

# Earth Rheology, Isostasy And Eustasy

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## EARTH RHEOLOGY AND LATE CENOZOIC ISOSTATIC MOVEMENTS

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The magnetic horizon\* by the Swedish writer Professor Carl Fredrik  
Hornbomstedt (Helsingfors-Lausanne, Switzerland), printed in 1975.

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## Interpretation of Postglacial Isostatic Adjustment

### Phenomena in Terms of Mantle Rheology

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#### Abstract

Non-linear mantle rheologies (i.e.  $\sigma^n$ ,  $\rho \sigma^n$ ) and linear mantle rheologies ( $\sigma = \eta$ ) where viscosities increase strongly with depth have a common response to loading and unloading in peripheral zones outside but near the loaded areas: peripheral bulges form on loading and a peripheral trough upon unloading. The response of a linear mantle rheology with constant viscosity as a function of depth is exactly opposite the response of the previous two cases in the peripheral zones and appears to be unique. The behavior of peripheral stresses is therefore a key to mantle rheology and/or viscosity structure. The uplift history of areas peripheral to the ice caps which covered Canada and Fennoscandia are examined for their implications to mantle rheology. In the Canadian case postdeformations of the earth's gravity field caused by the ice melting are taken into account. Because gravity changes affect local sea level they could affect the geological evidence of uplift which is reported in terms of the elevation of post sea levels. Geoid effects are shown not to change previous interpretations of peripheral Canadian uplift data. Peripheral uplift followed by subsidence indicates an  $\sim 10^{22}$  P Newtonian mantle. Geoid perturbations do make the observed lack of Hudson high sea level in the south Pacific a more severe constraint on global melting and lower mantle viscosity than it would otherwise be and suggest the viscosity of the lower mantle is non-linear less than  $10^{22}$  P. Recent data on the isostatic uplift of Fennoscandia shows the zero uplift isostatic rebound exhibited at one location along the Swedish east coast for the last 3000 years. It corrects this evidence to suggest a fairly substantial asthenosphere underlain by an  $\sim 10^{22}$  P mantle in the Fennoscandian shield area.

#### 1. Introduction

Direct evidence of the long term ( $\sim 1000$  year) rheological properties of the earth's mantle is provided by the manner in which the earth adjusted to the surface load (ice and water) redistributions of the Pleistocene. Particulary constraining evidence is provided by the adjustment behaviour of areas adjacent to the region actually loaded or unloaded. This is because the adjustment behaviour of these peripheral regions is completely different for important classes of mantle rheology and

for different common assumptions concerning the viscous structure as a function of depth into the mantle (Cathrs, 1971, 1975). To my knowledge Daly (1954) was the first to draw attention to the importance of the uplift behaviour of peripheral areas. Immediately he expected that mantle material squeezed out from under glaciated areas would naturally tend to produce peripheral bulges. When the glaciers melted these peripheral bulges would collapse. Peripheral trough might even form

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temporarily upon unloading, before the peripheral glacial isostatic equilibrium was restored.

Daly could not find the appropriate geological evidence for peripheral bulges. In fact he felt, if anything, the behaviour of the peripheral areas was opposite to that expected on the basis of the above bulge hypothesis. He therefore developed a 'punching' hypothesis whereby glacially loaded areas punched through the lithosphere but friction on the punched-through zones influenced the peripheral lithosphere and eliminated peripheral bulges by forcing mantle material to be redistributed at depth. The peripheral areas were dragged down somewhat by the lithosphere as the loaded regions sank and later uplifted, producing an initial synperiglacial peripheral behaviour. Daly felt such behaviour was indicated (weakly) in the geologic record. When the ice melted, uplift of the unloaded regions caused friction on the punched through zones and dragged the peripheral areas up somewhat above isostatic equilibrium before the uplift in the central areas slowed and the peripheral areas sank back down.

Soon after publication of Daly's book, Hasckell (1935, 1936, 1937) showed that a lithosphere was not required to produce the initial synperiglacial peripheral behaviour desired by Daly *via* eliminate peripheral bulges) but that such behaviour would result if the viscosity of the mantle under the load were constant to a depth of a load diameter or so. In such cases deep flow occurred naturally and did not have to be forced by a rigid lithosphere. Peripheral areas responded initially in a synperiglacial fashion with the central load area and later rose (loading) or sank (unloading) back to equilibrium level much as Daly felt the geologic evidence suggested. Indeed Hasckell argued the only way (with a linear rheology) to produce (loading) bulges of the types envisioned in Daly's bulge hypothesis was to restrict flow to a viscous channel beneath the load.

Recent studies of the mode of postglacial uplift peripheral to areas that deglaciated in North America have tended to confirm Daly's judgement regarding the lack of evidence for a peripheral (loading) bulge (Frost, 1971, p. 346).

and confirm his feeling that the peripheral uplift was in fact initially synperiglacial to the central uplift. Areas along the east coast of the U.S. uplifted during and immediately after the melting of the ice of the last glacial, and have more recently been sinking (Kaye and Barghorn, 1964; Mörner, 1968, p. 442-443).

Cathles (1971, 1975) calculated the response of a self-gravitating viscoelastic earth to the load redistribution of the last deglaciation and concluded the observed isostatic adjustment in North America and elsewhere is consistent with and suggests a mantle with uniform  $\sim 10^{22}$  poise viscosity from the base of the asthenosphere to the core mantle boundary. The uplift behaviour of the east coast of the U.S. formed an important part of the analysis. Independent methods of calculation have given similar results and implied similar conclusions (Petit and Andrews, 1976).

Cathles (1975) also argued the observed mode of uplift, particularly the uplift behaviour of peripheral areas, required a Newtonian as well as a constant viscosity mantle.

Both Cathles (1975) and Petit and Andrews (1976) calculate uplift curves. Although gravitational effects are calculated neither account for the influence of gravity on ocean sea levels. Farrell and Clark (1976) point out the potential significance of good (sea level) perturbations caused by the Pleistocene load redistributions. Since mantle material rising up under Canada would gravitationally attract ocean water and lead to a rising sea level, gravitational effects could alter the nature of the geological evidence, perhaps enough to affect the interpretation of the peripheral bulge behaviour.

Several articles (Poet and Giffels, 1973; Brunton, 1974; Crough, 1977) have inferred mantle rheology from the form of central uplift in Fennoscandia and found it to be non-linear ( $n = 3$ ), not Newtonian ( $n = 1$ ).

In this paper we will first analyse the peripheral bulge phenomena from the point of view of mantle rheology. We show that, for a non-linear rheology ( $n = 3$ ), peripheral bulges should be formed upon loading (throughs upon unloading) that are at least as large as diamural. Thus geologic evidence mitigating

against diamural Newtonian flow mitigates as strongly or more strongly against a non-linear mantle rheology.

We next present calculations for good changes as a function of time for Models 1 (10<sup>22</sup> poise mantle) and 4 (10<sup>21</sup> poise mantle, 1000 km depth) given in Cathles (1971, 1975). Because the last melting of the glaciers was slow with respect to the rate at which the land and ocean basins adjusted in the 10<sup>22</sup> poise mantle case (Model 1), good effects are significantly diminished by the isostatic adjustment which takes place during the load redistribution.

Some consequences of good changes are noted for Holocene sea levels in islands of the Pacific. Good changes in the 10<sup>21</sup> poise mantle model make the absence of Holocene high sea levels a more stringent constraint on mantle viscosity than it would otherwise be. Present data probably suggest both a gradual buildoff of glacial melting and a lower mantle viscosity somewhat less than 10<sup>22</sup> poise.

Finally we examine the peripheral response of areas near the Canadian and Fennoscandian glacial systems. The effects of good perturbations are taken into account in the Canadian case and shown to make little difference in previous interpretations. Deep flow in a near constant viscosity linear mantle appears to be required by available data. The evidence in Fennoscandia is far less constraining but probably also requires deep flow and a fairly fluid asthenosphere. It is shown that interpretation of only the central uplift history in Fennoscandia in terms of mantle rheology can be misleading since the conclusions depend entirely on the amount of uplift assumed to remain at present. On the other hand the apparent lack of migration of the zero uplift isobase in Fennoscandia over the last 9000 years suggests deep flow in an  $\sim 10^{22}$  poise mantle and a fairly substantial asthenosphere under the Fennoscandian shield.

## II. The Peripheral Bulge Response in a Non-linear Mantle

If mantle material, displaced by isostatic adjustment from under glacially loaded areas,

is to distribute itself under the world's oceans rather than pile up near the loaded areas as a peripheral bulge, the volume rate of adjustment of the oceans under the oceans melt water load must be at least as fast as the rate of adjustment of the land stress under the altered glacial load (see Fig. 1). Mathematically a 'no bulge' criterion may be written:

$$\frac{\partial h_{\text{OCEAN}}}{\partial t} < \frac{\partial h_{\text{PERIPHERAL}}}{\partial t} \quad \text{CRITERIA FOR NO PERIPHERAL BULGE} \quad (1)$$

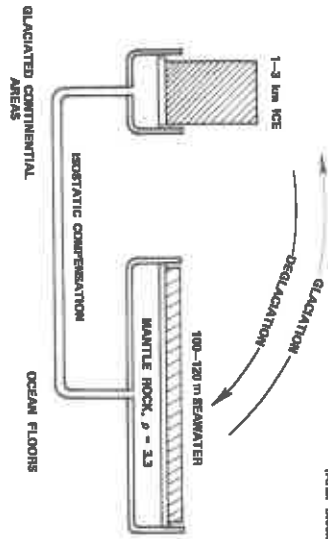
where  $A =$  mantle area near the earth's surface, and  $h =$  the displacement of the ocean basins or land from equilibrium. Using (1) it can be easily shown that the absence of a peripheral bulge can be naturally expected for Newtonian flow in a constant viscosity mantle but that if the upper mantle is more fluid than the lower mantle or if the rheology is non-linear, peripheral bulges must be expected upon loading and peripheral troughs upon unloading. The reasoning sketched below is tabulated in Table 1.

First consider linear rheologies. For any harmonic load regardless of the variation of viscosity with depth the rate of uplift ( $\partial h/\partial t$ ) and the amount of remaining uplift are related (Cathles, 1975, p. 43 ff):

$$\frac{\partial h}{\partial t} = \frac{h}{\tau} \quad (2)$$

If  $L$  is a characteristic dimension of the load or the wavelength of the load harmonic, the relaxation time,  $\tau$ , is proportional to  $L^2$ . If flow is confined to a diamural thin with respect to  $L$  (Takeuchi, 1963). Thus  $\partial h/\partial t$  is proportional to  $h/L$  or  $h/L^2$  with the result (since  $A_{\text{OCEAN}} \sim A_{\text{PERIPHERAL}}$ ) that the no bulge criterion is easily met in the case of deep flow (constant mantle viscosity) but not met in the case where flow is restricted to a diamural thin with respect to the dimensions of the Canadian glacial load area.

Brunton (1974) argued (and Crough, 1977, has verified by numerical calculations) that for a non-linear rheology with a creep coefficient



$$\frac{\Delta h_c}{\Delta T} < 1$$

$$\frac{A_c \Delta T}{A_o \Delta h_c}$$

Fig. 1. The criterion for no peripheral bulges is that the natural rate of ocean basin adjustment under the meltwater load be at least as rapid as the natural rate of adjustment of the glacially loaded areas. (After Bloom, 1967. Reproduced by permission of the Geological Society of America Inc. Copyrighted 1967)

Table I. Bulge criteria calculations for different rheologies and different flow geometries.  $\rho_m$  = density of the uppermost mantle

RHEOLOGY	1	1	3
$f$ or $\sigma^2$ , $n =$	1	1	3
FLOW GEOMETRY	Deep	Channel	Deep
UPLIFT DEPENDENCE			
$\frac{\Delta h_c}{\Delta T} \propto$	$AL$	$M/2^2$	$LP^2$
BULGE CRITERIA			
$\frac{A_c \Delta T}{A_o \Delta h_c}$	$\frac{L_c^2}{L_o^2}$	$\left(\frac{L_c}{L_o}\right)^2$	$\frac{L_c}{L_o} \left(\frac{h_c}{h_o}\right)^{-1}$
for $1/2 L_c \sim 1650$	$\sim 0.21$	$\sim 2\%$	$\sim 12\%$
for $1/2 L_c \sim 8000$			
$h_c \sim 2500$ m/ $\rho_m$			
$h_o \sim 100$ m/ $\rho_m$			

that is constant as a function of depth  $\Delta h/\Delta z \propto L_c^2$ . Table 1 shows that for reasonable parameter values the tendency to form peripheral bulges (loading) and troughs (unloading) should be even stronger for a non-linear ( $n = 3$ ) rheology than for flow restricted to a thin channel with linear rheology (i.e. 129 > 24). Any increase in creep coefficient with depth would increase the tendency toward 'channel' bulges and troughs by changing the flow geometry from  $L_c/L_o$  to  $(L_c/L_o)^2$ . The above analysis is very simple but two conclusions seem certain:

(1) Because of the difference in applied stress under glacially loaded (or unloaded) areas and oceans (meltwater efflux or influx) a non-linear rheology will have a very strong tendency to produce 'channel' type peripheral bulges (loading) and troughs (unloading). The equated dependence of adjustment rate on applied stress means the continental areas will adjust much faster than the ocean basins. Peripheral bulges will result because mantle material cannot get to the ocean basins fast enough.

(2) The tendency of a non-linear mantle to produce peripheral (loading) bulges will be at least as great as in the case of Newtonian flow restricted to a thin channel. Thus if available geologic data mitigates against channel flow (i.e. against mantle viscosities which increase with depth), the data mitigates even more strongly against an effective (i.e. so far as isostatic adjustment phenomena is concerned) non-linear mantle rheology.

III. Model Geoid Changes Attending Last Deglaciation

Fig. 2a-i gives the calculated changes in global geoid elevation of a model self-gravitating viscoelastic earth with a uniform  $10^{22}$  poise mantle (Model 1) as a function of model time for the case of a load redistribution similar to the one which occurred on the earth between 19,000 years ago and present. Details of how the ice load was redistributed to the world's oceans and details of the computational method and model are given in Cushman (1975). A feeling for the model

deglaciation is provided by the meltwater curve shown in Fig. 3.

Isostatic equilibrium was assumed to have been attained under the glacial load. The model ice masses began to melt 19,000 years ago. The ice melted first in Western Canada and southern and eastern Europe. As the ice melted and the oceans filled, isostatic adjustment began. The degree of isostatic disequilibrium reflected itself in perturbations of the geoid. Where the land surface lay below isostatic equilibrium (i.e. deglaciated areas) relative to the load distribution at the time, the geoid was depressed. Where the earth's surface lay above isostatic equilibrium (i.e. in the oceans) the geoid was elevated. The larger the scale of any given degree of isostatic disequilibrium, the greater the relative perturbation of the geoid.\* At any given time sea level parallels the geoid surfaces everywhere. No account was taken of the load redistribution that would be caused by the perturbation of the geoid, as done by Farrell and Clark, 1976. These effects are small. Considering the maximum geoid perturbation by the original meltwater increment ( $\sim 109$  m) was  $\sim 10$  m, the perturbation of the geoid due to the load redistribution caused by the geoid perturbation should be  $\leq 1$  m.

Fig. 2a-i shows the geoid was perturbed by the Wisconsin load redistribution as one might expect. At the start of deglaciation the perturbation was zero. As deglaciation and adjustment proceeded the perturbation of the geoid increased, reaching at  $T_{\text{model}} = 8000$  yrs BP a minimum of  $\sim -57$  m in central Canada and a maximum of  $\sim +11$  m in the Pacific south east of Australia. At present ( $T_{\text{model}} = 0$ ) the geoid is hardly deformed at all ( $-4.3$  m in central Canada and  $+0.9$  metres SE of Australia). The model earth is quite near isostatic equilibrium. The degree of geoid perturbation was greatly diminished by the fact with a  $10^{22}$  poise mantle the low order load harmonics all decay with decay times  $\geq 2500$  years whereas glacial melting took

\* Unsurprisingly the approximate formula for geoid perturbation on a perfectly rigid earth is followed surprisingly closely for all order numbers (i.e. see Cushman, 1975, p. 267 equation IV-34).

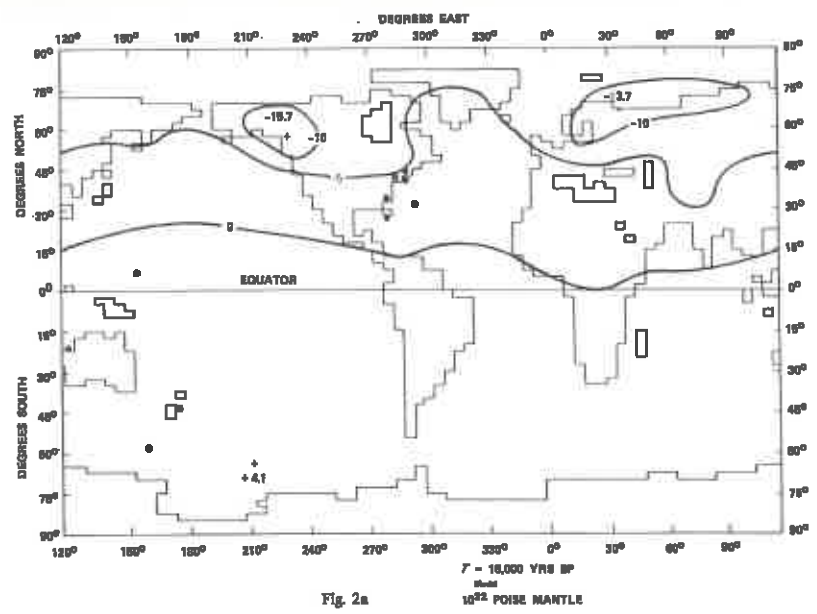


Fig. 2a

Fig. 2a-i. Displacements in metres of the geoid from equilibrium due to the load redistribution caused by the melting of the Late Wisconsin glaciers are given as a function of time. A constant viscosity  $10^{22}$  P linear mantle is assumed (Model 1). Ice melting began 19,000 years before present and continued to 6500 years before present (see Fig. 3). Details of glacial retreat, ice melting, and methods of computation are given in Cathles (1975). The isostatic adjustment which occurred as the ice melted substantially reduced the magnitude of geoid perturbations (see Fig. 4)

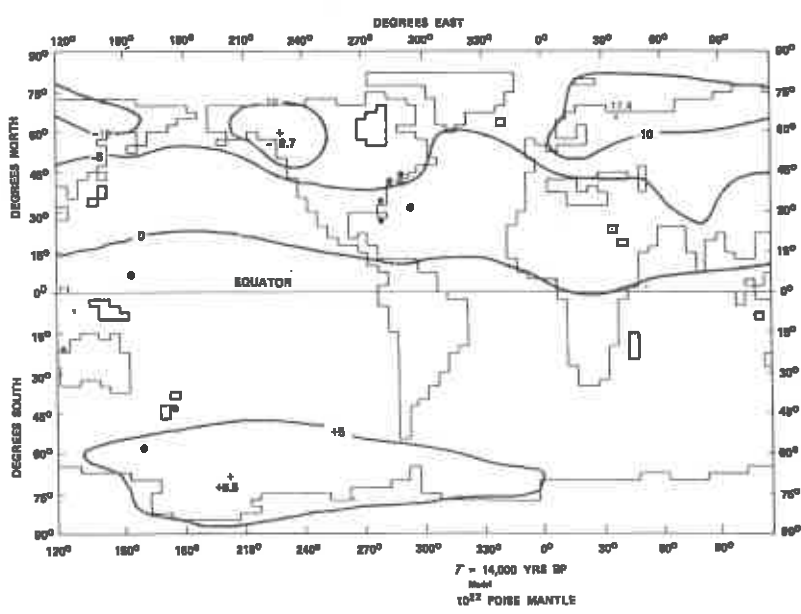


Fig. 2b

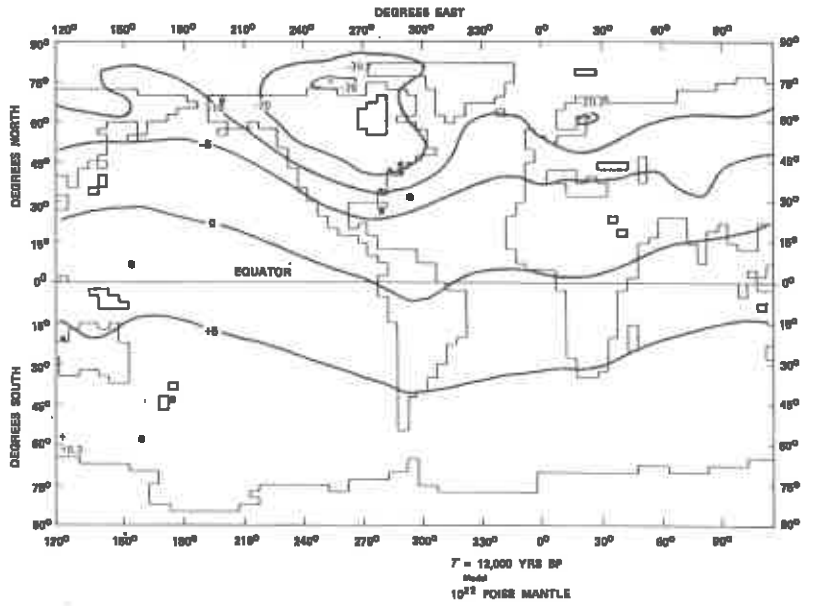


Fig. 2c

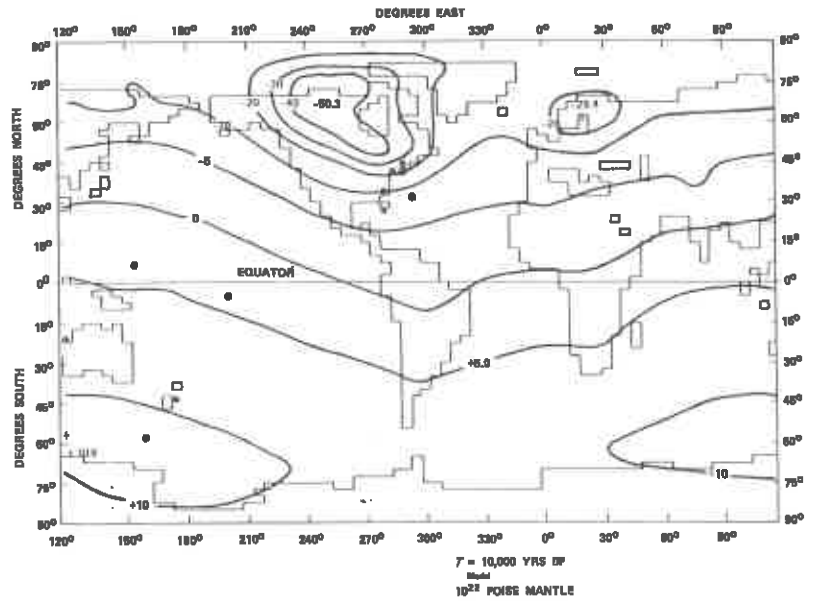


Fig. 2d

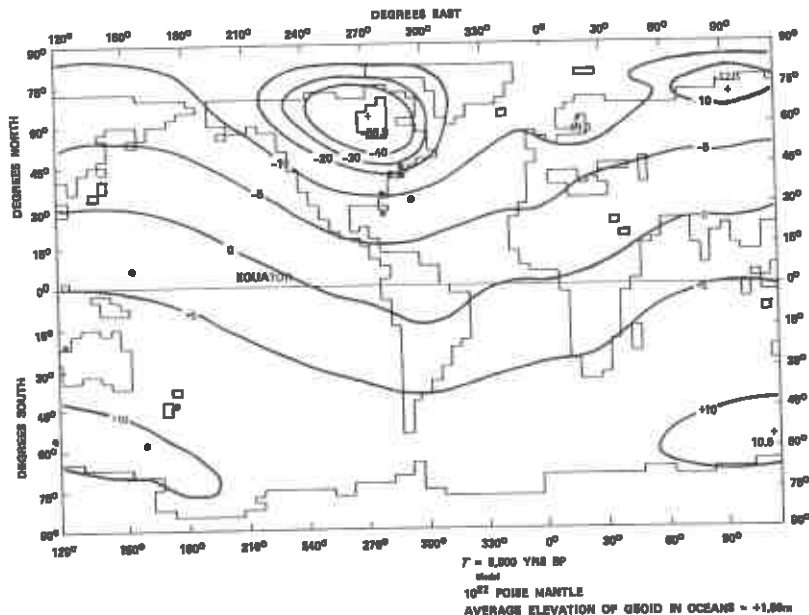


Fig. 2e

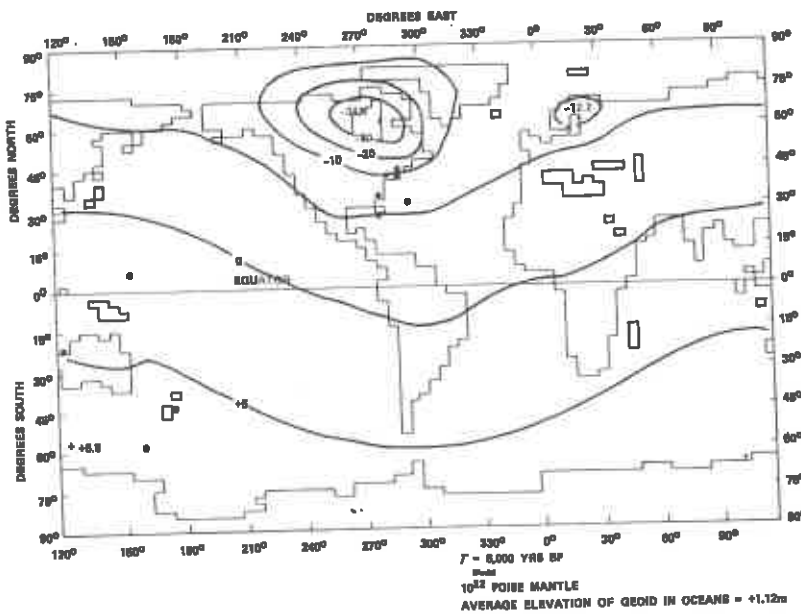


Fig. 2f

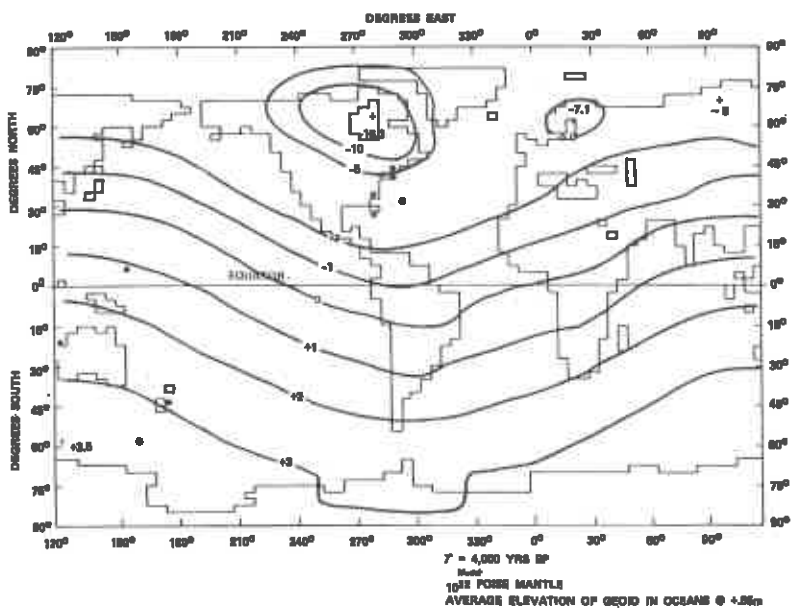


Fig. 2g

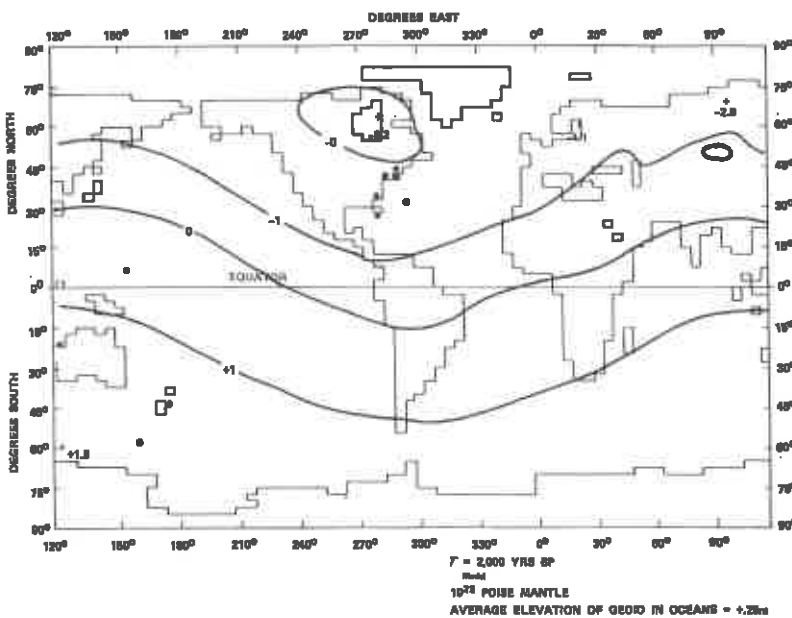


Fig. 2h



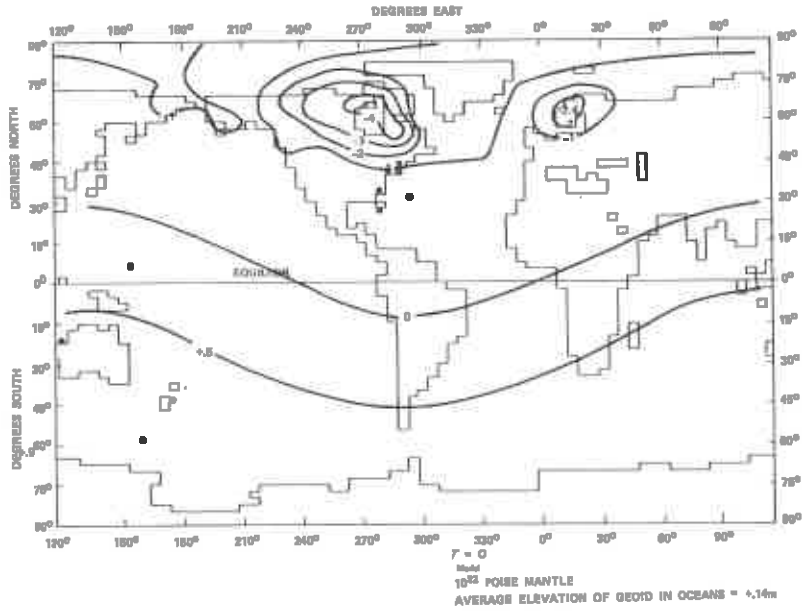


Fig. 2i

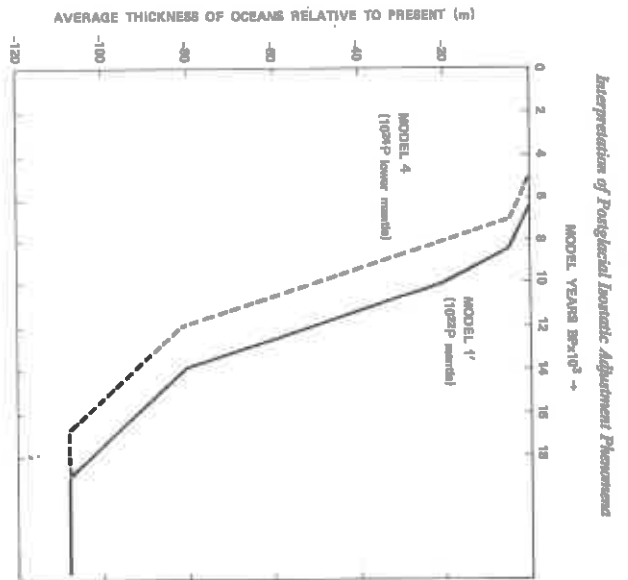


Fig. 3. Meltwater curves for the two models discussed in the text. The difference between the two model curves is roughly that present time is taken to be 2000 years later in Model 1 than in Model 4. In both cases the ice took  $\sim 10,000$  years to melt.

$\sim 10,000$  years to accomplish (Fig. 3). Thus the load harmonics that contribute most strongly to geoid deformation (the low order or long wavelength harmonics) had time to substantially decay during the load redistribution itself.

The deformation of the geoid for Model 4 ( $10^{22}$  poise mantle below 1000 km depth,  $10^{22}$  poise above 1000 km; somewhat different history of ice redistribution—see Fig. 3) is shown in Fig. 4 a and b. It is clear a  $10^{22}$  poise lower mantle substantially prevents isostatic adjustment of the ocean basins. The geoid perturbations are still very large even at  $T_{\text{model}} = -2000$  BP (corresponds to  $T_{\text{model}} = 0$  BP for the  $10^{21}$  poise model, Model No. 1).

#### IV. Hudsonian Sea Levels of Subarctic Pacific Localities ( $10^{22}$ Poise Mantle)

Fig. 5 a to d show model emergence curves for selected localities in the Pacific. These localities (indicated by dots in Fig. 2) have in

Since the full glacial load was applied to Canada for only about as long as Canada has now been deglaciated (20,000 years at the most) the geoid maps are clearly not appropriate. They assume isostatic equilibrium at the time of deglaciation. We should have assumed such equilibrium prior to glaciation and considered the effects of the glacial cycle of glaciation and deglaciation.

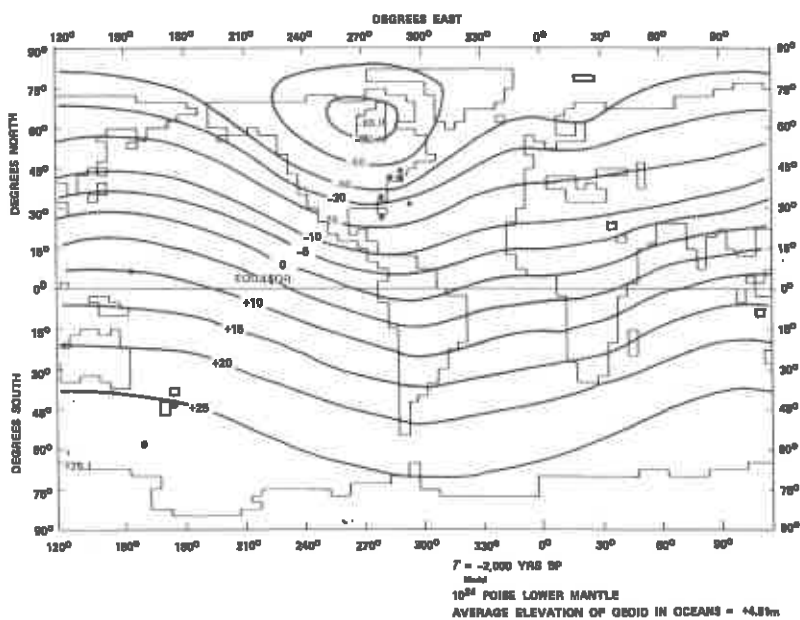
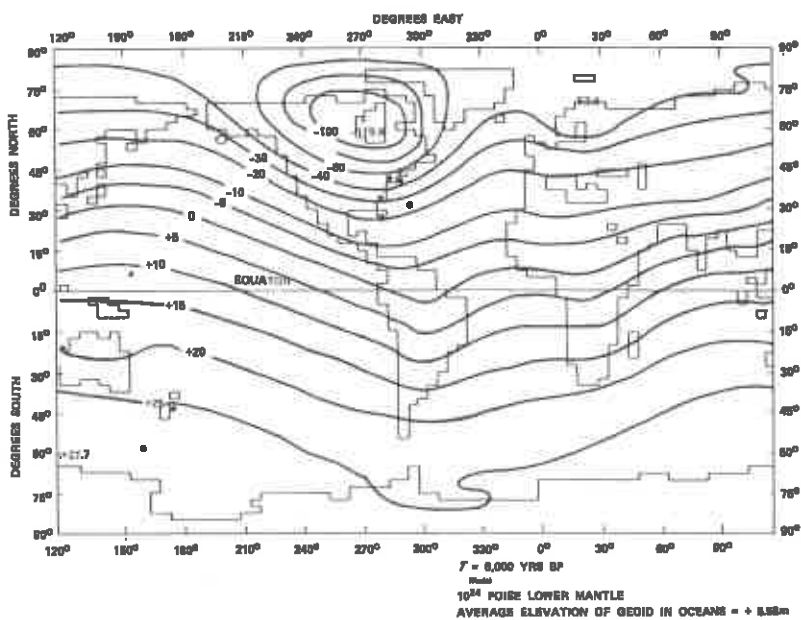


Fig. 4. Geoid perturbation in metres for Model 4 ( $10^{22}$  P upper mantle underlain by  $10^{24}$  P mantle below 1000 km depth). The high viscosity lower mantle stops isostatic adjustment of the ocean basins of the southern hemisphere over the time period analysed. Since the full glacial load was applied for at most 20,000 years the assumption of isostatic equilibrium at the start of deglaciation is clearly not correct. A load cycle of glaciation and deglaciation with isostatic equilibrium prior to glaciation must be considered to compute realistic geoid maps for slow responding models such as this one

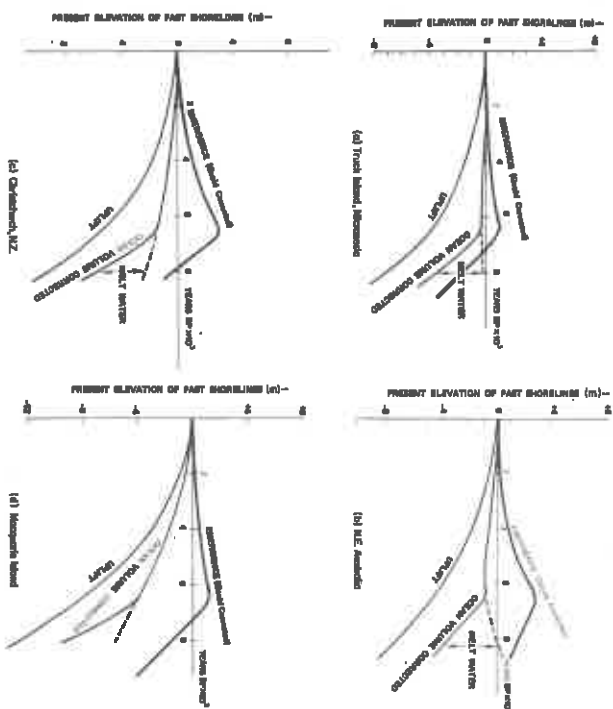


Fig. 5. Uplift curves for islands in the Pacific (see dots in Fig. 2) are corrected first for ocean beam anti-diffusion and meltwater influx (ocean volume corrected curve) and then for the tilting of the geoid ( $10^{22}$  P model of Fig. 2) to obtain an emergence curve. Geoid corrections make the observed lack of Holocene high sea levels (see Fig. 6) a more severe constraint on the last stages of ice melting and on lower mantle viscosity.

general sink over the last 8000 years under the influence of 109 metres of model meltwater influx into the oceans. (The total isostatic subsidence under such a load is  $109 \rho_w/\rho_m = 109/3.513 \text{ g/cm}^3 = 32.6 \text{ m}$ , where  $\rho_w = \text{density of water}$ ,  $\rho_m = \text{density of upper mantle}$ .) In most cases, however, the sinking of the localities is quite close to the average sinking of the ocean basins as a whole. Thus when changes of ocean volume are accounted for the localities behave like 'tip sticks' (Bloom, 1967) and the ocean level at the locality does not change much ('Ocean Volume Corrected' curve).

From Fig. 2 it is clear that the geoid is

tilting as time goes on. At  $T_{\text{model}} = 8000$  years BP it is elevated 7 to 10 m in the South Pacific. At  $T_{\text{model}} = 0$  it is elevated at most  $\sim 1$  metre. The degree of tilting increases as one moves north or south from approximately the zero line in the diagrams of Fig. 2.\* The tilting of the geoid must of course be taken into account in considering changes in sea level at any locality and represents the final corrections to obtain the emergence curves in Fig. 5.

\*Approximately from the zero line because geoid changes will be considered relative to the average geoid elevation over the oceans.

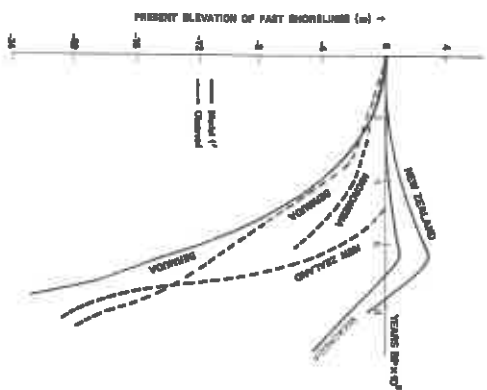


Fig. 6. Solated calculated (Model 1',  $10^{22}$  P mantle) emergence curves are compared to those observed. Observed curves are taken from a compilation by Mörner (1971). The relationship between the curves are similar in the observed and calculated cases. The curves may be brought into better agreement by tilting of the melting of the glaciers more gradually and by decreasing the viscosity of the lower mantle.

The geoid corrections clearly make the meltwater influx probably did not terminate as abruptly or at as early times as indicated in the Pacific a more stringent constraint on the details of glacial melting and lower mantle viscosity (rate of ocean basin adjustment).<sup>4</sup> Fig. 6 compares the model emergence curves for three islands (see next section for Bermuda curve) to the emergence curves observed. The relationships between the model curves is similar to that between the observed curves, but the form, particularly the Holocene high sea levels in the model curves, is not correct. Two solutions are indicated. First the

<sup>4</sup>There continues to be some discussion on whether or not Holocene high sea levels are observed in the South Pacific or not. See contributions by Mörner and Stouffer in this volume.

Fig. 7 compares computed and observed curves for the  $10^{22}$  P lower mantle model whose geoid changes are given in Fig. 4. Bermuda, on the peripheral trough around Canada, is

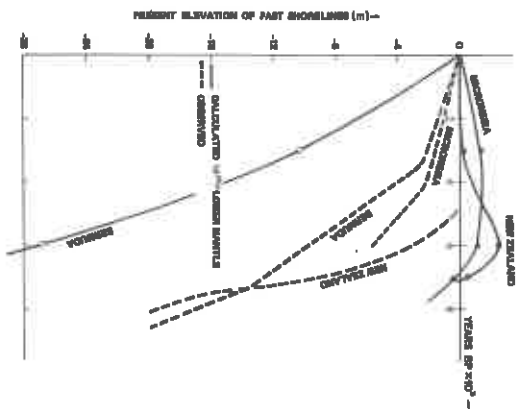


Fig. 7. Elevation curves calculated for Model 4 ( $10^{24}$  p mants below 1000 km depth,  $10^{22}$  p above) are compared to those observed. The sinking of Baxanda (which lies on the peripheral channel) through surrounding the Canada) is far more rapid than observed and the relationship between the sea level curves of Micronesia and New Zealand is not similar to that observed.

sinking far more rapidly than observed and the relationship between the Micronesia and New Zealand curves is not similar to that observed.

#### V. The Uplift Response of Areas Peripheral to the West Canadian Ice Cap

Calculation of grid changes attending isostatic adjustment allow us to correct the uplift curves of Model No. 1' ( $10^{24}$  p mants) and Model No. 4 ( $10^{24}$  p mants below 1000 km depth,  $10^{22}$  p above) presented previously (Cathie, 1975) for gravitational effects. The uplift history of localities along the east coast of the U.S. was taken by the author in previous work to be a particularly significant indicator of mantle viscosity and rheology.

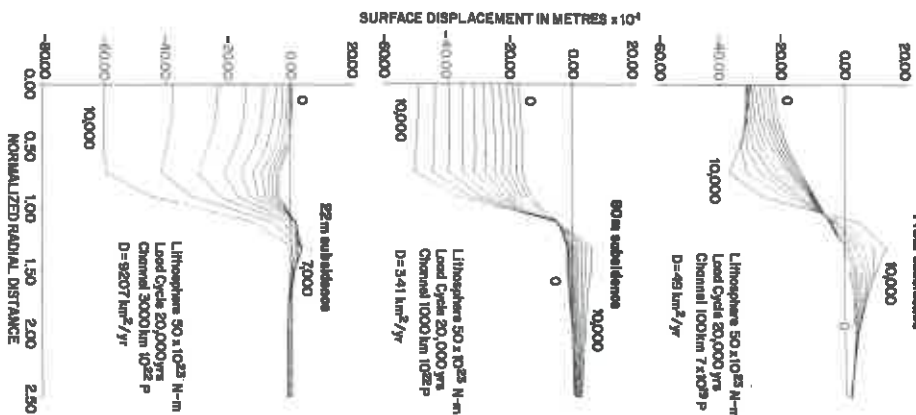


Fig. 8. Half space calculations (no gravity, no viscoelastic effects) for flow restricted to domains of different thicknesses are given. In all cases a load 1650 km in radius (Canadian ice cap limit) with square edges was placed on an infinite viscous channel of the indicated thickness overlain by a lithosphere with flexural rigidity  $50 \times 10^{25}$  N-m (i.e. an 88 km elastic layer with Young's modulus  $8.34 \times 10^{11}$  dynes/cm<sup>2</sup>) for 20,000 years and then removed. The time of removal is assumed to be 10,000 years before present. Uplift profiles are given every 1000 years from 10,000 BP to present. The deep flow phenomena of peripheral uplift followed by subsidence are evidenced in the last case where the channel thickness is equal to that of the entire mantle. For convenience the channel dimensions are also given in km<sup>2</sup>/year ( $D = \rho g R^2 / \eta$ ,  $\rho$  = density of upper mantle,  $g$  = gravitational constant,  $R$  = channel thickness,  $\eta$  = viscosity).

Fig. 8 reviews the significance of peripheral uplift behaviour. The uplift profiles in this figure are for a flat earth with a viscous channel of various thicknesses. A circular, square edged Canadian sized load 1650 km in radius is applied to the half space with a surface lithosphere for 20,000 years and then suddenly removed. Uplift profiles are given every 1000 years after the sudden displacement. Despite the simplicity of the calculation the contour of the second two models are reasonably close to the similar models discussed in this paper which include elastic and gravitational effects and a realistic gradual load removal.

The calculations of Fig. 8 show the magnitude of postglacial peripheral subsidence is diminished greatly as the thickness of the

channel in which flow is permitted is increased. In the last case the channel thickness is great enough (the entire thickness of the mantle) that the deep flow phenomena of peripheral

uplift followed by subsidence is evidenced. In this last case a maximum early post-glacial uplift of about 42 m is followed by only 22 m of subsidence. In the other cases the post-

glacial subsidence is 143 and 80 m respectively. Thus a uniform viscosity mantle is a good way to minimize the magnitude of post-glacial peripheral subsidence. In addition the last model is unique in having late glacial and early postglacial uplift followed by subsidence.

Sea level data along the east coast of the U.S. apparently indicate just such a late glacial and early postglacial uplift followed by subsidence (Cathles, 1975; Mörner, 1969, p. 442-443; Kaye and Barghoorn, 1964). This evidence alone indicates a ~ uniform

viscosity Newtonian mantle unless other phenomena can give an apparent late glacial uplift followed by subsidence.

The first model (100 km,  $7 \times 10^{21}$  poise channels) which shows more peripheral subsidence than central uplift, and a smaller central uplift than near edge uplift is clearly at odds with observations which show the greatest uplift in central Canada and small peripheral subsidence.

Fig. 9 addresses the question as to whether gravitational effects could help produce the

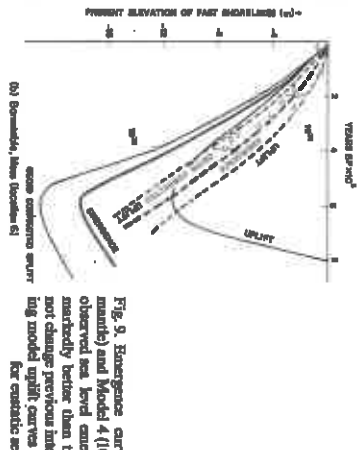
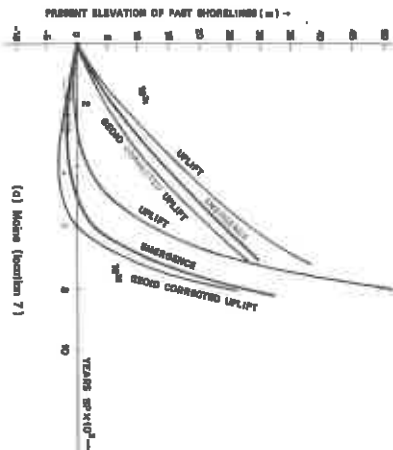
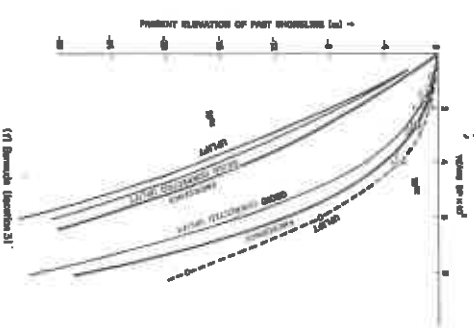
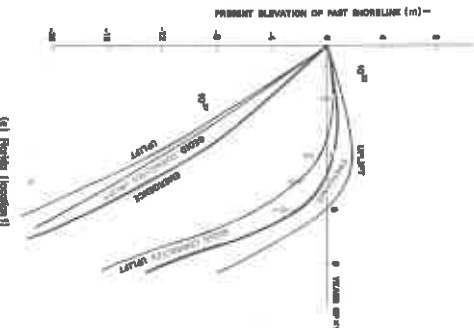
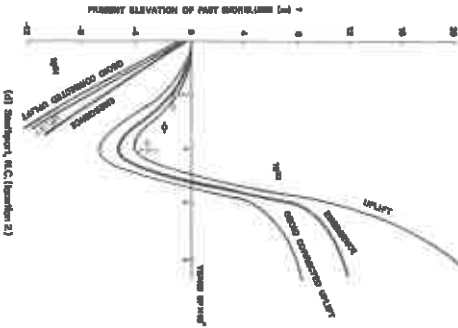
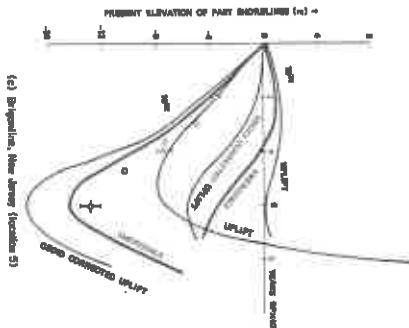


Fig. 9. Emergence curves calculated for Model 1' ( $10^{21}$  P mantle) and Model 4 ( $10^{24}$  P lower mantle) are compared to the observed sea level emergence data. The  $10^{21}$  P model agrees markedly better than the  $10^{24}$  P model. Gravitational effects do not change previous interpretation of this data made by comparing model uplift curves to observed emergence curves corrected for crustal sea level changes (Cathles, 1975)



observed peripheral uplift history. Previously calculated (Cathles, 1975) uplifts for Model No. 1' (10<sup>22</sup>P mantle) and Model No. 4 (10<sup>24</sup>P below 1000 km depth, 10<sup>22</sup>P above) are corrected first for the tilting of the geoid in the ocean basins (Fig. 2 and 4) and then for the isostatic adjustment of the ocean basins and freshwater influxes so as to obtain first a geoid corrected curve and then an emergence curve. The calculated emergence curves are compared to those observed along the east coast of the U.S.

Although geoid corrections can be 10 m or larger, corrections in the last few thousand years are small. The observed emergence curves are in markedly better agreement with the 10<sup>22</sup>P mantle model than with the high viscosity lower mantle model. Particularly critical data is provided by Maine, which shows late glacial and early postglacial uplift followed by subsidence (which can only be matched by a deep flow uniform viscosity model) and by the southern localities which show much less sinking than would be required by the unloading trough of a channel or high viscosity lower mantle model.

We conclude that gravitational effects will not substantially help high viscosity lower mantle models (i.e. channel models) fit the observed sea level data in the critical North American east coast area. A substantially uniform ~10<sup>22</sup> poise Newtonian mantle is still strongly indicated by data from this area even when geoid changes are accounted for in the calculations.

As discussed elsewhere (Cathles, 1975) once it is concluded the earth's mantle is Newtonian and of reasonably constant viscosity, other uplift evidence (the present rate of uplift in Canada and the history of that uplift) places quite rigid constraints on the viscosity of the mantle as a whole, and the common geomorphological practice of assuming an experimentally decreasing uplift is found to have a sound theoretical basis. Also the present rate of subsidence of the east coast of the U.S. is in good agreement with the observed present rate of subsidence assuming a present rate of sea level rise of 2 mm/year due to freshwater influx. Previously, without geoid

corrections, the present rate of sea level rise needed was 2.4 mm/year.\*

**VI. The Uplift Response of Aresen Fjordfjall to the Fennoscandia Ice Cap**

Several recent authors have inferred the mantle rheology is non-linear through analysis of the uplift of central Fennoscandia (Post and Griggs, 1973; Brenna, 1974; Crough, 1977). Post and Griggs (1973) for example point out that (if fluid properties do not change with depth) the central areas of an unloaded region will uplift in a manner such that

$$\frac{\partial h}{\partial t} = \frac{A}{c} \sigma^n \quad (3)$$

where  $n$  is the order of the stress-strain rate constitutive relation (i.e.  $c \dot{\epsilon} = \sigma^n$ , where  $\dot{\epsilon}$  = strain rate,  $\sigma$  = stress,  $c$  = creep coefficient),  $h$  is the amount of uplift remaining in the central area of uplift at any given time, and  $A$  is a constant which depends on  $c$ , the dimension of the unloaded area, the geometry of the zones where flow or creep is permitted, etc. Both Brenna and Crough give detailed expressions for  $A$ . We know the history of uplift in Fennoscandia quite well (Lidén, 1938; Lilbourey, 1971) and so know the velocity of central uplift,  $v = dh/dt$ , at past times. If we know, or could guess, the amount of uplift remaining at present we could add this amount to the uplift that has occurred between past times and present and so determine  $h$  as a function of time. Since (3) implies  $\log h = 1/n \log A + 1/n \log v$ , the slope of a plot of  $\log h$  vs  $\log v$  determines mantle rheology directly.

Fig. 10 shows a plot of  $\log h$  vs  $\log v$  assuming various amounts of isostatic uplift remain to be realized in central Fennoscandia. It is immediately apparent that the value of  $n$  deduced by such an analysis depends entirely

\* On assuming geoid effects a factor of error in Fig. 5b-c of Cathles (1975). In calculated uplift curves were based on a depth of 2.7 to the model. Since they should be based on the actual ice cap depth of 2.2, the error in the diagram is to their uplift curves which are between the observed and calculated (10<sup>22</sup> poise mantle model) rate of uplift along the US east coast (see text, but does not change any of the conclusions reached).

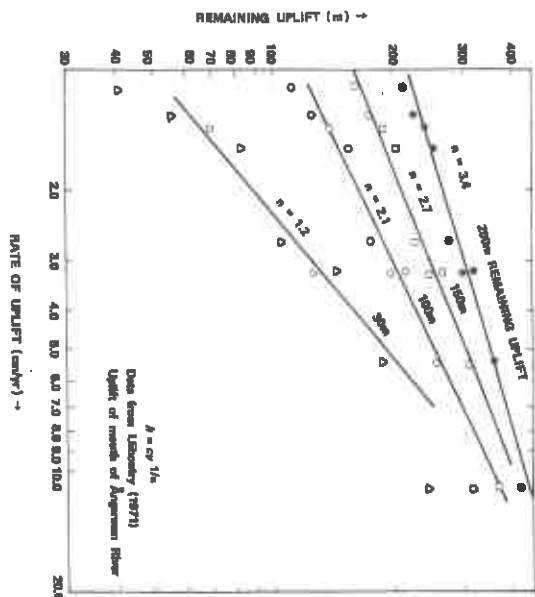


Fig. 10. Rate of uplift values for the month of the Ångerman River, Sweden, compared by Lilbourey (1971) are plotted against remaining uplift assuming various amounts of uplift remain at present. Values of the uplift that has occurred to present were obtained using Midson's (1969) subsidence curve to convert emergence data collected initially by Lidén (1938) to time added (for recent missing values) by Lilbourey (see also Swares, 1971). As discussed in the text this type of plot may be used to infer mantle rheology directly but results depend entirely on the amount of uplift assumed to remain at present.

on the amount of uplift assumed to remain at present. Fig. 11 shows the fit to the data is equally good whether  $n = 1$  or  $n = 3$ . The equivalent in that figure are simply the solutions to (3) with  $n = 1$  or 3 and constants redefined in a convenient fashion. The form for the solution of (3) with  $n = 3$  is exactly the same as the solution for the uplift of a surface depression shaped like gaussian trough with mantle flow restricted to a channel that is thin with respect to the load dimension. This problem was solved and applied to the Fennoscandian uplift by Van Bemmelen and Berlage (1935, p. 32, equation 12). The parameters values used in Fig. 11 are those suggested by Lilbourey (1971), who more recently made an analysis similar to Van Bemmelen and Berlage's.

At face value selected negative gravity anomalies in Fennoscandia of ~30 mgal could be interpreted as they have been in the past (Niskanen, 1939; Kästner, 1953) to suggest ~210 m of uplift remain. This would suggest a non linear mantle rheology. However, more recent gravity maps (Honkaniemi, 1964, 1966) show gravity anomalies have little correlation with present or past uplift. They correlate well with lithologic changes in the area. Most of the gravity anomalies can therefore reasonably (I think more reasonably)

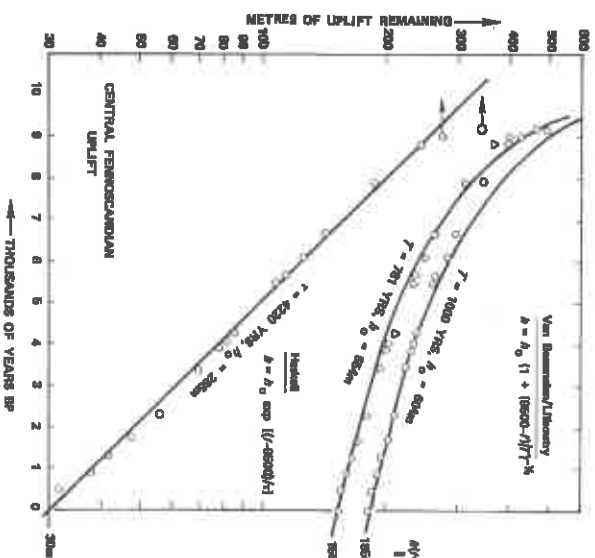
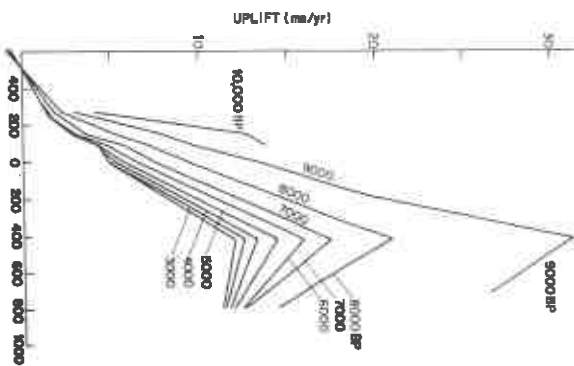


Fig. 11. Uplift data for mouth of Angerman River, Sweden (see Fig. 10) is plotted against theoretical curves for deep flow in a constant viscosity mantle (exponential curve) and for flow restricted to a thin channel or non-linear ( $n = 3$ ) flow. The two types of curves fit the data equally well provided the amount of uplift remaining at present is shown appropriately. Arrows represent possible correction for missing values in Liuhory's curve chronology (Sjaveri, 1971)

be attributed to density changes in the crust supported by an elastic lithosphere. Only a small large scale anomalies ( $\sim 3.5$  mgal) then remain, which suggest a central uplift of  $\sim 25$  m. This implies a Newtonian ( $n = 1$ ) mantle rheology (Fig. 10) Bailing (this volume) shows a negative gravity anomaly too broad to be supported crustally results if gravity is expressed as a residual from an empirical relation between bouguer gravity anomalies and elevation.

Since determination of the fraction of gravity anomalies in Finnoscandia attributable to presently remaining isostatic







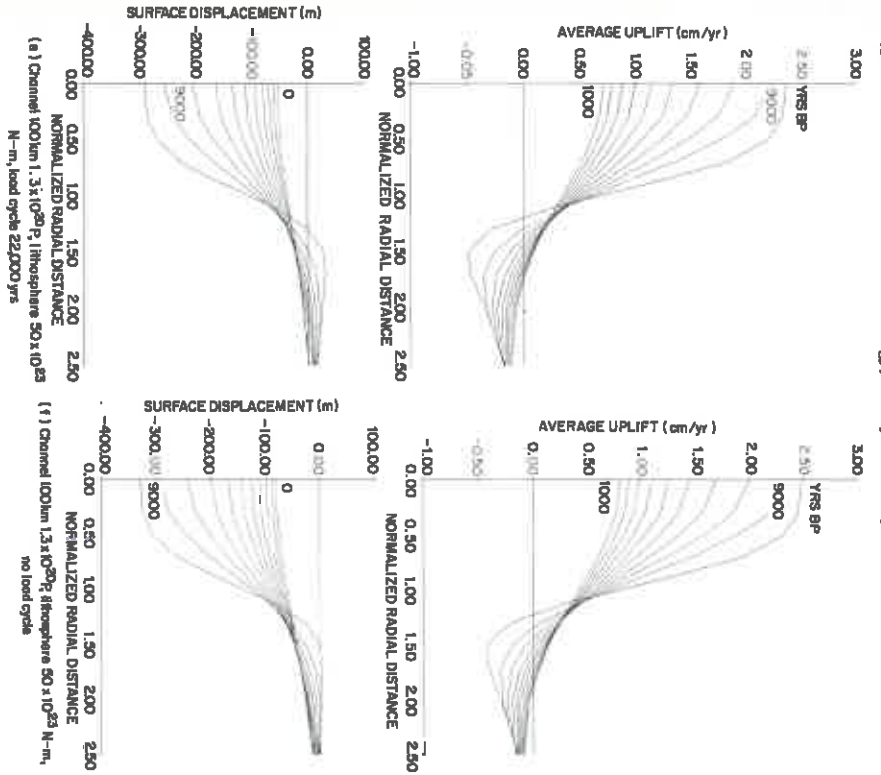


Fig. 14 shows the models of Fig. 13c and d (particularly 13e) produce decay spectra in good agreement with the decay spectra deduced from the Fennoscandian uplift data by McConnell (1968). Stationary zero uplift position is associated with a decay curve which

has nearly the same values of  $\tau$  over a range of  $\lambda$  values, i.e. with the change over from dominantly channel flow to dominantly deep flow. This is reasonable since, if the zero uplift isobase migrates inward for deep flow and outward for channel flow, it should remain

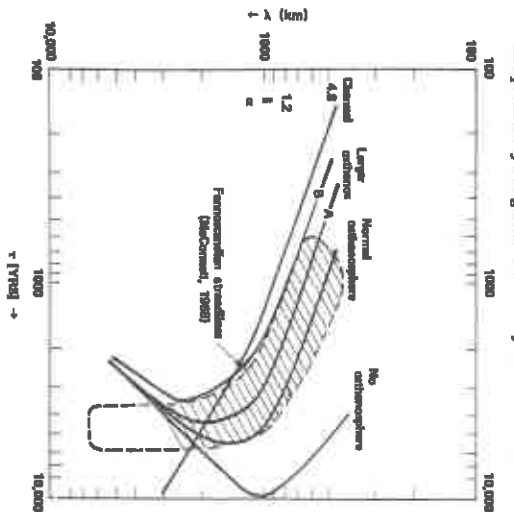


Fig. 14. The decay spectra of the model of Fig. 13e (curve A) is in excellent agreement with decay spectra inferred by McConnell (1968) from uplifted isobathlines in Fennoscandia. As discussed in the text absence of zero uplift isobase migration is associated with the transition from channel flow to deep flow, i.e. spectral curves which have smaller decay times for a range of wavelengths. The downward curving of McConnell's data, initially suggested deep flow. Channel flow decay spectra do not bend downward (Cahillie, 1973, p. 183).

lined for some suitable combination of the two. The important point is that deep flow is required to produce a stationary zero uplift isobase. Channel flow alone will not do. As shown by Morner in this volume the lack of migration of the zero uplift isobase is not restricted to the Swedish east coast but is shown in other profiles across the ice margin to the north west and east.

It should be cautioned that this analysis is oversimplified in the sense that no account of gradual ice melting or retreat is made. Although it is difficult to visualize how gradual removal can make a very substantial difference, the validity of the analysis could depend in large part on how significant the Salpausselkä etidland was in the total glaciation history phase.

#### VII. Conclusions

(1) The tendency of different flow geometries and rheologies to produce peripheral

- bulges upon loading and troughs upon unloading is evaluated. Non-linear rheologies ( $n=3$ ) should generate peripheral bulges around loaded regions that are at least as large as those produced when linear fluid flow is restricted to a thin channel (Table 1). Thus evidence against channel Newtonian flow is also evidence against effective (so far as glacial rebound phenomena are concerned) non-linear mantle rheology.
- (2) Perturbation of the earth's geoid that occurred during and after the melting of the last (Weichselian) glaciers is calculated for two mantle models (uniform  $10^{21}$  erg mantle in Fig. 3;  $10^{21}$  erg upper mantle,  $10^{22}$  erg mantle below 1000 km depth in Fig. 4). For the  $10^{21}$  erg mantle case the maximum perturbation of the geoid was  $-57$  m in central Canada and  $+11$  m in the south Pacific. Geoid perturbation is substantially reduced by isostatic adjustment which occurred as the glaciers melted.
- (3) Geoid perturbation makes the observed
- (4) Taking into account geoid perturbations does not affect the interpretation of critical sea level data along the east coast of the U.S. which directly requires a constant viscosity  $\sim 10^{21}$  Newtonian mantle (Fig. 9).
- (5) Interpretation of central Fennoscandian uplift in terms of mantle rheology give Newtonian or non-linear ( $n=3$ ) results depending upon the amount of uplift remaining in central Fennoscandia at present (Fig. 10).
- (6) The observation that the zero uplift isohase has remained fixed for at least the last 9000 years along the Swedish coast may indicate that Fennoscandia is underlain by a fairly fluid asthenosphere and a Newtonian mantle of constant  $\sim 10^{21}$  erg viscosity (Fig. 15c).

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- (4) Taking into account geoid perturbations does not affect the interpretation of critical sea level data along the east coast of the U.S. which directly requires a constant viscosity  $\sim 10^{21}$  Newtonian mantle (Fig. 9).
- (5) Interpretation of central Fennoscandian uplift in terms of mantle rheology give Newtonian or non-linear ( $n=3$ ) results depending upon the amount of uplift remaining in central Fennoscandia at present (Fig. 10).
- (6) The observation that the zero uplift isohase has remained fixed for at least the last 9000 years along the Swedish coast may indicate that Fennoscandia is underlain by a fairly fluid asthenosphere and a Newtonian mantle of constant  $\sim 10^{21}$  erg viscosity (Fig. 15c).
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