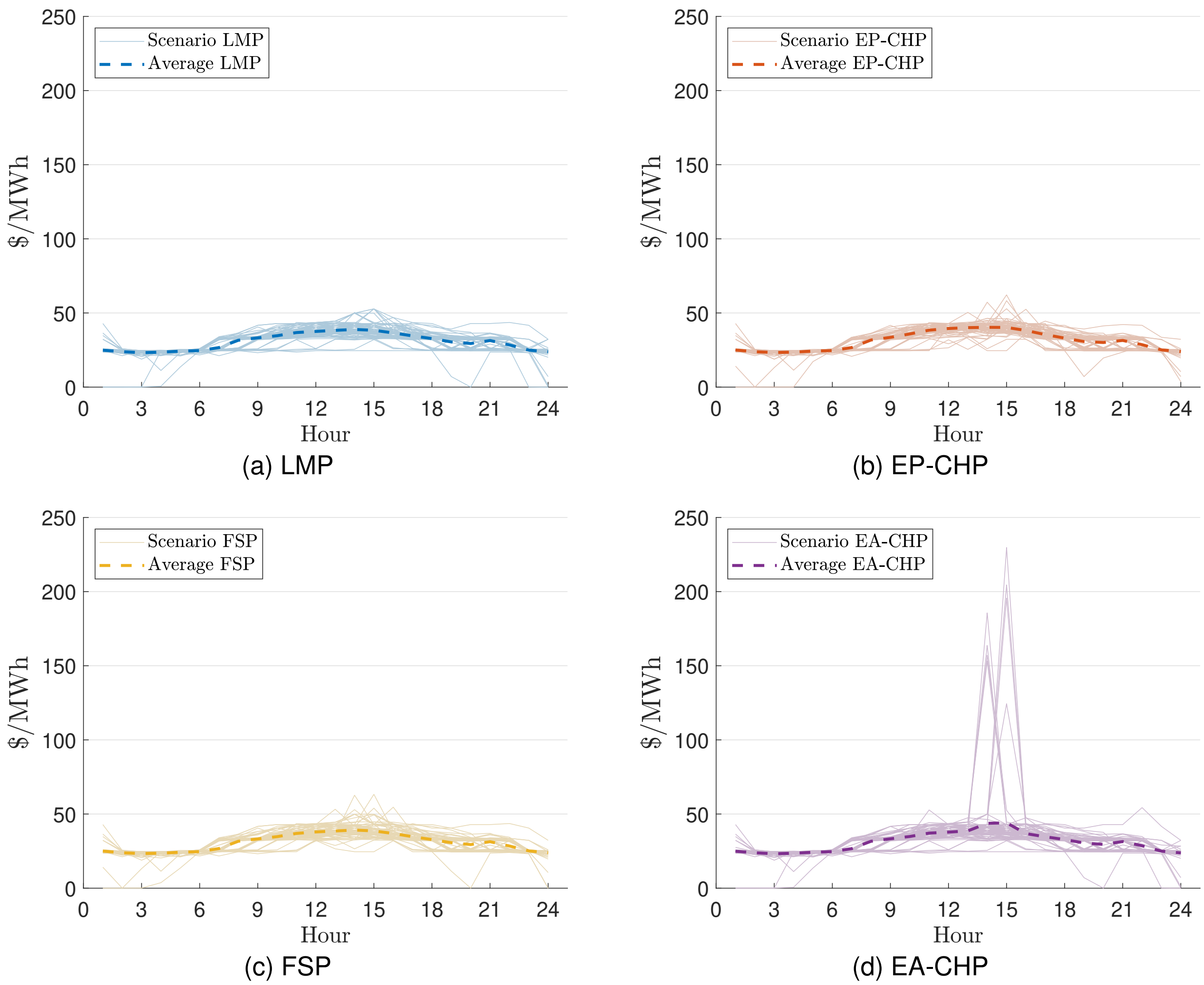

Fig. 3. Hourly Price in Each Scenario under Each Pricing Policy

idealized conditions of complete markets. In the example of Section II, introduction of day-ahead and intraday markets was enough to eliminate scenarios with losses under EA-CHP (Table IV). The forward markets defined in Part 1, however, are not sufficient to complete the market in the larger scale test system. Table VIII shows the expected make whole payments as a percentage of operating costs under each pricing scheme when trading is limited to these two contracts. In contrast to the results in Section II, we see even

TABLE VIII
TOTAL EXPECTED MAKE WHOLE PAYMENTS AS A PERCENTAGE OF EXPECTED OPERATING COST.

Settlements	LMP	EP-CHP	FSP	EA-CHP
One	0.4658%	0.1560%	0.4037%	0.4298%
Two	0.3943%	0.0842%	0.3645%	0.1708%
Three	0.3141%	0.0757%	0.2989%	0.1157%

that three settlements is not enough to drive the expected make-whole payments to the idealized values presented in Table VII. The inability of EA-CHP to outperform EP-CHP under conditions of incomplete trading is an indication of the underlying price volatility combined with incomplete forward markets. In Section II, because all generators except for the always-profitable generator 0 were block loaded, the quantity sold by committed generators was decoupled from the price. In the large-scale example, generators produce more when prices are higher, leading to a risk that cannot be hedged through fixed-volume swaps alone. The larger variability in prices in EA-CHP demonstrated in Figure 1 and quantified in Table VI means that market participants are unable to hedge completely in the modeled forward markets, driving relatively larger losses in a few scenarios. Comparatively, under EP-CHP such market participants experience smaller losses in several scenarios (but also smaller gains), driving expected make-whole payments lower.

The results in Tables VI and VIII suggest an important trade-off in the choice of a price formation policy. With greater price volatility, market participants may be exposed to a higher chance of losses. This potential for losses could in turn affect the offer behavior of risk-averse market participants, degrading efficiency in operations [7]. At the same time, price volatility is an important incentive to invest in resources that are flexible enough to take advantage of that volatility. Accordingly, suppressed volatility relative to the ideal could degrade efficiency on longer timescales [25]. Resolving this trade-off would entail both producing efficient underlying spot prices and ensuring the availability of a broader range of hedging instruments with low transaction costs.

IV. Conclusion

This article investigates the combined effect of uncertainty and non-convexity when evaluating policies for price formation in wholesale electricity markets. Our results emphasize the importance of correctly diagnosing the source of misaligned incentives for market participants. Policies developed on the basis of deterministic models or ex post analysis may be counterproductive, leading to poor incentives at the time generator commitment decisions must be made. In particular, uplift payments that appear "necessary" in a deterministic analysis may be revealed as inefficient subsidies in a stochastic analysis, while enhanced pricing schemes that neglect the effect of uncertainty may have the negative consequence of suppressing volatility in prices and hampering efforts to attract an efficient level of investment in flexible resources.

To help elucidate the economic phenomena the paper defines a new construct, ex ante convex hull pricing, that minimizes expected lost opportunity costs for market participants. Generators are nevertheless exposed to the possibility of realizing losses due to underlying non-convexity as well as the uncertainty inherent in electricity systems. Results from the case study indicate that intraday markets may help reduce the potential for losses, but more complete risk management would require introduction of option-like instruments enabling market participants to manage the positive correlation between price and quantity. In current markets, uplift payments and enhanced pricing schemes may have the effect of partially managing risk on behalf of participants. In doing so, however, they may introduce distortions and subsidies that reduce overall efficiency.

Despite its appealing theoretical properties, EA-CHP faces an important epistemic challenge to implementation in practice. While ex ante prices in the paper are determined through a stochastic program that includes all possible future states of the world, a real-world system would encounter scenarios in real-time that were not included in the model. With this complication, it is not clear how to produce EA-CHP prices in real time. Just as an inability to produce exact EP-CHP prices has not prevented market operators from implementing approximations, however, our results suggest that market designers should seek workable policies able to approximate the properties of EA-CHP. Conceptually simple approximations will be the most compelling candidates for use in practical settings, possibly being formulated as an ex post pricing policy with similar form as currently implemented fast-start pricing models. In addition to variants of schemes proposed in the deterministic context, an alternate route to forming prices that provide similar incentives to EA-CHP may be through the design of operating reserve demand curves [14]. Further tests on multiperiod stochastic models are needed to assess the empirical performance of competing proposals.

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V. Biography Section

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