Geophys. J. Int. (1997) **128**, 489–492

Comment on 'The inference of mantle viscosity from an inversion of the Fennoscandian relaxation spectrum' by J. X. Mitrovica and W. R. Peltier

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Accepted 1995 December 4. Received 1995 November 16; in original form 1994 August 4

Mitrovica & Peltier (1993; hereafter MP) have recently analysed the Fennoscandian uplift data using inversion techniques. They concluded that, although observations support the presence of a weak asthenosphere under Fennoscandia, the decay spectrum for the best-fit model (1.3×10^{19} Pa s asthenosphere overlying a 10^{21} Pa s mantle) found by previous workers (Fjeldskaar & Cathles 1991a,b; Cathles 1975, 1980, hereafter FC) lies outside the decay spectrum observed in Fennoscandia and is thus excluded by the observed data. We believe this conclusion results from the authors inadvertantly using a different asthenosphere thickness in their calculations than the one that they report. The purpose of this communication is to argue this case and to show that when proper asthenosphere thicknesses are used, the models of MP and FC are both compatible with the observed uplift data and that each in fact defines one edge of the observed uplift spectrum. The essential difference in the two end-member models is the elected trade-off between lithosphere thickness and asthenosphere viscosity. MP choose a stiff lithosphere and a minor asthenosphere, whereas FC choose the opposite. We believe lithosphere rigidity data (not discussed in MP) argue for a weak lithosphere in Fennoscandia and suggest that the FC model is to be preferred.

The relaxation spectrum for the best-fit model of FC, as presented in MP's Fig. 11, is shown by the dashed curve labelled LVZ in Fig. 1. It clearly lies below and outside the observed Fennoscandian relaxation data (shaded area). The decay spectrum for the LVZ model is, according to MP, based on loading of a 70 km lithosphere on a 90 km asthenosphere of viscosity 2×10^{19} Pa s overlying a 10^{21} Pa s mantle. The maximum relaxation time for the LVZ model exceeds 6000 years. However, the maximum relaxation time for FC's best-fit asthenosphere model that the LVZ curve is supposed to

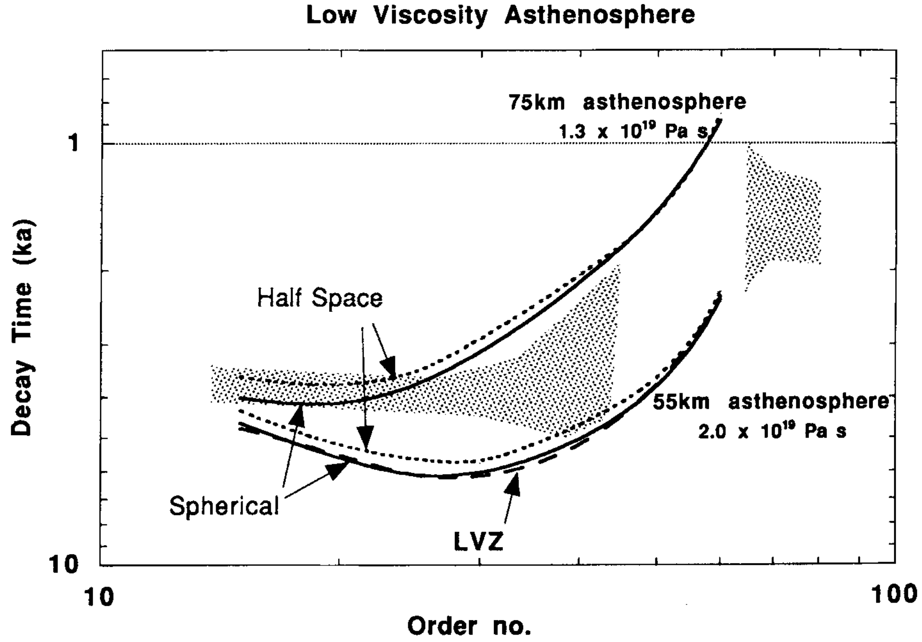

Figure 1. Decay time versus order number for models with a low-viscosity asthenosphere. The upper curves plot the calculated relaxation curve for our (FC's) best-fit published model. MP's version (LVZ) of our best-fit model, apparently erroneously calculated for a 55 km rather than 75 km thick asthenosphere, is the lowermost dashed line. The lower solid and dotted lines are the decay curves we have calculated for a 55 km thick asthenosphere. The shaded area is the Fennoscandian decay spectrum according to MP.

duplicate is less than 4000 years for the half-space solution (Fjeldskaar & Cathles 1991a; see also Cathles 1980). We have analysed the discrepancy and found that we can duplicate the LVZ curve if we make the model asthenosphere 55 km rather than 90 km thick. This is shown in Fig. 1. Another important point shown in Fig. 1 is that both self-gravitating spherical earth and half-space models with 55 km asthenospheres replicate the LVZ model, and the two merge at high-order numbers as they should. Thus our spherical, self-gravitating, viscoelastic earth model is compatible with and confirmed by a simple half-space model, whereas MP's model is not. There seems little doubt that MP have simply generated spectra for a slightly, but significantly, different asthenosphere thickness than they report. This could be inadvertently done if the model discretization was coarse with respect to the asthenosphere thickness, for example.

The spectra for FC's 'best fit' published model with an asthenosphere 75 km thick and a 70 km thick lithosphere (corresponding to a flexural rigidity of 40×10^{23} N m) is compared to the Mitrovica & Peltier miscalculation of this model in Fig. 1. The correct 75 km thick asthenosphere FC model bounds the upper range of the shaded observed decay spectra. Lithosphere thickness is an important parameter in this model. Cathles (1975) estimated the flexural rigidity of Fennoscandia to be between 10 and 50×10^{23} N m. Wolf (1987) concluded the flexural rigidity of the Fennoscandian lithosphere was

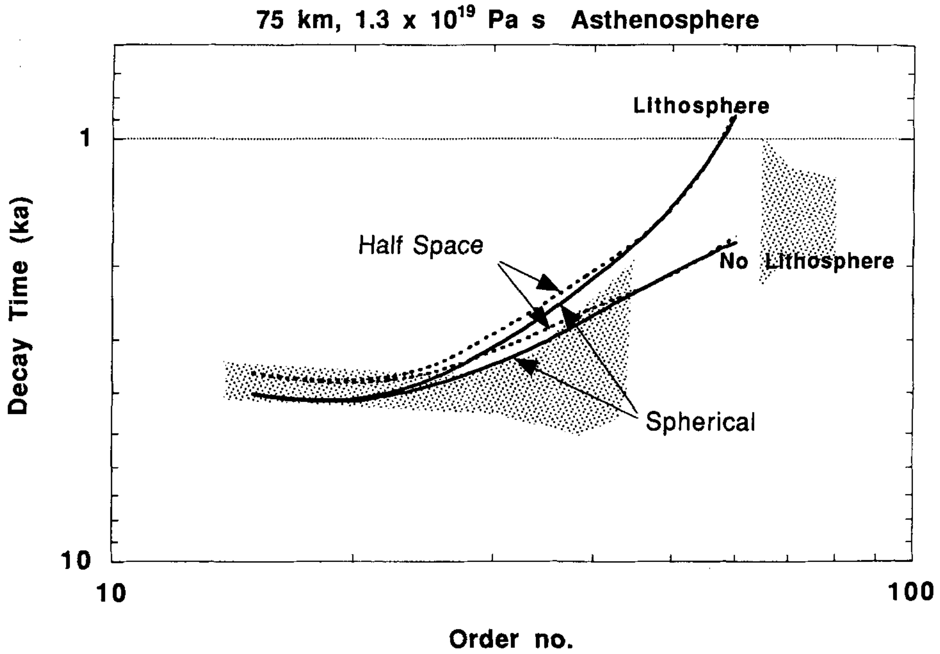

Figure 2. Decay time versus order number for a mantle of viscosity 1.0×10^{21} Pa s, overlain by a 75 km asthenosphere of viscosity 1.3×10^{19} Pa s. Half-space and spherical solutions are shown for cases with a lithosphere of 40×10^{23} N m and with no lithosphere. The upper (with lithosphere) curves are the same as in Fig. 1.

Table 1. Summary of models presented in Figs 1–3. Best-fit models are shown in bold type and bound the observed decay spectra in Fennoscandia on the bottom (MP–'LVZ2') and top (FC–'weak lithosphere'). The most important distinguishing feature of the preferred (bold) models of MP and FC is the flexural rigidity of the lithosphere.

	Adiabatic						Partly non-adiabatic	
	Fig. 1		Fig. 2	Fig. 3			Fig. 4	
	MP version of FC 'LVZ' curve	FC	FC 'weak lith.'	MP 'LVZ2'	No lithosp.	90 km asthen.	MP '2LAYE'	No lithosp.
Flexural rigidity of elastic lithosphere (10^{23} N m)	40	40	**0**	**40**	0	40	**40**	0
Asthenosphere thickness (km)	55 (not 90)	75	**75**	**55 (not 90)**	55	90	**0**	0
Asthenosphere viscosity (10^{21} Pa s)	0.02	0.013	**0.013**	**0.05**	0.05	0.05	–	–
Upper-mantle viscosity (10^{21} Pa s)	1.0	1.0	**1.0**	**0.5**	0.5	0.5	**0.37**	3.7
Lower-mantle viscosity (10^{21} Pa s)	1.0	1.0	**1.0**	**1.6**	1.6	1.6	**2.2**	2.2

markedly less than 50×10^{23} N m. Based on tilts of palaeo-shorelines, Fjeldskaar & Cathles (1991a) also concluded that the rigidity was less than 50×10^{23} N m. More recently, Fjeldskaar (1996) has shown that the flexural rigidity is less than 10×10^{23} N m, and that it is close to 10^{23} N m at the Norwegian coast. The effects of the lithosphere on the decay spectra are shown in Fig. 2. A mantle viscosity of 10^{21} Pa s overlain by a low-viscosity asthenosphere of 1.3×10^{19} Pa s and a lithosphere rigidity less than 10×10^{23} N m gives an excellent fit to the observed spectrum. This is our (FC's) current best-fit model for the Fennoscandian relaxation data. The three models shown in Figs 1 and 2 are summarized in Table 1.

MP suggest a modified version of LVZ that they feel gives a better fit to the observed decay spectrum. This model, termed LVZ2, has a lower mantle of 1.6×10^{21} Pa s, an upper mantle

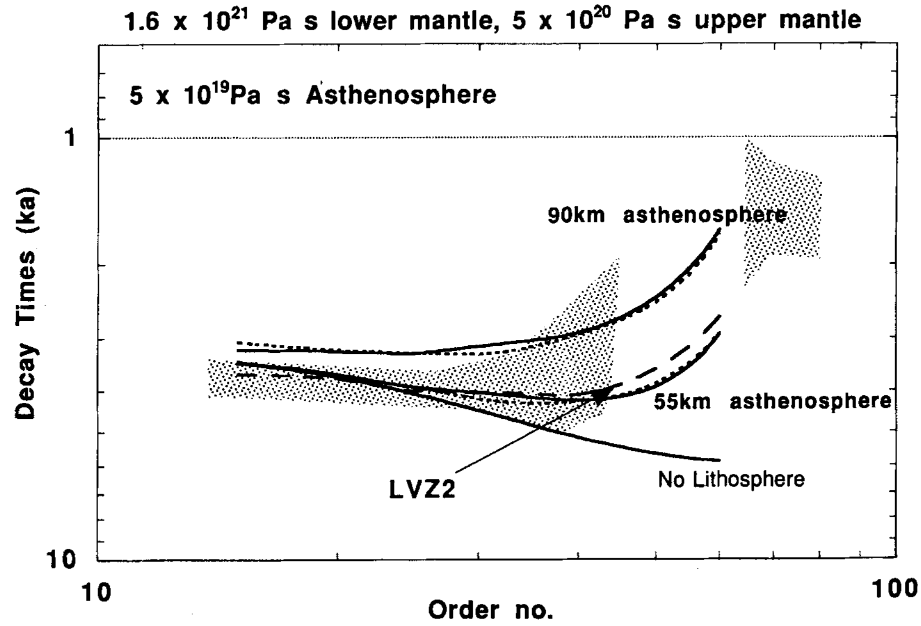

Figure 3. Decay time versus order number for models corresponding to MP's best-fit low-viscosity asthenosphere model, LVZ2. As in Figs 1 and 2, the half-space curves are solid; the self-gravitating, spherical earth curves are dotted.

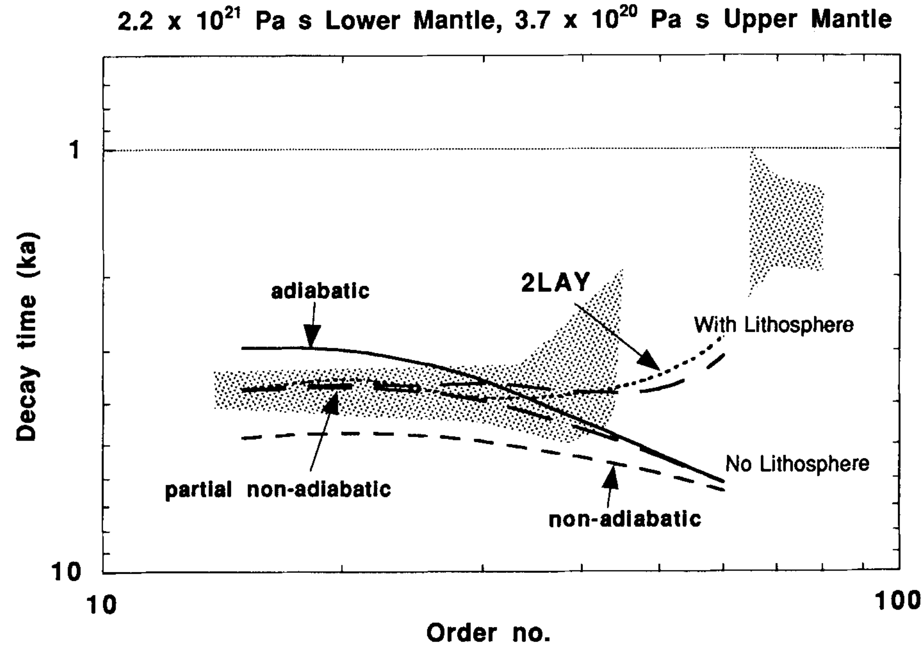

Figure 4. Decay time versus order number for models calculated in this paper corresponding to MP's best-fit two-layer model named 2LAY (dashed curve). Our curves are generated for adiabatic, partially non-adiabatic (0.5 g cm^{-3} non-adiabatic density increase between 335 and 670 km depth), and fully non-adiabatic models with no lithosphere (see Fjeldskaar & Cathles 1984). Curves are for a lithosphere of 0 and 40×10^{23} N m, as indicated. MP's 2LAYE model appears to have been calculated for a partially adiabatic mantle and provides a reasonable match to the observed decay spectra (shaded area) provided the flexural rigidity of the lithosphere in Fennoscandia is at least 40×10^{23} N m. All models calculated by us in Figs 1–3 are for fully adiabatic mantles.

of 5×10^{20} Pa s and an asthenosphere of 5×10^{19} Pa s. The LVZ2 decay spectrum is apparently also calculated with a 55 km thick rather than a 90 km thick asthenosphere, because, as shown in Fig. 3, we can accurately duplicate their results with a 55 km, but not a 90 km, asthenosphere. Fig. 3 also shows that the LVZ2 model (corrected to 55 km thick asthenosphere) gives a good fit with the observed decay spectrum for a lithosphere of rigidity 40×10^{23} N m. However, the good fit is dependent on a relatively thick lithosphere. With a thinner lithosphere, there is a significant misfit at short wavelengths (high-order numbers). These models are also summarized in Table 1.

FC's best-fit model (mantle viscosity of 10^{21} Pa s overlain by a low-viscosity asthenosphere of 1.3×10^{19} Pa s and a weak, $< 10 \times 10^{23}$ N m, lithosphere) and LVZ2 are both compatible with the observed uplift data, each defining one edge of the observed uplift spectrum. This is in agreement with a recent analysis of thickness and viscosity of the asthenosphere for the Fennoscandian uplift (Fjeldskaar 1994). The conclusion of that analysis was that models with combinations of lower-mantle viscosities ranging from 1 to 2×10^{21} Pa s, upper mantle viscosities ranging from 0.7 to 1.0×10^{21} Pa s and low-viscosity asthenospheres of thicknesses ranging from 25 to 100 km are compatible with the available uplift data.

Finally, in the same paper MP also investigate two-layer (no asthenosphere) partially non-adiabatic mantle models for Fennoscandia. They find that the asthenosphere could be replaced by a more fluid upper mantle overlying a more viscous lower mantle. Again, a thick lithosphere is required. The need for a thick lithosphere is obvious from the mismatch shown in Fig. 4 between the observed (shaded) relaxation times and the thin lithosphere decay spectrum at high-order numbers.

In conclusion, the important point we wish to make here is that despite claims to the contrary there is a simple trade-off between lithosphere thickness and asthenosphere viscosity. A 10^{21} Pa s mantle with a 75 km thick 1.3×10^{19} Pa s asthenosphere and a thin lithosphere (FC's favoured model) is equally compatible with the Fennoscandian relaxation spectrum data as models with a thicker lithosphere and a less fluid or thinner asthenosphere (MP's favoured models). The essential question is the flexural rigidity of the lithosphere in Fennoscandia. Diverse data addressed in the references cited above suggest the flexural rigidity of the lithosphere in Fennoscandia is much lower than that required by MP's favoured models and compatible with FC's favoured model. Certainly the best-fit FC model cannot be 'ruled out', as asserted by MP in the paper under discussion.

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