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A simple analytical method for calculating temperature perturbations in a basin caused by the flow of water through thin, shallow-dipping aquifers

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Abstract—Fluid flow in sedimentary basins can be driven by a variety of mechanisms. Commonly, flow is localized in relatively thin aquifers or structures and often these permeable units may be approximated by a series of straight planar segments with shallow dip. A simple formula is derived that describes temperature perturbations in a straight aquifer segment with shallow dip. The temperature is determined everywhere along the segment if the flow rate and temperature at one point is known. "Daisy chaining" the solutions allows easy calculation of the temperature perturbation in a basin caused by fluid flow through a shallow-dipping network of linear fault and aquifer segments. The approach may find many applications but is here illustrated by a number of examples related to the origin of Mississippi Valley-type Pb–Zn deposits. The cases considered emphasize the "salinity problem" encountered by models that hypothesize these deposits formed as the result of cross-basin hydrological flow.

INTRODUCTION

The temperature distribution in sedimentary basins is important for a number of reasons: (1) it controls how rapidly hydrocarbons mature; (2) it governs the rate of diagenetic alteration reactions such as the transition of smectite to illite; and (3) it controls the viscosity of oil and the solubility of silica and metals in aqueous solutions. Thus temperature is related to the diagenetic alteration of basin sediments and the initial (generation or dissolution) stages of basin hydrocarbon and mineral resources.

Work over the last 15 years has documented that fluid flow can significantly change the temperature distribution in basins. In some cases the fluid flow is reasonably steady and driven by variations in hydraulic head that are related to topography (Hitchon, 1984). In other cases the flow is driven by compaction and may be episodic (Bonham, 1980; Cathles and Smith, 1983). Theoretical analyses have been offered for the temperature perturbations caused by topography-driven fluid flow in basins with homogeneous permeability (Domenico and Palciauskas, 1973; Smith and Chapman, 1983). Torrance *et al.* (1980) consider the thermal effects of flow through a semi-circular aquifer. Their results are similar to those presented in this paper. Numerical calculations illustrate the thermal effects of fluid flow in geologically more realistic cases where the permeability is anisotropic and inhomogeneous (Smith and Chapman, 1983; Garven, 1985; Bethke, 1986). Analyses have also been presented for compactively driven flow (Cathles and Smith, 1983; Bethke, 1986).

A practical drawback of the models so far presented is that they are either numerical or theoretically complicated. No simple analytical model has been published that would allow an interested party to calculate the temperature perturbation due to fluid flow in some aquifer system of interest. The main purpose of this paper is to present a simple equation that allows the calculation of steady-state temperature perturbations due to flow in an aquifer that consists of segments with shallow dip. This expression is not applicable in all cases and cannot answer all the questions we might have about fluid flow in sedimentary basins. It can, however, contribute to our understanding of many cases where flow is confined, mainly to very permeable units with shallow dip, and it can clarify some questions regarding flow in deep aquifers that must be resolved if we are to understand fluid flow in basins. The analytical expression is physically derived in the next section. Mathematical details are given in the appendix. The model is then illustrated through a variety of heuristic applications.

THE MODEL

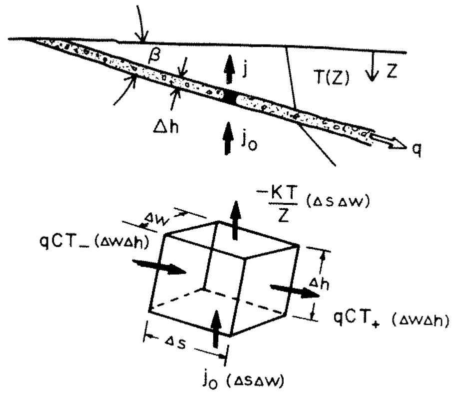
Consider flow into a shallow-dipping aquifer as shown in Fig. 1. The initial aquifer segment is considered to be straight and thin compared to the average depth of the segment.

In the absence of flow through the aquifer, the temperature gradient would be linear and "normal" for the heat flow in the area. For example, if the thermal conductivity of the basin sediments, K, was 4 mcal/cm s °C (or TCU), and the "normal" heat flow, j_0, 1 mcal/cm^2 s (or HFU), the thermal gradient would be 25°C/km. Mathematically, we can write:

$$j_0 = -K\, \partial T/\partial z \qquad (1)$$

so that $\partial T/\partial z = -j_0/K$.

If water flows from the surface into the aquifer, temperatures along the aquifer will be reduced. If the flow is very rapid, the temperature will be depressed



$$\frac{\partial T}{\partial Z} = \left(\frac{j_o}{K} - \frac{T}{Z}\right)\frac{K}{Qc\tan\beta}$$

FIG. 1. Diagram showing heat balance that determines steady-state temperature in an aquifer.

to close to the average ambient surface temperature to substantial depths. At steady state, the thermal gradient will still be "normal" below the aquifer, but it will be depressed above the aquifer. The thermal gradient above the aquifer will be linear. At any point the gradient above the aquifer will simply be the difference between the temperature at the surface and the temperature in the aquifer divided by the depth to the aquifer:

$$\partial T/\partial z|\text{above the aquifer} = (T(z) - T_s)/z = \Delta T(z)/z. \quad (2)$$

Note, we have defined ΔT as the temperature relative to the average ambient surface temperature at the basin location. Because T_s is a constant, ΔT can be substituted for T in Eqn (1). The linear temperature gradients above and below the aquifer envisioned in our simple physical model are illustrated in the left hand portion of Fig. 1. If the flow is very fast, the temperature gradient above the aquifer will be close to zero.

Because the temperature gradient below the aquifer is greater than that above the aquifer there is a net heat flux into the aquifer. At steady state this heat flux must be balanced everywhere by heat fluxes caused by the movement of fluid through the aquifer. The appropriate balance is indicated in Fig. 1. The fluid entering any volume element along the aquifer must be cooler than the fluid leaving so that the net heat introduced by the "kink" in the thermal gradient is carried off. Mathematically:

Net heat flux due to "kink" in thermal gradient = $\{(K\Delta T/z) - j_0\}\Delta s\Delta w \cos\beta$.

Net heat flux due to fluid flow = $qc\Delta h\Delta w\{\Delta T_- - \Delta T_+\}$.

Setting the two expressions equal to one another, heat balance requires:

$$(\Delta T_+ - \Delta T_-)/\Delta s = (j_0/K - \Delta T/z)K\cos\beta/(q\Delta hc).$$

In the above expressions ΔT_+ is the temperature of the fluid leaving the volume element, ΔT_- the temperature entering the element, β the dip of the aquifer, s the distance down the aquifer from its outcrop in cm, z the depth measured downward, q the mass flux through the aquifer in grams of fluid per cm^2 s, and c the heat capacity of the water in the aquifer in cal/g °C; Δw is the width of the volume element of the aquifer and Δh its height as shown in Fig. 1.

If we note that the distance down the aquifer from its outcrop is related to the depth of the aquifer, then $z = s\sin\beta$, and the above expression can be modified:

$$\partial\Delta T/\partial z = (j_0/K - \Delta T/z)\alpha \quad (3)$$

where

$$\alpha = K/(Qc\tan\beta).$$

Note, we have converted the finite difference to a partial derivative, and have defined a new quantity $Q = q\Delta h$. Q is the total flow of fluid through the aquifer per unit strike length perpendicular to the cross-section in Fig. 1.

Equation (3) can be solved by substitution of variables; the techniques are shown in the appendix. The result is the following simple formula giving the temperature in the aquifer as a function of depth:

$$\Delta T = (j_0 z/K)(\alpha - B\{z_c/z\}^{(\alpha+1)})/(\alpha + 1) \quad (4a)$$

where

$$B = (\Delta T_c/\Delta T_{0,z_c})(\alpha + 1) - \alpha. \quad (4b)$$

The temperature at one point along the aquifer segment must be known to apply Eqn (4a). This particular temperature is assumed to be at depth z_c and is designated ΔT_c in Eqn (4b). $\Delta T_{0,z_c}$ is the "normal" temperature (relative to the ambient surface temperature) at depth z_c. $\Delta T_{0,z_c}$ can be easily calculated from Eqn (1). Because we have assumed that lateral conduction of heat is insignificant, β must be a small angle, probably <10°. However, β can be negative as well as positive. Outflow as well as inflow can be calculated. For inflow, α is positive, and it is negative for outflow. A value of $\alpha = -1$ must be avoided (by taking $\alpha = -1.01$ or -0.99, for example) but otherwise Eqn (4a) is quite general. Finally, if $\beta = 0$ Eqn (4a) must be replaced:

$$\Delta T = \Delta T_{0,z_c} + (\Delta T_c - \Delta T_{0,z_c})\exp(-xK/Qcz_c). \quad (5)$$

Equations (4a) and (5) may be used to calculate the temperature in a basin in which flow is through a segmented aquifer. For example, we can calculate the perturbation caused by flow through the aquifer loop in Fig. 2 as follows: first, the temperature in segment 1 can be calculated from Eqn (4a) by taking $\Delta T_c = 0$ and z_c small. This is equivalent to assuming water enters the aquifer at shallow depths and at

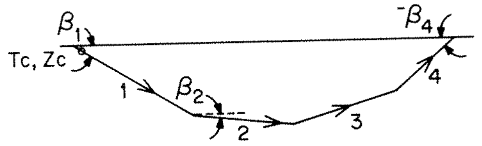


FIG. 2. Example of flow in a segmented aquifer. The steady-state temperature distribution along, above and below such an aquifer can be calculated using Eqn (4a) as discussed in the text.

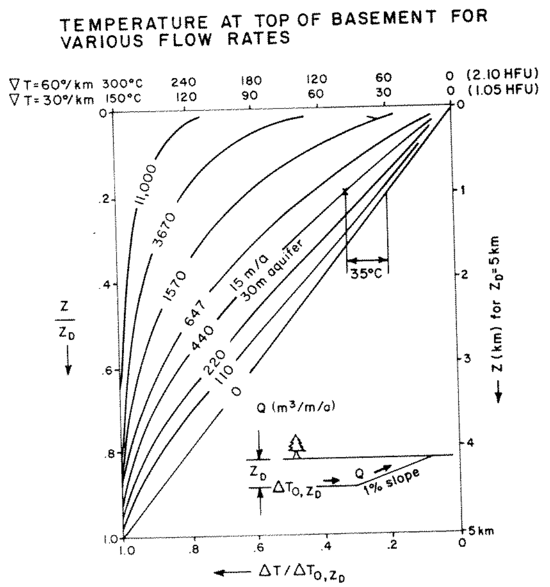


FIG. 3. Result of fluid flow out of an aquifer with 1% slope, assuming the aquifer fluids are in thermal equilibrium at z_D.

temperatures very near ambient. Note, β_1 is positive because, by definition, it is measured clockwise (downward) from the horizontal. The temperature at the end of the first segment at z_1 is ΔT_{z1}. These values can be used with β_2 as inputs to Eqn (4a) to calculate the temperature distribution in segment 2. If the second segment is horizontal, Eqn (5) rather than (4a) should be used. The temperature at the end of the second segment at depth z_2 can be used with β_3 (which in this case is negative) in Eqn (4a) to calculate the temperature distribution in the third segment, and so forth. By such "daisy chaining" of aquifer segment solutions, the temperature distribution along any aquifer can be calculated.

The temperature above the aquifer can be found by linear extrapolation to $\Delta T = 0$ at the surface. The temperature distribution below the aquifer can be found by extrapolating downward along the "normal" geothermal gradient.

EXAMPLES

It is instructive to consider a few examples of the use of Eqns (4a) and (5).

First, consider the case, perhaps applicable to the compactive expulsion of fluid from a basin, where the fluid enters the "outflow" aquifer in thermal equilibrium with the normal temperature of the deep parts of the basin, as illustrated in the insert to Fig. 3. Because by hypothesis $\Delta T_D = \Delta T_{0,z_D}$, $B = 1$ in Eqns (4b) and (4a) becomes the following particularly simple expression:

$$\Delta T/\Delta T_{0,z_D} = [(z/z_D)^{\alpha} + \alpha(z/z_D)]/(\alpha + 1). \quad (6)$$

The results are plotted in Fig. 3 for $\beta = -0.01$ radians, $K = 3.5 \times 10^{-3}$ cal/cm s °C and $c = 1$ cal/g °C. In the diagram, Q is converted to the convenient units of m³ of fluid throughout per m strikelength of aquifer per year. It can be seen from Fig. 3 that as the fluid throughout increases, an increasing fraction of the aquifer approaches temperatures of $\Delta T_{0,z_D}$.

The temperatures actually attained at any depth depend, of course, on what particular values of $\Delta T_{0,z_D}$ and z_D are appropriate for the case at hand. The top and right axes of Fig. 3 show temperatures and depths for $z_D = 5$ km, and $\Delta T_{0,z_D} = 150$ or 300°C. These values correspond to an unperturbed basin temperature gradient of 30 and 60°C/km, respectively, and a "normal" or unperturbed heat flow of 1.05 and 2.10 mcal/cm² s if $K = 3.5$ cal/cm s °C. These particular values suggest that if a ΔT of 100°C is desired at 1 km depth (x in Fig. 3), a minimum flow rate of 440 m³/m/a would be required. This corresponds to a Darcy flow rate of 15 m/a in a 30 m thick aquifer.

The ratio nature of the temperature and depths of Eqn (6) are particularly useful. For example, we can treat $\Delta T_{0,z_D}$ and z_D as variables for some fixed values of ΔT and z. Suppose we want to examine the flow rates required to produce a ΔT of 100°C at a depth of 1 km in basins with various initial (unperturbed) temperature gradients. We can proceed as shown in Fig. 4. The right axis of Fig. 4 shows the values of z_D that correspond to the ratio on the left with $z = 1$ km. The top axis shows the values of $\Delta T_{0,z_D}$ that correspond to the ratios on the bottom axis and $\Delta T =$ 100°C. The dashed lines on Fig. 4 show different unperturbed temperature gradients compatible with the top and left axes. At 20°C/km, for example, the temperature at the base of a 5 km deep basin would be 100°C and at the base of a 10 km deep basin the temperature would be 200°C. This defines the dashed line marked 20°/km. Similarly, for a temperature of 40°C/km the temperature at the bottom of a 2.5 km deep basin will be 100°C, at the bottom of a 5 km deep basin 200°C. If we do not consider basins hotter than 400°C in their deepest portions we can define the T_D = 400°C cutoff in Fig. 4. It can be seen from Fig. 4 that, provided the unperturbed thermal gradient is <60°C/km, flow rates of at least 440 m³/m/a are required to produce temperatures 100°C above ambient at depths of 1 km by flow through a basal (or deep) aquifer in a basin with a slope of 1%. Greater flow rates are required for smaller unperturbed ther-

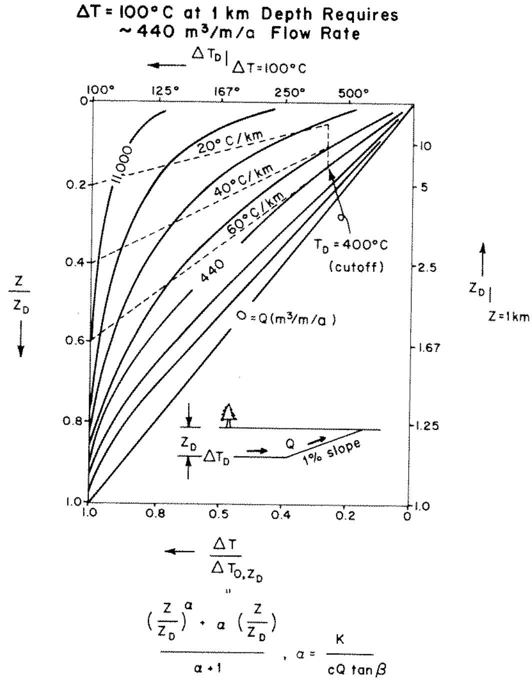


FIG. 4. Increasing basin depth within reasonable limits does not allow the margin to be heated with smaller discharge rates.

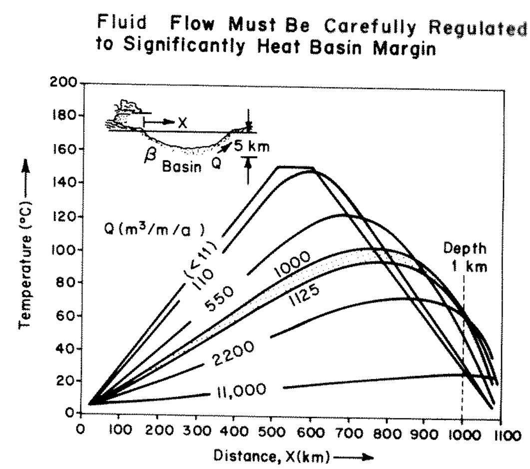


FIG. 6. Cross-basin hydrologic flow cools the intake margin and the basin as a whole as it warms the discharge margin.

mal gradients. This is the same result as was reached in Fig. 3, but more general because the sensitivity of the results to basin depth and different initial temperature gradients are considered in the diagram.

The results can be used in various ways. For example, Fig. 5 shows the maximum fluid expulsion rate during the accumulation of sediments in a basin roughly the size of the Illinois Basin is three orders of magnitude smaller than the 440 m^3/a estimated above. If temperatures at the margins of shallow-sloped basins are to be perturbed, as suggested above, the compactive expulsion of fluids must be episodic. These matters have been discussed at greater length in CATHLES and SMITH (1983). We might note, parenthetically, that in that paper, Eqn (3) was solved by finite difference techniques with results identical to the analytical solution given in Eqn (6).

As a second example consider flow through a basal aquifer across a sedimentary basin as illustrated in the insert in Fig. 6. Temperatures are computed by "daisy chaining" Eqns (4a) and (5) as described above, and starting with a temperature near ambient at a very shallow depth. The basin is 5 km deep and

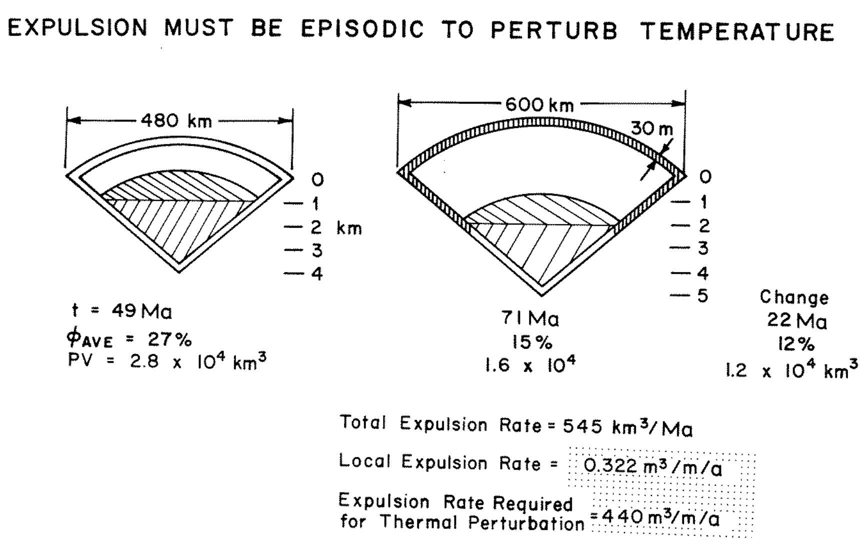


FIG. 5. Idealized expulsion of brines from a basin roughly the size of the Illinois Basin. If reasonable porosity vs depth relations and basin subsidence and filling rates are chosen (see CATHLES and SMITH, 1983 for details) the hatchured sediment volume will expel fluids at a rate (0.32 $m^3/m/a$) three orders of magnitude smaller than that required to perturb the temperatures at basin margins to the degree suggested by Mississippi Valley-type Pb–Zn deposits.

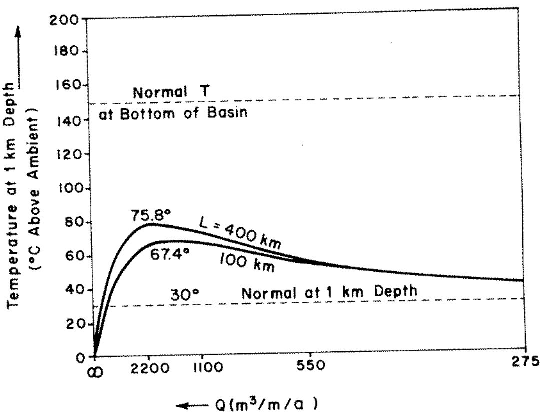


FIG. 7. The maximum warming at the discharge margin of the basin illustrated in Fig. 6 is never more than half the initial temperature in the deepest parts of the aquifer loop. Increasing the length of the flat bottom of the aquifer helps little because most of the heat is lost as the aquifer nears the surface.

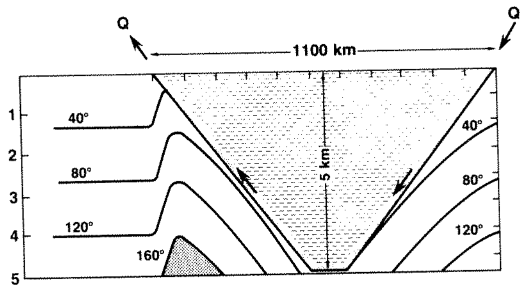


FIG. 8. Temperature anomalies are produced in the basement under the elevated temperatures along the discharge margin. It is important to appreciate that no flow goes through this anomaly. If it did the flow would tend to erase the anomaly.

there is a 100 km long flat segment at its base. The margin slopes are $\beta = +1\%$ (inflow) and -1% (outflow). Again, particular values of Q are calculated for $K = 3.5$ TCU and $c = 1$ cal/g °C. The initial unperturbed thermal gradient is taken to be 30°C/km.

Several features of interest are displayed by Fig. 6. As the rate of flow through the aquifer increases the inflow margin is cooled and the outflow margin heated. As flow increases, the basin as a whole is cooled. When flow becomes very fast the entire basin is cooled to close to ambient temperatures. It should be noted that at no location along the basal aquifer does fluid flow ever increase temperature above the maximum temperature that the deepest part of the aquifer would have in the absence of fluid flow.

Figure 7 shows aquifer temperatures as a function of flow rate on the outflow margin where the aquifer is 1 km deep (dashed line in Fig. 6). Figure 7 shows that the temperature increase reaches a maximum at a flow rate of about 2200 m^3/a and then decreases as flow rate increases further and the entire basin is cooled. The maximum temperature achieved at 1 km depth on the shallow basin flank is only about half the unperturbed (no flow or "normal") temperature at the bottom of the basin. This result has been previously pointed out by TORRANCE *et al.* (1980). Furthermore, because most of the heat is lost on the discharge flank, increasing the flat, deep part of the basin increases the maximum exit temperature only slightly.

As discussed previously, the temperature in a basin with temperature-perturbing flow in a segmented aquifer can be determined by extrapolating ΔT from the aquifer to 0 at the surface and along the "normal" (unperturbed) temperature gradient at depth. Figure 8 shows this for the "optimum" temperature perturbation along the aquifer in Fig. 6. Perhaps the most dramatic temperature anomaly in Fig. 8 is the isolated hot anomaly at the discharge margin of the basin. Because of the blanketing effects of the heated aquifer, temperatures in the "basement" (i.e. the strata below the aquifer) at the far end of the discharge margin are raised about 50°C above "normal". This is enough to produce accelerated maturation of hydrocarbons, accelerated diagenesis, perhaps some aquathermal pressuring. It should be remembered that in our model there is *no flow* through this "basement" temperature anomaly. The isolated hot spot at the discharge margin is produced because fluid flow has elevated the temperature of overlying strata and thermal steady state requires a normal geothermal gradient below the aquifer.

Figure 8 helps us understand similar discharge flank temperature anomalies in published numerical calculations, an example of which is given in Fig. 9.

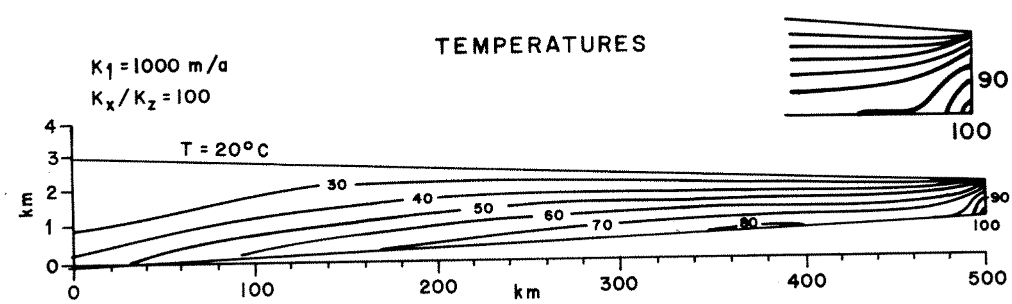


FIG. 9. Example of the steady-state temperature anomalies that numerical calculations indicate will form at the margins of basins subject to cross-basin hydrological flow. The origin of the anomaly in this diagram is similar to that in Fig. 8. The calculated anomaly shown in this diagram is from GARVEN (1985, Fig. 6).

These anomalies are also "conductive" in nature, owing their existence to the fact they underlie a heated cap. This can be seen most easily by considering the convective heat flux through the 90°C isothermal contour. Because fluid must cross the 90°C isothermal contour both entering and leaving the "anomaly" there is *no* net fluid transport of heat into the anomaly. The anomaly therefore owes its existence to conductive heat flux from below and is not produced by whatever flow occurs through it. In fact, fluid flow into the anomaly tends to reduce or erase it, because any inflow represents the introduction of colder water. The anomaly might, in fact, be enhanced by permeability distributions that reduced flow through the "deep anomaly" while maintaining the temperature of the "cap". It is important to appreciate the nature of the flank anomaly when considering Mississippi Valley-type mineralization. For example, periodic incursions (perhaps due to faulting) of flow into a distal margin anomaly, such as is shown in Figs 8 or 9, might lead to temporary increases in temperature and the intensity of Pb–Zn precipitation above the anomaly. These fluctuations could produce banding in the mineralization that it is otherwise tempting to attribute to episodic basin dewatering.

Other matters can be addressed from diagrams such as Fig. 6. For example, the permeability of the basal aquifer required for any flow rate can be easily estimated provided geological estimates can be made of the hydraulic head difference driving the flow. Darcy's law relates the volumetric fluid flux to the hydraulic head gradient: $V = \mathcal{K}\nabla h$, where V is the Darcy velocity in cm^3 water per cm^3 cross-sectional area of the aquifer perpendicular to fluid flow per second, $\mathcal{K}$ is the hydraulic conductivity of the aquifer (for some fluid at some temperature such as 25°C) in cm^2/s, and ∇h is the gradient in the hydraulic head (the height of the water table above sea level) in cm. Because $Q = V\Delta h$, $\mathcal{K} = Q/\Delta h \nabla h$. If a difference of hydraulic head of 1 km is to drive fluid flow at $Q = 440$ $m^3/m/a$ through a 1100 km long aquifer 30 m ($=\Delta h$) thick, as shown in Fig. 4, then the permeability of that aquifer must be 15 000 m/a (or 9 darcies, taking the viscosity of water to be 0.002 P).

The degree to which temperature differences between the intake and discharge limbs can assist fluid circulation can also be easily estimated. From Fig. 6 the maximum difference in average temperature between the intake and discharge limbs produced by fluid flow is about 30°C. Assuming a coefficient of thermal expansion of water of 10^{-3}, the intake column of water in the aquifer has a density about 0.03 g/cc less than the discharge limb. This density difference over 5 km is equivalent to about a 0.15 km difference in head. The temperature difference between the intake and discharge limbs will thus increase the flow rate produced by a head difference of 1 km by about 15%.

The time required to flush a brine from the aquifer

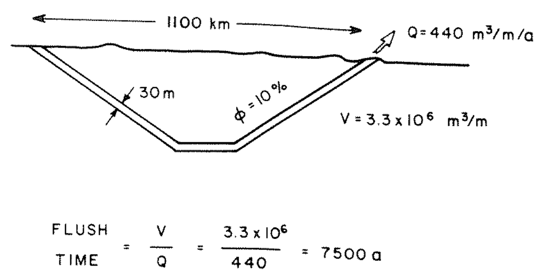


FIG. 10. Cross-basin hydrological flow rates of the magnitude needed to perturb basin margin temperatures would quickly flush the original brines from the aquifer system unless the brines are replenished by halite dissolution.

can also be estimated. A Darcy flow rate of 15 m/a through a 1100 km long aquifer with 10% porosity will be flushed (or replaced) once after 7500 a, twice after 15 000 a, etc. (see Fig. 10). Unless the salinity is "buffered" by the dissolution of salt in the aquifer, the maximum salinity of the aquifer (at the discharge point) will be reduced to half its initial value after the first flush, 1/4 after the second, and so forth. If CARPENTER (1974) is correct in maintaining that the chemistry of oil field brines requires them to have acquired their salinity from the evaporation of seawater and not from the dissolution of halite, the short flush time of brine from an aquifer system such as is shown in Fig. 4 is a strong argument against a cross-basin flow origin for Mississippi Valley-type Pb–Zn deposits. It might be commented that recent work by KNAUTH and BEEUNAS (1986) goes a long way toward removing isotopic arguments for the flushing of basin brines by the cross-basin flow of meteoric water.

Finally, the model embodied in Eqn (4a) provides a simple starting point for estimating chemical changes related to fluid flow in an aquifer system (e.g. WOOD and HEWETT, 1982).

DISCUSSION

The main purpose of this paper is to present a simple equation describing the temperature perturbations produced by fluid flow in a segmented aquifer system where the segments all have shallow dip. This is done in Eqns (4a) and (5). It is hoped that this particularly simple model may prove useful to others engaged in research in fluid flow in basins.

Use of the equation is illustrated through a number of examples. The samples are chosen for their relevance to the origin of Mississippi Valley-type Pb–Zn deposits. They provide some insight into the temperature anomaly many numerical calculations show is produced on the discharge flanks of basins subject to cross-basin fluid flow. They emphasize the "salinity problem" of hydrological flow models. Once initiated, cross-basin hydrologic flow is not easy to turn off quickly. It is hard to see how saline brines can have remained in the deep aquifers of basins associated with Mississippi Valley-type deposits if cross-basin hydrological flow produced these deposits.

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APPENDIX

Equation (3) in the text can be solved by substitution of variables. For simplicity, we will represent ΔT by T in this appendix.

Let $T/z = v$. Differentiating this expression we see $\partial T/\partial z = v + z\partial v/\partial z$. Substituting both expressions in Eqn (3) of the text we obtain:

$$v + z\partial v/\partial z = j_0\alpha/K - v\alpha$$

which can be rearranged:

$$\frac{\partial v}{j_0\alpha/K - v(\alpha - 1)} = \frac{\partial z}{z}.$$

Each side can be integrated. Remembering the constant of integration, substituting T/z for v, converting the log equation to exponential form, and using the temperature, T_c, at some particular depth, z_c, to determine the constant of integration results in Eqns (4a) and (4b) given in the text.