I. Advection — Lagrangian + Eulerian Coordinates

So far we have described static situations: stress and strain rate. As soon as there is motion we have other needs such as describing the trajectory of motion, and other possible reference frames. We might choose fixed space (through which the fluid particles move) which is called the eulerian reference frame, or a frame attached to the material particles (the Lagrangian reference frame, for example). In the latter case, the frame attached to the material particles [illegible] maintains of a material element. That is

Statics

Vectors + tensors — transform w/ rotation matrices

Eigenfunctions
Invariants } looking in diff frames can be useful

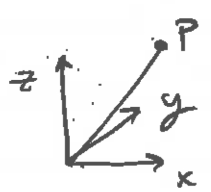
Review

① $\dot{\varepsilon}$ is only place

→ ② talk to much about streamlines

⇒ ③ Lagrangian method of antisymmetric part better — 2x axis rotation macroscopically small but microscopically large so that from a fluid motion point of view they are small but they contain enough molecules that their physical properties are representative of the fluid as a whole.

Now consider the scalar density field. Space is filled with a substance whose density varies from place to place, and the substance is moving.



The density is clearly a function of location, $\underline{x}$, and, if the substance is moving, (so density is a function of) time also, so:

$$\rho = \rho(x, y, z, t).$$

Using the chain rule

$$\underset{\text{Total variation}}{d\rho} = \underbrace{\frac{\partial \rho}{\partial t} dt}_{\text{time variation } (\Delta x_i = 0)} + \underbrace{\frac{\partial \rho}{\partial x} dx + \frac{\partial \rho}{\partial y} dy + \frac{\partial \rho}{\partial z} dz}_{\text{spatial variation } (\Delta t = 0)}$$

We can divide by dt ("d" because this is an time "independent" variable which depends only on itself and therefore can be specified independently):

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}\frac{dx}{dt} + \frac{\partial \rho}{\partial y}\frac{dy}{dt} + \frac{\partial \rho}{\partial z}\frac{dz}{dt}$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \underline{u} \cdot \underline{\nabla} \rho$$

Change in density of a fluid "particle" (if we move with the particle)

time variation at a point in space

space variation across ~~point~~ point in space (note spatial derivative is frozen in time – the same in Lagrangian and Eulerian coordinates, and $\underline{u}$ is the velocity of the particle in Eulerian space) [At $\Delta t = 0$ Lagr space = Eulerian space.]

Generalizing

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \underline{u} \cdot \underline{\nabla}$$

$$\boxed{\frac{D}{Dt} = \frac{\partial}{\partial t} + \underline{u} \cdot \underline{\nabla}} \qquad (4.1)$$

Notation in book: ~~special~~ capital D indicates material derivative, d indicates independent variable which depends only on itself.

The Eulerian description is more common, but the Lagrangian description is more fundamental { As we saw in deformation tensor derivation

For example, in a steady flow situation fluid properties seems to be increasing with time because the property is increasing upstream. But this does not indicate anything is changing in the fluid and does not require a physical or chemical process to be operating. It is just advection.

What is fundamental, from a physics view point, is whether the fluid properties are changing in material coordinates, whether $\frac{D\alpha}{Dt} > 0$.

The relation between material and Eulerian time derivatives (eqn 4.1), is central to Fluid Dynamics!

II. Streamlines and Streaklines

There are various ways to describe the trajectories of fluid particles. One way is by drawing streamlines, which are lines tangential to the flow direction at an instant of time.

Because an increment of a streamline, $d\underline{s}$, is tangent to the flow, there is no component of flow perpendicular to $d\underline{s}$, $\underline{u} \times d\underline{s} = 0$, and we can define a streamtube that will "contain" fluid like a pipe. Clearly this is very useful – flow through a set of pipes is much easier to understand than a whole flow field.

Streakline – current location of all fluid particles that have passed through a fixed spatial point at previous times. Streaklines are different from streamlines if flow is unsteady.

release dye at a point

III Streamlines

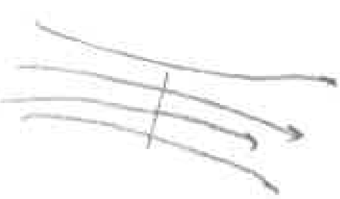
along a streamline

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$\underline{u} \times d\underline{s} = 0$ because u is then parallel to ds

stream tube

$\underline{u} \equiv -\hat{k} \times \nabla\psi$ in 2D



gradient $\perp$ to flow = velocity

very useful for visualization

Geostrophic flow — pressure balances coriolis force — is non-divergent

and streamlines are isobars — plot pressure at 300 mb (hPa)

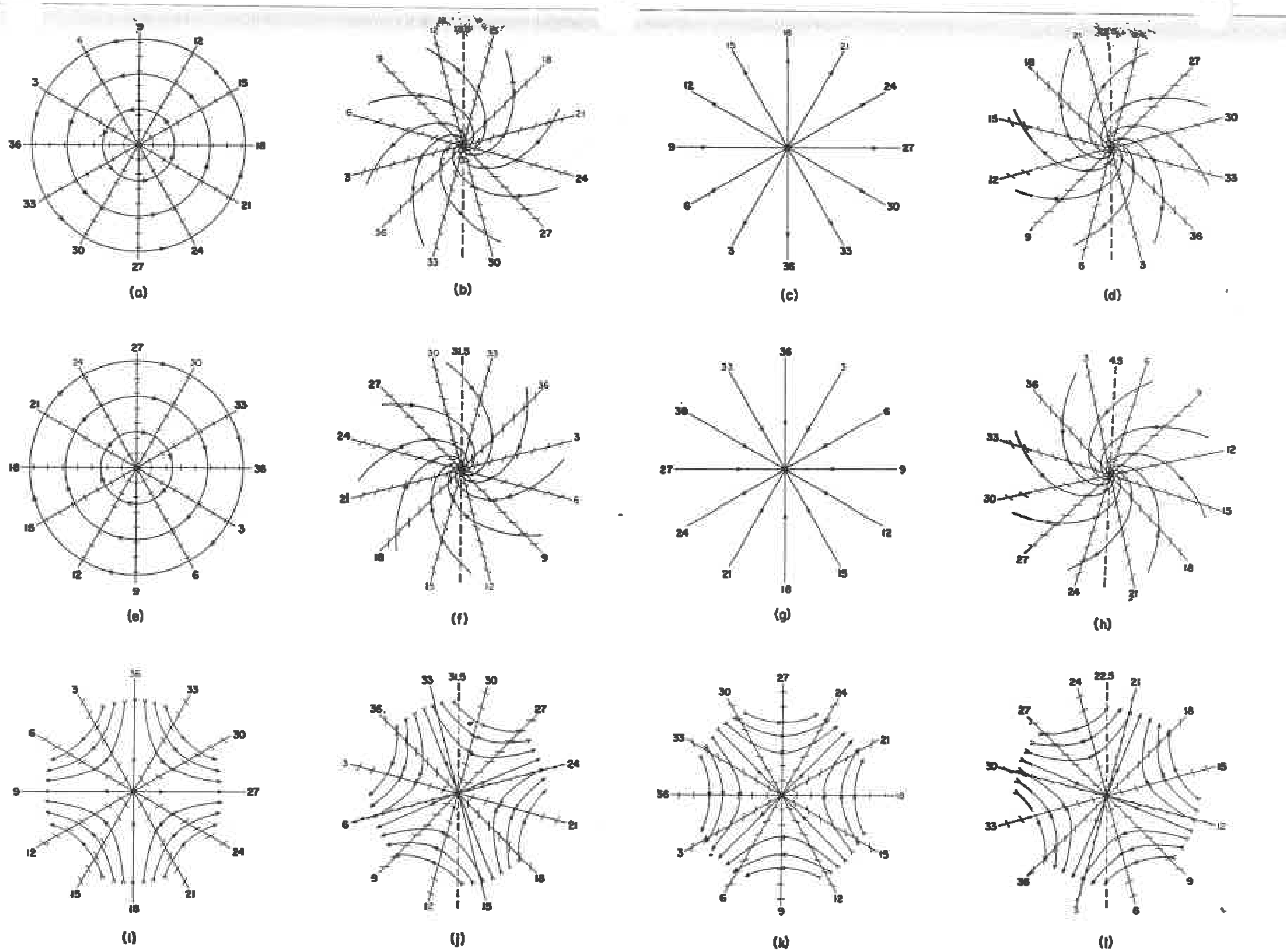


FIG. 10.04.—Singular points in the patterns of streamlines and isogons. Short line segments on the isogons indicate slope of streamlines. (*a*) Pure counterclockwise rotation; (*b*) diverging streamlines with counterclockwise rotation; (*c*) pure divergence; (*d*) diverging streamlines with clockwise rotation; (*e*) pure clockwise rotation; (*f*) converging streamlines with clockwise rotation; (*g*) pure convergence; (*h*) converging streamlines with counterclockwise rotation; (*i, j, k, l*) hyperbolic singular points.

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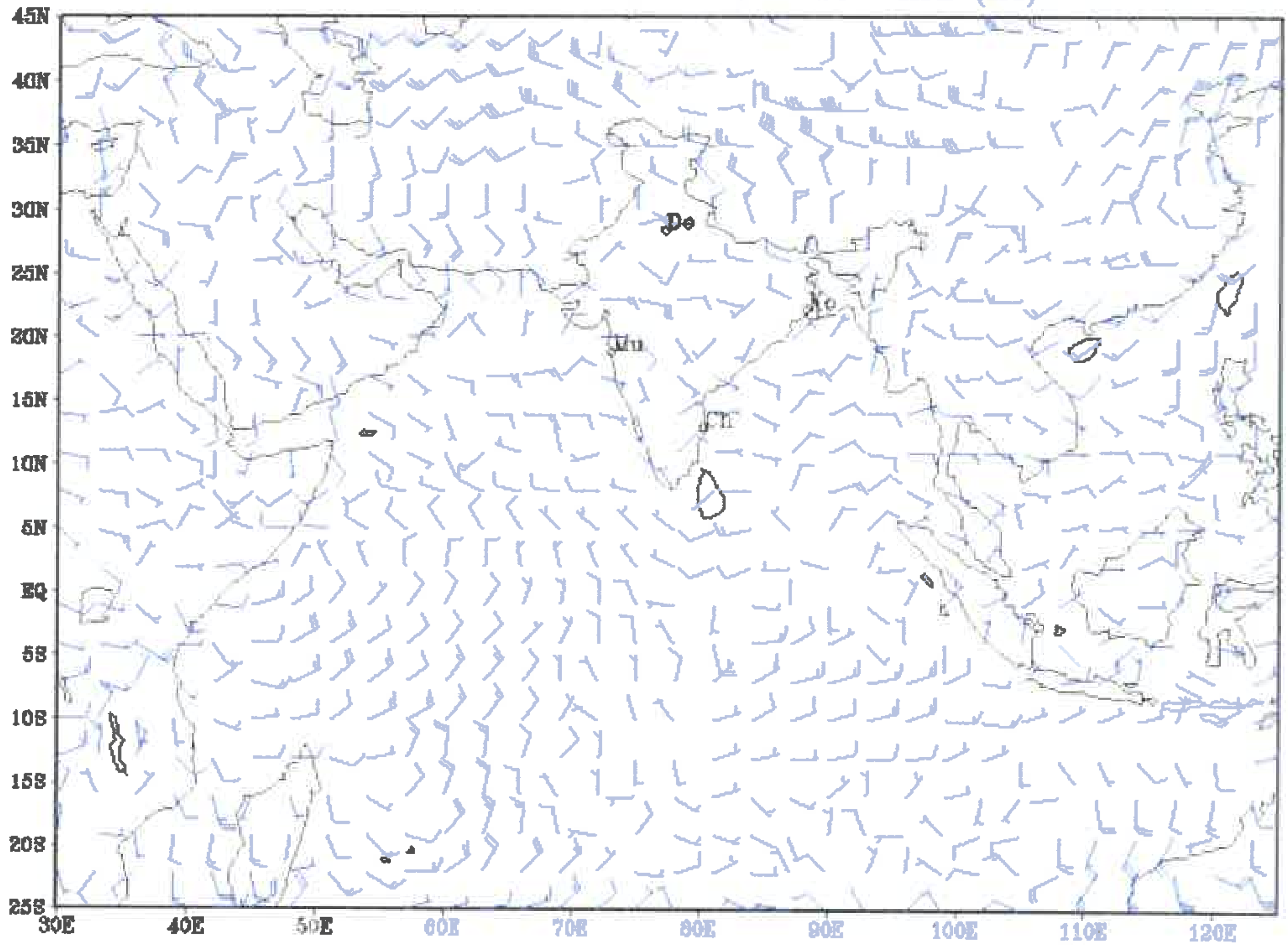
RSMC(IMD) New Delhi 925 hPa Analysis
00 UTC of 11 Feb 2010 wind (kt)



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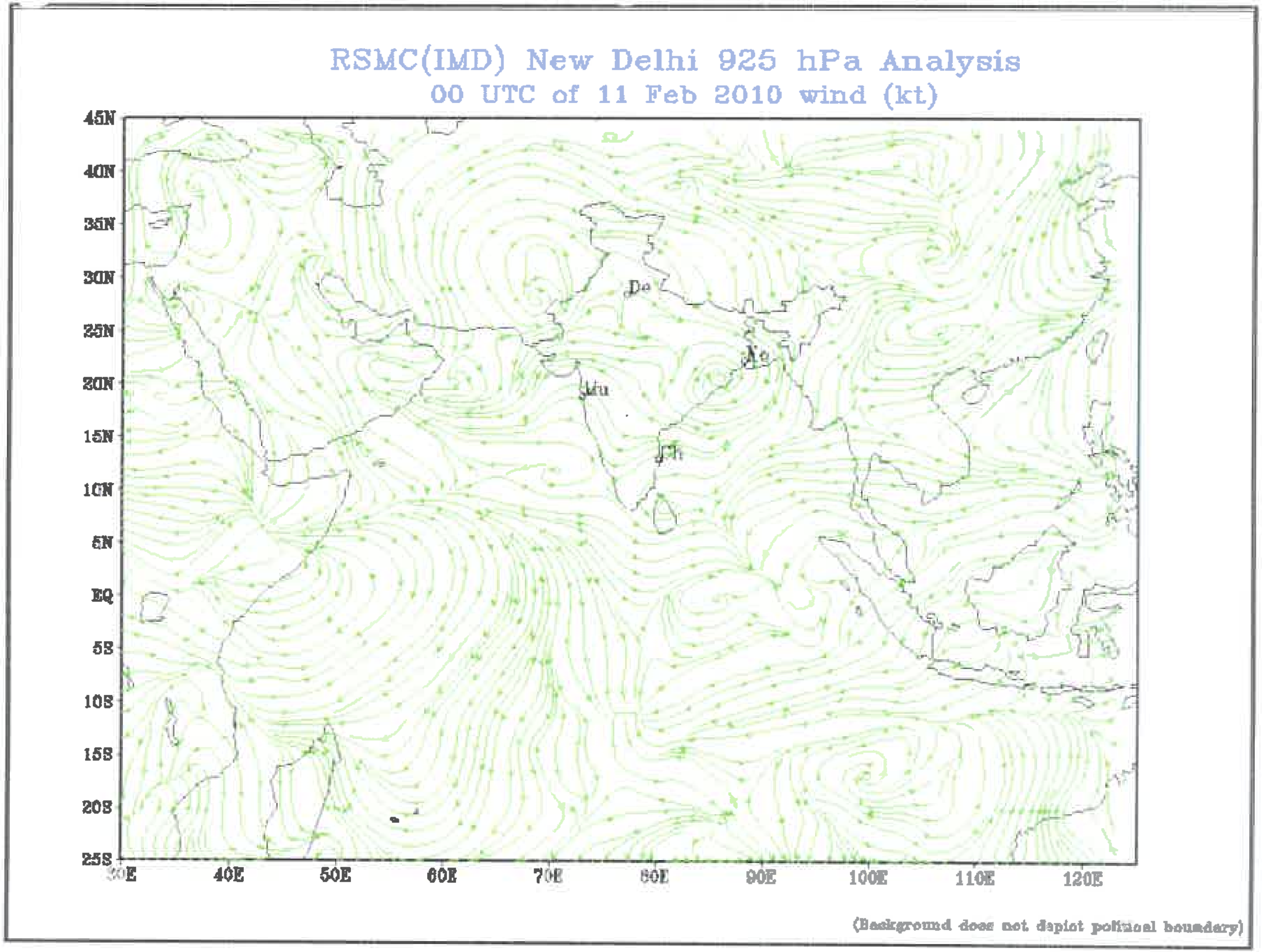
RSMC(IMD) New Delhi 925 hPa Analysis
00 UTC of 11 Feb 2010 wind (kt)
45N
40N
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