

A Model of the Dump Leaching Process that Incorporates Oxygen Balance, Heat Balance, and Air Convection

L. M. CATHLES AND J. A. APPS

A one dimensional, nonsteady-state model of the copper waste dump leaching process has been developed which incorporates both chemistry and physics. The model is based upon three equations relating oxygen balance, heat balance, and air convection. It assumes that the dump is composed of an aggregate of rock particles containing nonsulfide copper minerals and the sulfides, chalcopyrite and pyrite. Leaching occurs through chemical and diffusion controlled processes in which pyrite and chalcopyrite are oxidized by ferric ions in the lixiviant. Oxygen, the primary oxidant, is transported into the dump by means of air convection and oxidizes ferrous ion through bacterial catalysis. The heat generated by the oxidation of the sulfides promotes air convection. The model was used to simulate the leaching of copper from a small test dump, and excellent agreement with field measurements was obtained. The model predicts that the most important variables affecting copper recovery from the test dump are dump height, pyrite concentration, copper grade, and lixiviant application rate.

THE leaching of low-grade copper-bearing waste has been practiced either by accident or through design for several hundred years. During the last fifty years, increasing attention has been paid to the systematic leaching of low-grade waste resulting from the open pit mining of porphyry copper deposits in the western United States. By now, this activity is yielding an important secondary source of domestic copper.¹ Indeed, some mining operations have been planned and are operating exclusively from the production of copper obtained from leaching. Many of these operations are exploiting oxide copper deposits where copper is readily leached by the application of dilute sulfuric acid.

Low-grade waste discarded as a result of open pit mining of porphyry copper deposits is dumped in gullies surrounding the deposit. The disposal site is determined primarily by the convenience of the site to the mining operation, and is not usually based on considerations necessary for optimum leaching. In the western United States several billion tons of waste has accumulated in this manner.

During the last decade, many people have become conscious of the fact that this enormous resource of copper is not being exploited effectively because insufficient attention is being paid to those factors which could lead to improved design and layout of waste dumps. It is believed that if the leaching process were completely understood, then it would be possible to design and leach copper from waste dumps in a far more efficient manner than is currently being practiced. The problem is a large one. Not only must the chemistry of leaching be understood, including both kinetic and thermodynamic aspects, but the effect of

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heat generation, fluid flow and other transport phenomena relating to the leaching process must also be considered. A leaching system cannot be considered in a steady state, because all factors involved in the leaching process change progressively as a function of time.

In this paper we have developed a one-dimensional model of the nonsteady-state dump leaching system. We have applied this model to a small test dump constructed and leached at the Utah Copper Division of Kennecott Copper Corporation. To our knowledge only two other attempts have been made to integrate the diverse aspects of dump leaching into a coherent all-embracing model.^{2,3} While we do not feel that the model presented in this paper is the final answer to a clarification of the dump leaching process, we believe that it forms a basis upon which subsequent research in this area might be coordinated.

A MODEL OF THE DUMP LEACHING PROCESS

Initial Assumptions

Sulfides must be oxidized before their metal values may be put into solution. The conceptual basis of the model of dump leaching presented here is simply that the exothermic sulfide oxidation reactions generate heat and consume oxygen from the air, and by so doing drive air convection through the dump. This air convection is the only significant source of oxidant to the dump.

A system is envisioned in which a countercurrent interlocking flow of air and water passes through an aggregate of rock fragments, as shown in Fig. 1.

The oxygen leaves the gas phase within the dump by dissolving in the liquid phase where it oxidizes ferrous to ferric iron through the agency of bacteria. The ferric iron diffuses into the ore fragments and

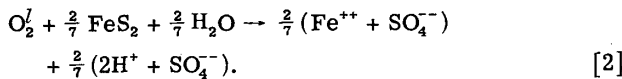
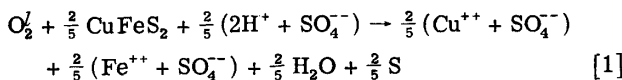
oxidizes the sulfide minerals:* Acid, Fe⁺⁺ and heat

*At 1/5 atmosphere P_{O₂} and the temperatures involved in dump leaching, oxygen is not very soluble in water (<8.6 × 10⁻³ g/l). Typical ferric iron concentrations in leach dumps run ~1 g/l. These relative concentrations ensure Fe⁺⁺⁺ will be the oxidizing agent in the diffusion controlled processes envisioned above.

are produced along with Cu⁺⁺.

For the purposes of our model, we have assumed the formation of a leached rim which is separated from the unreacted core by a sharp boundary, as shown in Fig. 2. As the leached rim grows the rate of leaching drops because of longer diffusion paths and a shrinking reaction zone. Evidence for a shrinking core has been supported observationally by Braun, Lewis and Wadsworth⁴ and by Madsen, Wadsworth and Groves.⁵ Theoretical arguments also support the existence of a sharp boundary for the conditions of our model (see Eq. [1]). For those reasons we will later employ the mathematical formulation of the so called "shrinking core model", as developed by Braun, Lewis and Wadsworth. However, we recognize that there are many conditions in which a sharp boundary between the leached rim and the unreacted core boundary is not observed because of variable reaction rates of the sulfides, acid gangue interaction, sulfide concentration, grain size of sulfides and gangue minerals, and porosity of the rock. A generalized model, taking account of several of these factors has been developed by Bartlett.⁶

Most low-grade waste, from which copper is leached, is derived from the outer pyritic halo of porphyry copper deposits, where the copper-bearing sulfide is chalcopyrite.⁷ We assume that chalcopyrite and pyrite are the principal sulfide minerals and that they oxidize in a waste dump environment in the following manner:



Evidence that these are the oxidation mechanisms for the two sulfides comes from studies by Wadsworth.⁸ Observations by Stephens⁹ show that sulfur is a product of the oxidation of sulfides in waste dumps. It can be seen that for every mole (64 g) of chalcopyrite leached, 5/2 mole (5/2 · 32 g) of oxygen will be consumed. If *FPY* moles of pyrite are leached per mole of sulfide copper, an additional 7/2 *FPY* moles (7/2 · 32 · *FPY* g) of O₂ will be consumed. Thus for every

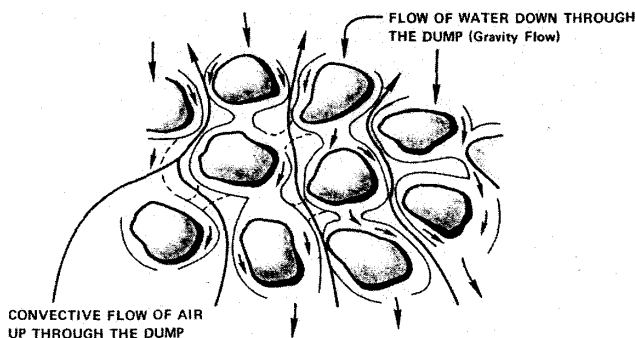


Fig. 1—Countercurrent interlocking flow of air and water through a leach dump. The flow of water is usually intermittent.

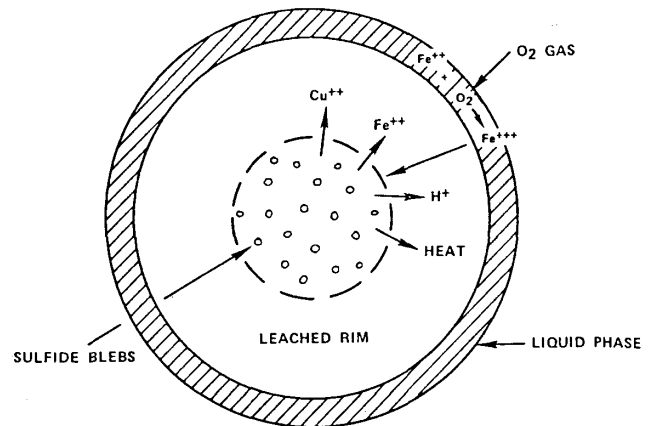


Fig. 2—Idealization of the leaching of a single waste particle.

gram of chalcopyrite copper leached, the following number of grams of O₂ will be consumed:

$$\text{g O}_2 \text{ consumed} = (1.25 + 1.75 \text{ FPY}) \text{ g chalcopyrite Cu leached} \quad [3]$$

Actually the amount of oxygen consumed per gram of chalcopyrite copper leached is somewhat greater than this, if account is taken of the oxidant required to precipitate, as jarosite (KFe₃(SO₄)₂(OH)₆), the iron exchanged for copper during cementation (2.5 lb Fe/lb Cu) and the iron produced in leaching the chalcopyrite and pyrite. The precipitation of cementation iron *within* the dump must clearly be taken into account even though the source of the iron is outside the dump.

Further, if the excess acid produced by the oxidation of pyrite is neutralized by reaction with gangue of biotite composition* additional iron is generated, oxidized,

*Biotite has been found to be more reactive by a factor of ~100 than other gangue minerals in a porphyry copper intrusive. Calcite, the only other highly reactive mineral likely to occur, is usually present only in minor amounts.

and precipitated. With these additions Eq. [3] becomes:

$$\text{g O}_2 \text{ consumed} = (1.75 + 1.91 \text{ FPY}) \text{ g chalcopyrite Cu leached} \quad [4]$$

Waste material typically contains 10 to 100 moles of pyrite for every mole of sulfide copper. Thus pyrite is by far the most important oxidant consumer if it is oxidized in proportion to its molar ratio to sulfide copper.

Because leach solutions cannot carry significant oxidant with them as they move through the dump, air is the main source of oxidant within a dump. A liter of air contains 0.28 g O₂. Fig. 3 shows that Eq. [4] requires far more air than water to flow through a waste dump if the effluent solutions are to contain the copper concentrations typically observed. For the particular dump we shall consider, at least 80 times more air passed through the dump than water. That is, for each liter of leaching solution leaving the dump with a net gain of 0.25 g/l (2 lb. Cu/1000 gal.) copper, 80 liters of air are required to supply the oxidant necessary for the chemical reactions involved.

Eqs. [1] and [2] not only tell us the amount of oxidant consumed per gram of copper leached, but also the heat generated per gram of copper leached. (The enthalpy of reaction, ΔH_R, of Eq. [1] is approximately

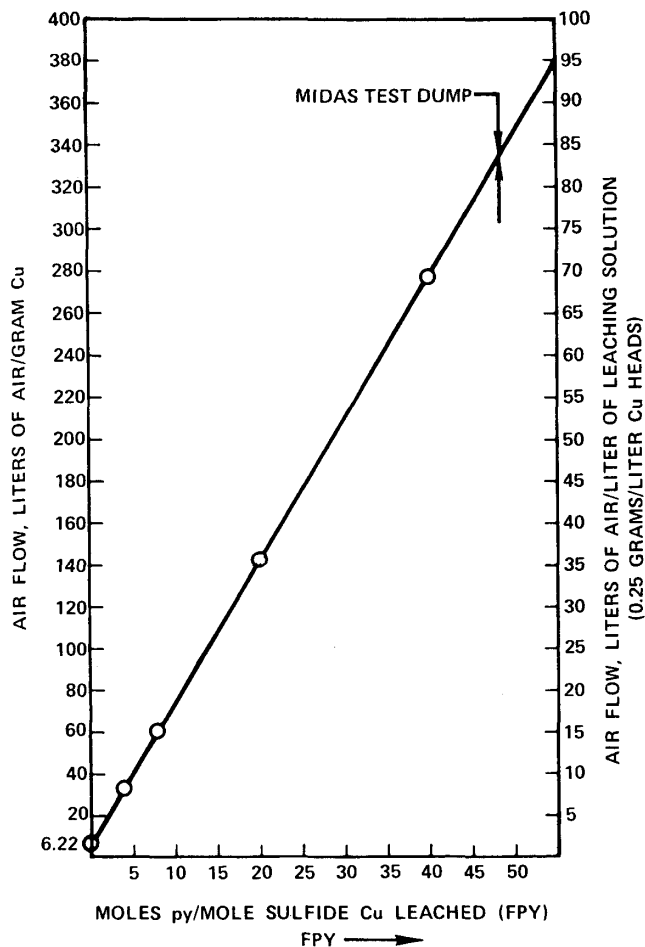


Fig. 3—Graph relating the air flow through a dump to the ratio of the moles of pyrite leached to sulfate copper leached (FPY), for a typical effluent copper concentration.

—108.8 kcal,* ΔH_R for Eq. [2] is -94.9 kcal). If we

*1 kcal = 4.1868 kJ.

again take into account the heat consumed in the precipitation of jarosite, require acid and iron balance, and assume 2.5 lb. Fe are exchanged per pound of Cu at the precipitation plant:

$$\text{kilocalories produced} = (2.89 + 5.41 FPY) \text{ g chalcopyrite Cu leached} \quad [5]$$

Again it can be seen pyrite oxidation will, in all probability, be the most significant source of heat. The rate at which a waste dump heats up is a direct measure of FPY .

Eqs. [4] and [5] contain the fundamentals of a model of the dump leaching process. Sulfide oxidation reactions consume oxygen from the air in a dump. Since O_2 is a heavy component in air, the oxygen depleted air is lighter. Since water vapor is a light component of air, saturation of the air inside a dump with water vapor will also produce buoyant forces. Buoyant forces tend to produce air convection. Furthermore, the oxidation reactions are exothermic, which also promotes air convection. A ΔT of $20^\circ C$ produces buoyant forces two or ten times larger than complete oxygen depletion or complete water vapor saturation, respectively.

For normally observed permeabilities, air convection rather than diffusion is the principal mechanism

of O_2 transport into a waste dump. In this respect our model differs from that of Harris,² whose pseudo-particulate leaching case assumes that O_2 transport into a dump occurs primarily by diffusion through the interstices of the particles.

In the next section we generalize Eqs. [4] and [5] slightly to take into account copper sulfides other than chalcopyrite. We then develop the rate equations relating copper extraction with oxygen uptake and heat generation. We show that these rate equations are governed by the chemical and diffusional processes occurring during the leaching of waste particles in the dump. Finally, we derive three equations describing oxygen balance, heat balance and convective air flow which are the basis for our one dimensional model.

Formulation of a One Dimensional Model

Because sulfide leaching is usually dominated by the leaching of pyrite, Eqs. [4] and [5] are quite easy to generalize. Provided FPY is taken as the moles of pyrite oxidized per mole of sulfide copper oxidized, and provided FPY is greater than ~ 4 , Eqs. [4] and [5] will hold to good approximation even if sulfide minerals other than chalcopyrite are present.

We assume sulfide oxidation takes place in a dump only where the air filled pores of the dump contain oxygen and that the oxidation proceeds at a rate independent of the actual oxygen concentration. Unpublished studies of the bacterial oxidation of waste dump leaching solutions with air, conducted by the second author, have shown that the bacterial oxidation rate of ferrous iron is essentially independent of oxygen concentration in the air until the concentration falls below 1 pct. More recent studies by the second author involving the uptake of oxygen by wetted mine waste show that oxygen uptake is substantially independent of oxygen partial pressure for the same range of oxygen concentrations.

By contrast nonsulfide copper is leached with acid alone. Acid generated by pyrite oxidation anywhere in the dump is recirculated through the dump in normal operation. Therefore, nonsulfide copper leaching should take place everywhere in the dump at a rate independent of the presence or absence of nearby oxygen.

Suppose the fraction of sulfide copper remaining in the dump after some leaching is X_S , and the fraction of nonsulfide copper in the dump is X_{NS} . Let the original sulfide copper grade be G_S and the original nonsulfide copper grade be G_{NS} . Then the rate at which copper is leached from the dump, α_{Cu} , may be expressed as:

$$\alpha_{Cu} = \rho_R(1 - \Phi) \left(G_S \frac{dX_S}{dt} + G_{NS} \frac{dX_{NS}}{dt} \right). \quad [6]$$

Similarly the rate of oxygen consumption, α_{O_2} , and the rate of heat generation α_A , may be expressed, from Eqs. [4] and [5]:

$$\alpha_{O_2} = \rho_R(1 - \Phi) G_S \frac{dX_S}{dt} (1.75 + 1.91 FPY) \quad [7]$$

$$\alpha_A = \rho_R(1 - \Phi) G_S \frac{dX_S}{dt} (2.89 + 5.41 FPY) \quad [8]$$

ρ_R , the density of the rock waste, is commonly about 2.7 g/cm³; Φ , the interblock porosity of the dump is ~25 pct; $\rho_R(1 - \Phi)$, the bulk density of the dump as a whole, is equivalent to 1.7 tons/yard³.

The rate of leaching, dX_S/dt and dX_{NS}/dt , may be described also in terms of leaching from a waste particle. Let us suppose that the leaching of a sulfide-bearing particle is governed by an equation of the form:

$$\frac{D'_{ox} \Phi_R l}{T_R^l} \nabla^2 [Ox]_l^R - k_{ox} a_{sulf}^R [Ox]_l^R = 0 \quad [9a]$$

and that $[Ox]_l$ just outside the ore particle is a known function of time. The symbols given are defined in Table I.

For a simple one-dimensional case, Eq. [9a] becomes:

$$\frac{D'_{ox} \Phi_R l}{T_R^l} \frac{\partial^2 [Ox]_l^R}{\partial x^2} - k_{ox} a_{sulf}^R [Ox]_l^R = 0 \quad [9b]$$

Satisfying the boundary conditions $[Ox]_l^R|_{x=0} = [Ox]_l^R$ and $[Ox]_l^R|_{x \rightarrow \infty} = 0$, the solution is

$$\frac{d[Ox]_l^R}{dx} = - \frac{[Ox]_l^R|_{x=0}}{\delta} e^{-x/\delta} \quad [10]$$

$$\text{Where } \delta = \text{reaction skin depth} = \sqrt{\frac{D'_{ox} \Phi_R l}{k_{ox} T_R^l a_{sulf}^R}} \quad [11]$$

The reaction skin depth is the distance into the particle where the oxidant concentration has fallen to 1/e its initial value. Since by hypothesis the rate of oxidation reaction is proportional to oxidant concentration, δ is a measure of the distance into the ore particle that significant chemical reaction takes place.

Using values given in Table I, $\delta = 0.142$ cm. The reaction skin depth is therefore thin, relative to the average size particle diameter. Leaching of a particle can, therefore, be described in terms of the so-called shrinking core model which is of a similar form to that developed by Braun, Lewis and Wadsworth.⁴

$$\frac{dX_S}{dt} = \frac{-3X_S^{2/3}}{6\tau_{DS}X_S^{1/3}(1 - X_S^{1/3}) + \tau_{CS}} \quad [12]$$

$$\frac{dX_{NS}}{dt} = \frac{-3X_{NS}^{2/3}}{6\tau_{DNS}X_{NS}^{1/3}(1 - X_{NS}^{1/3}) + \tau_{CNS}} \quad [13]$$

where τ_D is the time required to leach a waste particle completely when the process is solely diffusion controlled, and τ_C the time to leach a waste particle completely when the process is controlled by the decreasing surface area of the shrinking unleached core. τ_C and τ_D can be computed theoretically from the following relations:

$$\tau_{CS} = \frac{Ka}{k_{ox} a_{sulf}^R \delta [Ox]} \quad [14]$$

$$\tau_{DS} = \frac{T_R^l a^2 K}{6[Ox] D'_{ox} \Phi_R^l} \quad [15]$$

Using the values given in Table I, $\tau_{CS} = 903$ mo, and

Table I. Parameters Used for the Determination of τ_{DS} and τ_{CS}

Parameter	Description	Value
a	Radius of waste particle	1.5 cm
a_{sulf}^R	Surface area of sulfide mineralization per unit volume of waste	$\approx 80 \text{ cm}^{-1}$
D'_{ox}	Diffusion constant of Fe ⁺⁺⁺ in water	$\approx 2 \times 10^{-5} \text{ cm}^2/\text{s}$
FPY	Moles of pyrite leached per mole and sulfide copper leached	47
G_S	Sulfide copper grade	0.18 wt pct
K	Oxidant required to leach a unit volume of waste particle	0.444 g/cm ³
k_{ox}	First order rate constant for the oxidation of pyrite by Fe ⁺⁺⁺	$\approx 10^{-7} \text{ cm/s}$
$[Ox]$	Concentration of Fe ⁺⁺⁺ in leaching solution	10^{-3} g/cm^3
T_R^l	Tortuosity of diffusion channels	≈ 5
δ	Reaction skin depth (see Eq. [11])	Calculated value = 0.142 cm
Φ_R^l	Porosity of waste through which diffusion can take place	4×10^{-2}

$\tau_{DS} = 1590$ mo.* In addition, τ_C and τ_D may be given

*1 mo = 31 d.

a temperature dependence:

$$\tau(T) = \tau(T = 0^\circ\text{C}) \text{EXP} \left(\frac{1000 \cdot E^*}{R} \frac{T}{(273)(273 + T)} \right) \quad [16]$$

This introduces activation energies E_{DS}^* , E_{CS}^* , E_{DNS}^* , E_{CNS}^* . From the literature¹⁰ reasonable guesses for E_{DS}^* and E_{DNS}^* would be 5.0 kcal/mol. E_{CS}^* and E_{CNS}^* might range from 14.0 kcal/mol to 20.0 kcal/mol, the activation energies reported for the leaching of pyrite^{11,12} and for chalcopyrite.¹³

Given values for τ_{DS} , τ_{CS} , τ_{DNS} , τ_{CNS} , Eqs. [12] and [13] determine the rate of leaching at any point in the dump at any stage of leaching. X_{NS} and X_S can be updated after each increment of model leaching. Model time increments may be taken as short as desired.

The most serious approximation in Eqs. [12] and [13] is probably the assumption that the dump is composed of waste particles only of one size. This may not be as serious an approximation as it might at first seem, given the tendency of small ore particles to clump together and leach as if they were a larger aggregate, and the tendency for large ore particles to have large enough cracks that they leach like somewhat smaller particles.¹⁰

Furthermore, recent work has also shown that copper recovery rates from operating dumps at Kennecott's Bingham mine can be correlated quite well with laboratory studies of copper recovery from similar material when the mean particle size of the waste, as found in the dump, is compared with laboratory leaching studies.

The heat balance in a waste dump may be described by:

$$\rho_T C_T \frac{\partial T}{\partial t} = -(\rho_l C_l V_l + \rho_g C_g V_g) \cdot \nabla T + Q_A + K_T \nabla^2 T \quad [17a]$$

where ρ and C are the density and heat capacity of the

total dump (subscript T) and the liquid (subscript l) and gas (subscript g) phase of the dump. V_l is the Darcy liquid velocity (i.e. cm^3 water/ cm^2 dump surface-s passed through the dump). V_g is the darcy air velocity through the dump (i.e. cm^3 air/ cm^2 dump area-s). K_T , the thermal conductivity of the dump is taken to be 5×10^{-3} cal/ $\text{cm} \cdot ^\circ\text{C} \cdot \text{s}$. For calculation, $\rho_T C_T = 0.6$, $\rho_l C_l = 1.0$, and $\rho_g C_g = 1.3 \times 10^{-3}$ ($0.126 + 0.0283T$), where T is the temperature of the dump. This last expression takes into account the thermal effects of evaporation. It is assumed the air in the dump is always saturated with water; account is taken of the increase in water saturation values with increasing air temperature, T , and the effect, through the latent heat of vaporization, this would have on the heat capacity or heat carrying ability of air.

For a one dimensional dump (i.e. air and water flow restricted to be vertical only), Eq. [17b] simplifies to:

$$\rho_T C_T \frac{\partial T}{\partial t} = (\rho_l C_l V_l - \rho_g C_g V_g) \frac{\partial T}{\partial z} + \alpha_A + K_T \frac{\partial^2 T}{\partial z^2} \quad [17b]$$

One dimensional convective air flow through a dump may be described:

$$V_g = \frac{k_{AVE}}{\mu_g} \frac{\Delta P}{H} \quad [18]$$

H is the height of the dump, μ_g is the viscosity of air = 1.9×10^{-4} poise.* ΔP is the pressure drop across the

*1 poise = 0.1 Pa·s.

dump. ΔP may be expressed as:

$$\Delta P = \rho_{00} g_0 H \sum_i \bar{H}_i (\alpha(T_i) T_i + \beta (1 - [\text{O}_2]^g)) \quad [19]$$

Here ρ_{00} is the density of air at STP, g_0 is the gravitational constant, $\bar{H}_i = H_i/H$, is the normalized thickness of the i th incremental level of the dump. $\alpha(T_i)$ is the temperature dependent coefficient of thermal expansion which, like the heat capacity, includes the effects of changing water vapor saturation. β is a coefficient which describes the decrease in air density due to oxygen depletion ($\beta = 2.83 \times 10^{-2}$). $[\bar{\text{O}}_2]^g = [\text{O}_2]^g / [\text{O}_2]_{STP}^g$. k_{AVE} , the average permeability of the dump, may be expressed:

$$k_{AVE} = \frac{1}{\sum_i \bar{H}_i / k_i} \quad [20]$$

Any distance, z_i , from the base of the dump, where fresh air is assumed to enter, the oxygen concentration in the dump will be:

$$[\bar{\text{O}}_2]^g = 1 - \frac{\alpha_{\text{O}_2} z_i}{V_g [\text{O}_2]_{STP}^g} \quad [21]$$

Eqs. [17b], [18], and [21] represent a model of the dump leaching process that includes both physics (air convection) and chemistry. The equations can be solved using an implicit finite difference scheme in which the dump is considered to be broken into N layers. The method used was to start the dump leaching at some starting temperature and loop between Eqs. [18] and [21] until a steady state O_2 profile and air velocity was attained. Then Eq. [17b] was used to determine the temperature of the dump at $t + \Delta t$. Δt was generally taken to be one month. The average rate of fluid

application was used, an approximation that has been shown valid so long as the leach cycle is less than three months. As will be discussed later the ambient temperature was varied seasonally in a manner appropriate to the location of the dump (temperature measurements were available from a mine station). The surface temperature of the dump was also varied seasonally but at a higher average temperature and over a more restricted range. Air convection kept the dump surface warmer than the surroundings. Snow was observed to melt more quickly on the dump than in the surrounding areas.

Given a set of parameters and operating procedures (rate of application of water), the finite difference model computes the leach history of the model dump. The dump is considered to be broken into N layers. The percent copper leached per month (or the effluent copper heads) can be computed easily:

$$\frac{\text{fraction Cu}}{\text{mo}} = \frac{dX_{TOT}}{dt} \left(2.68 \times 10^6 \frac{\text{s}}{\text{mo}} \right) = 2.68 \times 10^6 \sum_{\text{dump}} \frac{(G_{NS} \frac{dX_{NS}^i}{dt} + G_S \frac{dX_S^i}{dt})}{N(G_S + G_{NS})} \quad [22]$$

$$\text{HEADS [g/1 Cu]} = \rho_R (1 - \Phi) H$$

$$\sum_{\text{dump}} \frac{(G_{NS} \frac{dX_{NS}^i}{dt} + G_S \frac{dX_S^i}{dt})}{NV_l} \cdot 1000 \quad [23]$$

The cumulative pct leached, $1 - X_{TOT}$ is just:

$$\text{fraction Cu leached} = 1 - \frac{(G_{NS} X_{NS} + G_S X_S)}{G_{NS} + G_S} \quad [24]$$

The next section compares the rate of leaching and the cumulative leaching of a test dump to the rate of leaching and cumulative leaching computed by the model through Eqs. [22] and [24].

CALIBRATION OF THE MODEL

Fig. 4 shows a cross section of the Midas test dump, built of mine waste with normal size distribution by the Utah Copper Division of Kennecott Copper Corporation at Bingham Canyon, Utah. The dump is about 400 ft long, 200 ft wide. The average depth is 20 ft with a

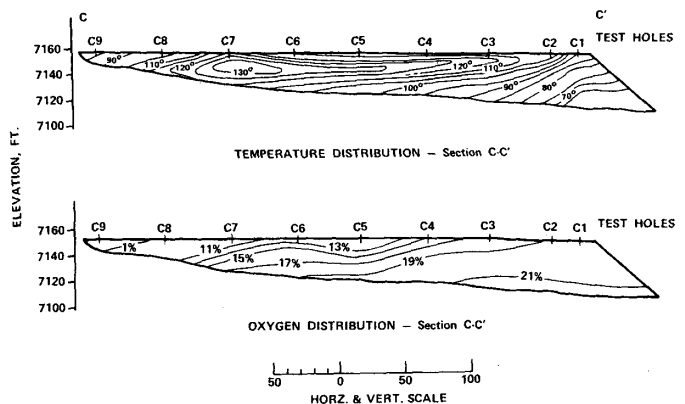


Fig. 4—August 1969 temperature and oxygen distribution in a section through the Midas test dump.

maximum depth of 40 ft. Fifty-eight leaching ponds cover the top of the dump. The waste tonnage beneath the ponds is about 93,000 tons (assuming 1.7 tons/yard³). The waste itself is 60 pct quartzite and 40 pct biotite granite. The average grade of the waste is 0.145 pct copper. 80 pct of the copper was sulfide, dominantly chalcocopyrite; the rest was nonsulfide copper.

Leaching of the dump began on April 9, 1969. Prior to this, there had been some runoff through the dump but very low copper extraction. Fig. 5 shows that leaching after water application was slow at first, increased rapidly to a maximum about five months after the start of leaching, and then fell steadily, with some fluctuations that appear correlated with the season (maximum in summer). Fig. 4 shows that by August 1969 the internal dump temperature had risen to 130°F (54°C). There was substantial oxygen depletion as the air convected through the dump. It can also be seen that the air convected in along the high permeability base of the dump and then up through the dump—the one dimensional model is appropriate for this case.

As time went on the location of maximum dump temperature shifted from the far end of the dump (as shown in Fig. 4) to about the same distance from the near end.

The parameters used in the model are listed in Table II.

The following thermal boundary conditions were chosen. The base of the dump was fixed at 20°C. The top surface temperature was allowed to vary:

$$T(\text{mo}) = 10^\circ\text{C} - 10 \cdot \cos((\text{mo} - 1)\pi/6)$$

where mo runs from 1 to 12 and is the number of the calendar month. Thus the top surface of the dump was assumed to vary seasonally between 32°F and 68°F, (0°C and 20°C), a slightly more restricted and hotter range than the ambient temperature variation of 19°F to 63°F (-7°C to 17°C).

The temperature at the base of the test dump was observed to fluctuate somewhat. The assumption of a constant 20°C basal temperature is a matter of convenience and is probably subject to some error. Both boundary conditions are plausible. Subsequent work has shown these assumptions to be quite reasonable.

The starting temperature of the model dump was 10°C. The Midas test dump was built in winter so the dump was initially at least this cold.

Fig. 6 compares the calibrated model leaching history to the leaching history of the Midas test dump shown in Fig. 5. The match in general is quite good. In addition to the leaching history similarity, the model dump reached 51°C internal temperature by August 1969 and then decreased in temperature to about 14°C, as did the far end of the Midas test dump. In August 1969 the effluent oxygen concentration was 9 pct, in good agreement with observation (see Fig. 4). The values of τ_{CS} and τ_{DC} are quite close to the values anticipated from Eqs. [14] and [15] (compare Tables I and II).

The initial rise in extraction rate is due to the heating up of the dump. This feature is not peculiar to the Midas test dump, and can be observed in the leaching history of many dumps. The fall in leaching rate after the first seven months of leaching is due to the fact the more accessible copper has been leached and Fe⁺⁺ must diffuse through already leached areas to reach the remaining copper. The fall in dump temperature also contributes to the decline in leaching rate.

VARIATIONS FROM THE BASE MODEL

It is of interest to vary the model parameters to see what effect they may have on the rate of copper extrac-

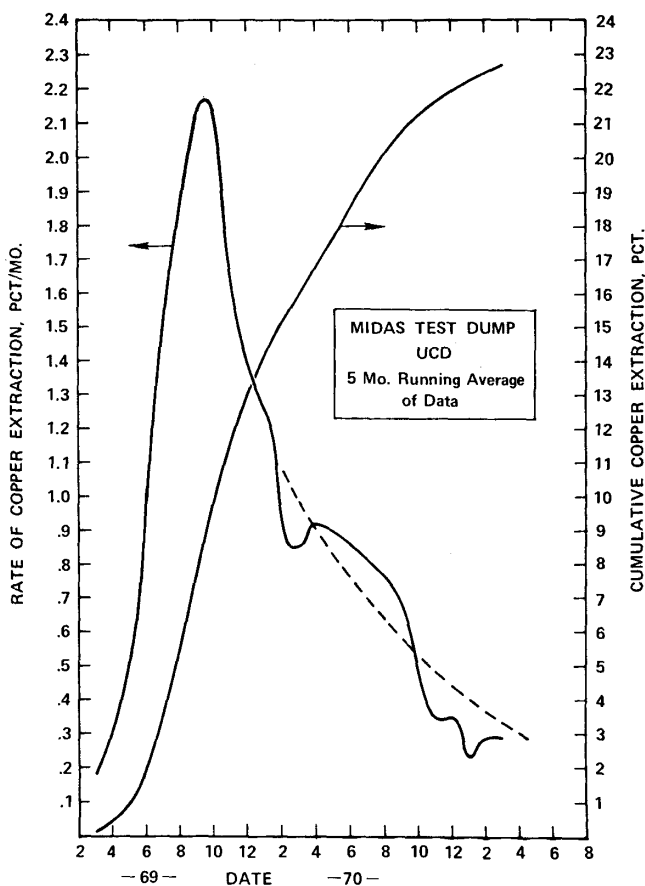


Fig. 5—The rate of extraction and cumulative extraction of copper from the Midas test dump, using a five-month running average.

Table II. Parameters Used for the Model Shown in Fig. 6

Parameter	Description	Value
H	Height of dump	670 cm (22 ft)
FPY	Moles pyrite leached/mole Cu leached	47
k'	Dump permeability k_{AVE}	10^{-5} cm ² (1000 darcys)
V_i	Rate of water application	2.26×10^{-5} cm ³ /cm ² dump·s (0.02 gal/ft ² ·h)
G_S	Dump sulfide copper grade	0.116 wt pct
G_{NS}	Dump nonsulfide copper grade	0.029 wt pct
τ_{DS}	Diffusional sulfide leach time (20°C)	1700 mo
τ_{CS}	Leach time for sulfide copper under surface area rate control (20°C)	200 mo
τ_{DNS}	Diffusional nonsulfide leach time (20°C)	500 mo
τ_{CNS}	Leach time for nonsulfide copper under surface area rate control (20°C)	300 mo
E_{DS}^*, E_{DNS}^*	Activation energies for diffusion	5.0 kcal/mol
E_{CS}^*, E_{CNS}^*	Activation energies for chemical leaching reactions	18.0 kcal/mol

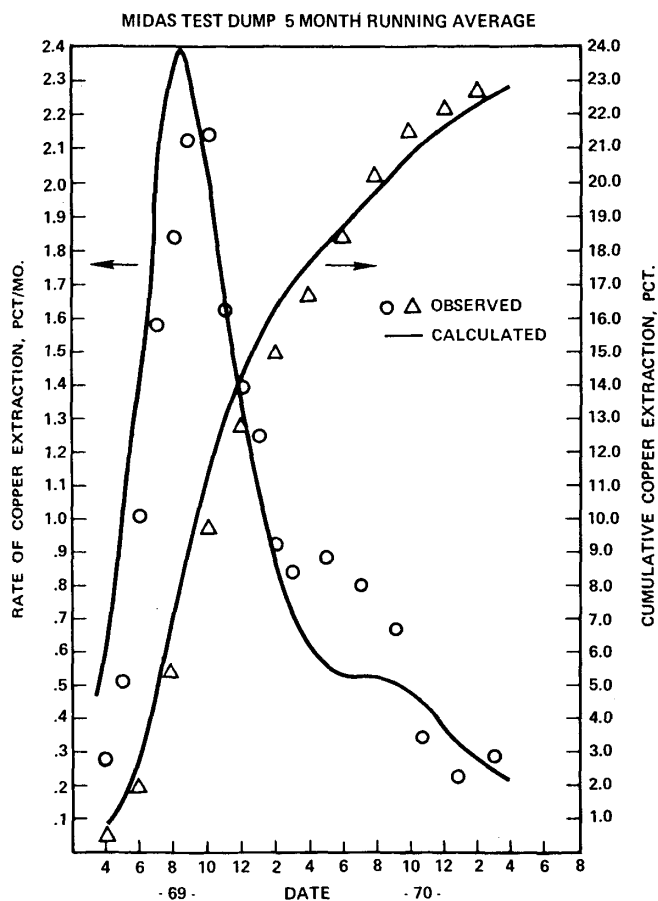


Fig. 6—Comparison of observed and calculated Midas test dump copper leaching behavior.

Table III. The Effect of Parameter Variation on Copper Recovery After 24 Months of Leaching

Parameter	Variation	Increase in Copper Recovered in 24 Months, Pct
H ✓	+10	8.9
FPY ✓	+10	6.2
τ_{DS}	-10	6.2
G_S ✓	+10	4.8
E_{DS}^*	+10	2.7
τ_{CS}	-10	1.8
V_t ✓	-10	1.3
τ_{DNS}	-10	1.3
E_{CS}^*	+10	0.9
G_{NS}	+10	0.9
τ_{CNS}	-10	0.4
E_{DNS}^*	+10	0.4
Starting Temp	+10	0.4
E_{CNS}^*	+10	0.0
k'	+10	0.0
	-10	-0.4

tion. Table III lists the parameters of Table II and shows the percent increase in copper extraction after twenty four months of leaching that results from a 10 pct alteration in the listed parameter. The parameters that are checked affect the rate of leaching primarily by allowing the dump to attain higher temperatures.

The reader is cautioned that the variations in leach rate shown in Table III are based only on what is in the model. Much may go on in a waste dump that has not

been included, as yet, in the model. Secondly, not all the parameters listed in Table III are mutually independent. For example, increasing FPY at a constant sulfide copper grade will cause τ_{DS} to increase substantially. Lastly, the combination of parameters that successfully models the Midas test dump is not necessarily a unique set or the correct set. Data from more than one test dump is needed to resolve these uncertainties. The most critical uncertainties are probably the chemical activation energies and FPY (see Table II).

The lack of dependence of leach rate on permeability simply indicates the dump was shallow and permeable enough not to be oxygen starved anywhere. Had the dump been thicker (~100 ft.), a significant dependence of leach rate on permeability would be noted.

CONCLUSIONS

From the discussion presented it can be concluded:

- 1) Air convection is an important part of the dump leaching process and must be accounted for in any successful model of this process (Fig. 3).
- 2) Exothermic oxidation reactions heat up waste dumps with time (Fig. 4). Any leaching model that is to be applicable to real dumps must account for the temperature dependence of the leaching process.
- 3) A simple model that requires energy, mass and momentum balance, and that derives rate control from a temperature dependent shrinking core model and a single "average" waste particle diameter (Eqs. [12], [13], [16]) has proved remarkably successful in accounting for the most important observed features of the leaching history of a well studied test dump (Fig. 6).
- 4) Dump height, lixiviant application rate and dump permeability are the most important factors affecting the rate of copper leaching that are accessible to operational alteration. (Table III and discussion in text).

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NOMENCLATURE

- a : radius of a waste particle, [cm]
 a_{sulf}^R : surface area of sulfide mineralization per unit volume of waste, [cm⁻¹]
 C_g : heat capacity of gas phase in dump including effects of variable water saturation, [g/cm³]
 C : heat capacity of mobile liquid phase in dump, [cal/g °C]

C_T	: heat capacity of dump as a whole, [cal/g °C]
D_{ox}^l	: diffusion constant of oxidant in water, [cm ² /s]
E_{DS}^*, E_{DNS}^*	: activation energies describing, through Eq. [16], the temperature dependence of τ_{DS}, τ_{DNS} , [kcal/mole]
E_{CS}^*, E_{CNS}^*	: activation energies describing, through Eq. [16], the temperature dependence of τ_{CS}, τ_{CNS} , [kcal/mole]
FPY	: moles of pyrite leached per mole of sulfide copper leached [-]
G_S	: initial copper sulfide grade of dump, [wt fraction Cu]
G_{NS}	: initial copper nonsulfide grade of dump, [wt fraction Cu]
g_0	: gravitational acceleration, [cm/s ²]
H	: height of dump, [cm]
H_i	: thickness of <i>i</i> th layer of dump, [cm]
\bar{H}_i	: dimensionless thickness of <i>i</i> th layer of dump $\bar{H}_i = H_i/H$, [-]
ΔH_R	: enthalpy of reaction, [kcal]
K_{AVE}	: oxidant required to leach waste particle, [g/cm ³ particle]
K_T	: thermal conductivity of dump as a whole (total dump), [cal/cm °C · s]
k'	: intrinsic permeability of the dump, [cm ²]
k	: average permeability of the dump (1 Darcy $\approx 10^{-8}$ cm ²), [cm ²]
k_{ox}	: chemical rate constant characterizing reaction of oxidant and sulfide minerals, [cm/s]
N	: number of layers into which dump has been arbitrarily broken for sake of computation (usually 30), [-]
[Ox]	: concentration of oxidant, [g/cm ³]
$[O_2]_{STP}^g$: concentration of oxygen in air under standard conditions of temperature and pressure, [g/cm ³]
$[O_2]^g$: concentration of oxygen in gas phase of dumps, [g/cm ³]
$[\bar{O}_2]^g$: normalized oxygen concentration in gas phase of dump $[O_2]^g = [O_2]^g/[O_2]_{STP}^g$, [-]
ΔP	: pressure drop across (bottom to top) the dump (10^6 dynes/cm ² \sim 1 atmosphere), [dyn/cm ²]
R	: gas constant (~ 2), [cal/°C · mol]
\mathcal{R}_A	: rate of heat generation, [kcal/cm ³ dump · s]
\mathcal{R}_{O_2}	: rate of oxygen consumption, [g O ₂ /cm ³ dump · s]
\mathcal{R}_{Cu}	: rate of copper leaching [g Cu/cm ³ · dump · s]
T	: temperature of dump at any particular location. Temperature of water, rock, and gas phases assumed identical, [°C]
T_R^l	: tortuosity of diffusion channels, [-]
V_g	: darcy gas velocity through dump, [cm ³ gas/cm ² dump area · s]
V_l	: darcy velocity of water passing through the dump. Average rate over application periods and rest cycles is used. [cm ³ water/cm ² dump surface · s]
X_{NS}	: fraction of initial nonsulfide copper remaining in dump or given layer of dump, [-]
X_S	: fraction of initial sulfide copper remain-

ing in the dump or a given layer of the dump, [-]

z_i : distance of center of *i*th layer of dump from base of dump, [cm]

GREEK LETTERS

α	: coefficient of thermal expansion of air including effects of changing water vapor saturation. [-]
β	: coefficient describing the change in air density due to oxygen depletion. (See Eq. [19]), [-]
δ	: reaction skin depth, [cm]
μ_g	: viscosity of the gas phase in the dump, [ρ]
ρ_R	: density of waste particles, [g/cm ³]
ρ_T	: density of the dump as a whole ("Total" dump) ($\rho_T = \rho_R[1 - \Phi]$), [g/cm ³]
ρ_l	: density of liquid phase of dump (water), [g/cm ³]
ρ_g	: density of gas phase in dump including effect of variable water vapor saturation, [g/cm ³]
ρ_{∞}	: density of air at standard temperature and pressure, [g/cm ³]
τ_{DS}, τ_{DNS}	: time to leach typical waste particle completely of sulfide or nonsulfide copper assuming rate of leaching is limited by diffusion of oxidant or acid into the particle, [mo]
τ_{CS}, τ_{CNS}	: time to leach typical waste particle completely of sulfide or nonsulfide copper assuming the rate of leaching is controlled by the shrinking surface area of the sulfide or non-sulfide copper ("chemical" control). [mo]
Φ	: interblock porosity of dump (usually ~ 25 pct), [-]
Φ_R^l	: porosity of waste through which diffusion can take place, [-]

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