Lots here

magnetic field
Obser
Th
Electricity in Atm
Cosmic Rays

Lecture 10 : Planetary + Solar Magnetic Fields

1st #1

A. The Earth

Science requires observation + Theoretical interpretation.

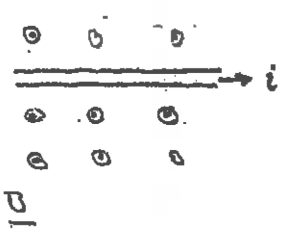
The magnetic fluxes + Atm
The electric connection

We have the fluid dynamic tools to interpret. Now we need some observations. We start with the earth's magnetic field.

The magnetic field is *defined* as the *force* exerted on one coulomb of charge, q_0, moving with velocity $\underline{u}$ through the magnetic field, $\underline{B}$:

$$(10\text{-}1) \qquad \boxed{\underline{F}\,[\text{nt}] = q_0\,[\text{coul}]\,\underline{u}\,(\text{m/s}) \times \underline{B}\left[\frac{\text{nt}}{(\text{coul})(\text{m/s})}\right]}$$

$$= \left[\frac{\text{nt}}{\text{amp}-\text{m}}\right] \equiv \text{weber/m}^2$$

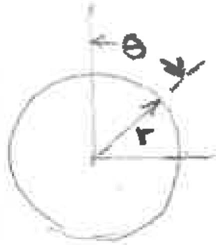


B is wrapped around wire

$$1\ \text{weber/m}^2 \equiv 10^4\ \text{gauss}$$

Earth's magnetic field ~ 0.5 gauss $= 0.5 \times 10^{-4}$ weber/m^2

$$1\ \text{gamma} \equiv \gamma = 10^{-5}\ \text{gauss} = 10^{-9}\ \text{weber/m}^2$$



$$B_\theta = -\mu_0 \frac{1}{r}\frac{\partial V_m}{\partial \theta} = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sin\theta$$

$$B_r = -\mu_0 \frac{\partial V_m}{\partial r} = \frac{\mu_0}{4\pi}\frac{2m}{r^3}\cos\theta$$

$m = i\pi r^2$

$B_\theta = \frac{\mu_0 i}{4 r}$

$B_0' = \frac{\pi i \times 10^{-7}}{r}$

at $r = R$

$$B_\theta = B_0 \sin\theta$$

$$B_r = 2B_0 \cos\theta$$

$$B_0 = \frac{\mu_0}{4\pi}\frac{m}{R^3} = 3\times10^{-5}\ w/m^2$$

$$= \underline{0.307 \text{ gauss}}$$

$R_{earth} = 6,370$ km

$\mu_0 = 4\pi \times 10^{-7}$ weber/amp-m

$m_E = 7.94 \times 10^{22}$ amp-m^2

$$B_{Total} = \sqrt{B_\theta^2 + B_r^2} = B_0\sqrt{\sin^2\theta + 4\cos^2\theta}$$

$\sin^2\theta + \cos^2\theta = 1$

$$= B_0\left(1 + 3\cos^2\theta\right)^{1/2}$$

$B_{Tot}(\underset{pole}{\theta = 0}) = .614$ gauss $\quad$ pole

$B_T(\underset{equat}{\theta = 0}) = .307$ gauss $\quad$ Equator

Actual: $\quad B_{N.Pole} = 0.615$ gauss @ 75°N 101°W

$B_{S.Pole} = 0.725$ gauss @ 67°S 143°W

Horiz Fld $\quad \sim 0.250 \le B_{equator} \le 0.420$ gauss

Declinat) - ... $\quad B_{overall} \sim 0.5$ gauss $= 50,000\ \gamma$

at any instant of time. compasses do not point to the poles but up to 30° from them.

The cause of compass direction variations are largely due to the westward drift of the anomalies in the dipolar field (westward drift of the magnetic vortices). Drift is fairly regular and the field changes thus somewhat predictable!

[VF]

Magnetic field is in fact decreasing in intensity at ~ 4 % per 100 years (0.04 %/yr). In fact we now know that the polarity of the earth's magnetic field has changed irregularly in the past. This is a major recent story in the solid earth sciences that led to the discovery of plate tectonics (Morgan + Vine) — and one I witnessed happening as a graduate student.

Flip my field

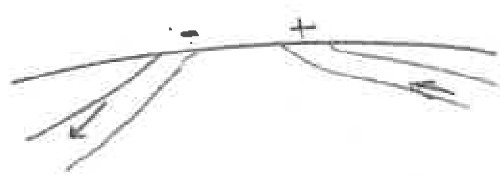
B. The Sun

Insight comes from stars: All stars have "dipole" magnetic fields, but the polarity of the sun's dipole field flips every 11 years. Maunder Butterfly

diagram

- start at higher latitude
- move toward equator
- each spot lasts several weeks
- spots are outbreak of strong magnetic field
  - solar flares
  - 1000's of gauss

- Spots are pairs of spots with different polarity (bipolar groups)



Toroidal field outbreak

- during each 11 yr cycle one polarity to west, another to east. Next cycle the polarity has changed

- solar poloidal field ~ 1 to 2 gauss but rapidly varies + variations in N + S hemispheres do not correlate. ∴ poloidal field is

REVIEW

Maxwell's Equations

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0}$$

$$\underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$c^2 \, \underline{\nabla} \times \underline{B} = \frac{\partial \underline{E}}{\partial t} + \frac{\underline{j}}{\epsilon_0}$$

$$\nabla \cdot B = 0$$

$$\underline{j} = \sigma \left(\underline{E} + \underline{u} \times \underline{B} \right)$$

r is the distance from the centers of mass of the two, and G is the gravitational constant ($G = 6.673 \times 10^{-11}$ nt-m^2/kg). We can define a gravitational field strength as the force per unit mass:

force for an infinitesimal mass (so as not to perturb field)

$$\underline{g} = \lim_{m \to 0} \frac{\underline{F}}{m} = -\frac{GM\hat{r}}{r^2} \;\underline{\underline{m}}\; m/\sec^2$$

= "test particle"

Note gravity is an acceleration, and in fact gravitational mass is the same as the inertial mass, which is one of the fundamental connections in physics. $\underline{g}$ is the gravitational force field.

Dicke + Braginsky experiment!

By the same logic we can define an electric field $\underline{E}$, and a magnetic field $\underline{B}$ that will exert force on a unit of charge. From Coulomb's law

$$\underline{F}_{\text{electrostatic}} = \frac{-1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

(11')

(11-6)
$$\boxed{B = \lim_{q \to 0} \frac{F}{q u}}$$ q = test particle

$\underline{\underline{m}}$ $\frac{nt}{\text{coul} \cdot \text{m/sec}}$

coul/sec $\equiv$ 1 ampere

$= \frac{nt}{\text{ampere-m}} = \frac{\text{webers}}{m^2}$

In fact an ampere is defined as the current w/ produces force of 1 nt for a 1 m length of wire in a magnetic field of 1 weber/m²

There is no such thing as a magnetic monopole so:

(11-7)
$$\boxed{\underline{\nabla} \cdot \underline{B} = 0}$$

How is $\underline{B}$ generated?

A current in a wire generates a magnetic field

$$B = \frac{1}{4\pi\epsilon_0 c^2}\frac{2I}{r}$$

$$B = \frac{I}{2\pi r \epsilon_0 c^2}$$

coul/m²-sec

$$\int_S (\nabla \times B) \cdot da = \int_S \frac{j}{\epsilon_0 c^2}\, da = \frac{I}{\epsilon_0 c^2}$$

$$\int_\Gamma B \cdot dl = 2\pi r B = \frac{I}{\epsilon_0 c^2}$$

(11-8)
$$\boxed{c^2 \, \nabla \times B = j/\epsilon_0}$$

line integral of tangential component

Finally consider how current, j, is induced in a fluid. Let σ = resistance per unit volume and remember $E = iR$ or $\sigma E = j$.

Remember also that moving through a field B can generate E.

Considering all we can infer that:

Siemens m^{-1}

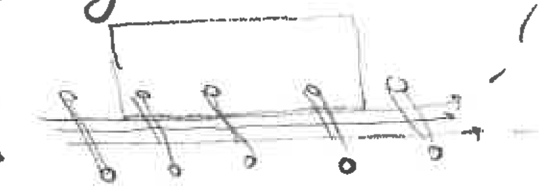
$ohm^{-1}\, m^{-1}$

$ohm = volt\; amp^{-1}$

Intrinsic prop of material

(11-10) $$\boxed{j = \sigma(\underline{E} + \underline{u} \times B)}$$

One quick example: Magnetic field in solenoid. Field is low outside solenoid:



From 11.8 $$\nabla \times B = \frac{j}{c^2 \epsilon_0}$$

$$\oint_\Gamma B \cdot dl = B_0 L = \frac{NI}{\epsilon_0 c^2}$$

$$\boxed{B = \frac{N/L\; I}{\epsilon_0 c^2}}$$

$$E = \frac{V}{L}$$

$$j = \frac{I}{A}$$

$$\sigma = \frac{j}{E} = \frac{I/A}{V/L} = \frac{IL}{VA}$$

$V = IR$ $\quad R^{-1} = \frac{I}{V}$

$(m^{-1}$ Resistance$)$

$$\boxed{R = \frac{L}{A\sigma}}$$

$$\sigma = \frac{L}{AR} \equiv ohm$$

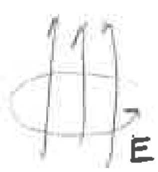
$\equiv ohm^{-1}\, m^{-1}$

Lecture 12 The Elsasser Dynamo

A. Theory

Now let's apply Maxwell's equations to magnetic dynamo. The key, as we discussed earlier, is the magnification of the magnetic field by the fluid stream that occurs in a turbulently convecting fluid. To work, magnetic lines of force should be frozen into fluid and carried with it when it moves. This should occur naturally in conductive fluids of large dimensions. All this should be obvious from the equations.

B Idea



To make these matters clear we should seek an equation for the magnetic field alone. As the magnetic field decays it should generate an electric field vorticity which will drive current loops which sustain the original field. A magnetic field

Then equation and using $\underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$

$$\underline{\nabla} \times \underline{\nabla} \times \underline{B} = \frac{\sigma}{c^2 \epsilon_0} \left(\nabla \times E + \nabla \times (\underline{u} \times \underline{B}) \right)$$

$$\underline{\nabla}(\underline{\nabla} \cdot \underline{B}) - \nabla^2 \underline{B} \qquad\qquad -\frac{\partial B}{\partial t}$$

$\underline{\nabla} \cdot \underline{B} = 0$ no magnetic dipoles

$$\frac{\cancel{B}}{L^2} = \frac{\sigma}{c^2\epsilon} \cancel{B}\, s^{-1} \qquad \nu_m \equiv \frac{c^2 \epsilon_0}{\sigma} \stackrel{m}{=} L^2 s^{-1}$$

$$\frac{s}{L^2} \stackrel{m}{=} \frac{\sigma}{c^2 \epsilon}$$

magnetic diffusivity

Thus:

$$\boxed{\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) + \nu_m \nabla^2 B} \qquad (11\text{-}8)$$

must have dimensions cm^2/sec

Reynolds = inertial force / viscous force

where ν_m is the magnetic ~~diffusivity~~ defined:

$\nu_m = \dfrac{c^2 \epsilon_0}{\sigma}$. Now we can define a magnetic

Peclet number equal to the ratio of the last two

Peclet = rate adv / rate diffus

terms:

$$R_m \equiv \frac{\nabla \times (u \times B)}{\nu_m \nabla^2 \underline{B}} . \qquad \frac{\text{advection}}{\text{diffusion}}$$

If we let $\nabla \sim \frac{1}{\lambda}$, where λ is the scale over which $\underline{B}$ varies, we see

$$R_m = \frac{\lambda u}{\nu_m} \qquad (11\text{-}9)$$

(5)

$R_m = \frac{\lambda u}{\eta_m}$

$\eta_m = \frac{c^2 \epsilon_0}{\sigma}$

$R_m = \frac{\sigma \lambda u}{c^2 \epsilon_0}$

Values of $R_m > 1$ mean the magnetic field will decay very little. R_m can be made arbitrarily large as σ becomes large or λ (the dimensions of the system) become large. Thus large, convecting, rotating conductors may generate magnetic fields.

Put some numbers in R_m

σ_{cu} (p14) $= \frac{1}{1.7\times10^{-8}} = 6\times10^{7}$ siemens m^{-1}

$\frac{\text{volts}}{\text{amp}} = \frac{\text{Joul}\cdot\text{coul}^{-1}}{\text{coul sec}^{-1}}$

0.3 S cm^{-1}

$$\lambda \sim 3000 \text{ km} = 3\times10^{6} \text{ m}$$
$$\sigma \sim 10^{5} \, (\text{ohm-m})^{-1}$$
$$\epsilon = 8.85\times10^{-12}$$
$$c = 3\times10^{8} \text{ m/sec}$$
$$u \cong 100 \text{ km/yr} = \frac{10^{5}\text{m}}{3.15\times10^{7}\text{ sec}} = 3\times10^{-3} \text{ m/sec}$$

Fe $= 10^{7} (\Omega\text{-m})^{-1}$?

$(\text{coul}^{2}\,\text{nt}^{-1}\,\text{m}^{-2})^{-1}$

see magnetic secular range $1^{\circ}/\text{yr}$

$$R_m = \frac{\lambda u \sigma}{c^2 \epsilon_0} = \frac{(3\times10^{6}\text{m})(3\times10^{-3}\text{ m/s})(10^{5})}{(3\times10^{8})^2 (8.85\times10^{-12})}$$

$$\cong 1130$$

Thus magnetic field should advect!

next ① Coulomb-J-Integral $\frac{D}{Dt}\int_A B\cdot da = 0$

② Alfen waves