Cowling's Theorem glow

diversion

Elsasser shows how field is frozen in

many conductors.

$$\nabla \times B = \frac{j}{c^2 \epsilon_0}$$

$$\nabla \times B = \frac{\sigma}{c^2 \epsilon_0} \left(\underline{E} + \underline{u} \times \underline{B} \right)$$

$$\nabla \times B = \frac{1}{\nu_m} \left(\underline{E} + \underline{u} \times B \right)$$

Thus

$$\underline{E} = \nu_m (\nabla \times B) - \underline{u} \times \underline{B}$$

From Maxwell's eqn $\nabla \times E = -\frac{\partial B}{\partial t}$

$$-\frac{\partial B}{\partial t} = \nabla \times E = \nu_m (\nabla \times \nabla \times B) - \nabla \times (u \times B)$$

$$\frac{\partial B}{\partial t} = -\nu_m \nabla \times \nabla \times B + \nabla \times (u \times B)$$

$$\underbrace{\int_a \frac{\partial B}{\partial t} da = -\nu_m \int_\Gamma (\underline{\nabla} \times \underline{B}) \cdot d\underline{l} + \int_\Gamma (\underline{u} \times \underline{B}) \cdot dl}$$

where we have used Stokes Theorem.

$$\frac{D}{Dt} \int_a B \cdot da$$

Glossa

7

change of flux through a of surface is fixed — flux added to material volume by motion

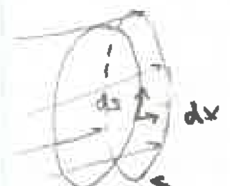
$$\underbrace{\int_a \frac{\partial B}{\partial t}\cdot da - \int_\Gamma (u\times B)\cdot dl}_{} = -\nu_m \int_\Gamma (\nabla\times B)\cdot dl$$

$$\frac{d}{dt}\int_a \underline{B}\cdot \underline{da} = \nu_m \int_\Gamma (\nabla\times B)\cdot \underline{dl}$$

(logic)

0 on $\nabla = \frac{1}{\lambda} \to 0$

large scale



Sweetie

$u\times B = 0$ if u is $\parallel$ to B

∴ $u\times B \neq 0$ if part of B is not parallel to u

Thus u points the bits spilled out

flux out edge of disk

normal to disk edge is $ds\times dx$

flux out edge is $\int_a (ds\times dx)\cdot B = dt\int_a (u\times B)\cdot ds$

flux out edge is flux lost to area due to its displacement — ∴ change sign

result is that $\int_a + \int_\Gamma = \frac{d}{dt}\int_a B\cdot da$.

Thus

Cowling's Integral

$$\boxed{\frac{d}{dt}\int_a \underline{B}\cdot \underline{da} = 0} \qquad (11\text{-}10)$$