Nonparametric Demand Forecasting and Detection of Energy Aware Consumers

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***Abstract*—To increase the reliability of the power grid and reduce the risk of power supply failure, demand-side management (DSM) is of central importance. In this paper, a nonparametric test is applied to detect if the demand behavior of consumers is consistent with time-of-day electricity tariff initiatives. The test is based on Afriat's theorem in economics and has the unique feature that it provides necessary and sufficient conditions to detect if the price-demand behavior is consistent with utility maximization (i.e., the test detects demand-responsive consumers) without prior knowledge of the consumer's utility function. For consumers that are responsive to time-of-day pricing initiatives, a nonparametric learning algorithm is used to forecast power demands for unobserved electricity tariffs. The nonparametric learning algorithm can be used in anticipatory control structures in a DSM framework to achieve power usage objectives. Real-world data from Ontario's power system and numerical examples illustrate the accuracy of the nonparametric test and nonparametric learning algorithm for forecasting consumer demand.**

***Index Terms*—Afriat's theorem, artificial neural network (ANN), demand-side management (DSM), revealed preferences, smart grid, utility maximization.**

I. Introduction

An IMPORTANT problem in the power grid is the reduction of peak power demand. It is known that the probability of power supply failures increases during peak periods of demand [1], [2]. To ensure reliable power delivery, utility companies design the power grid to deliver enough power at peak demand; however, peak demand only accounts for a fraction of the daily power demand of the network [1], [3]. Given the limited energy resources available, and the emergence of new types of demands such as plug-in hybrid electric vehicles, there is a substantial need for novel demand-side management (DSM) strategies to promote efficient energy usage. Currently, an effective DSM strategy is that of smart pricing [1], [2]. Anticipatory control is an emerging DSM strategy, in which consumer power demands are forecasted for unobserved electricity prices, then specific prices are selected to meet power usage objectives (e.g., reducing peak power demand).

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To achieve power usage objectives depends on several factors including the energy consumption behavior of consumers, the consumers power budget, and environmental conditions. Consumer behavior can be modeled using utility functions as illustrated throughout the literature on DSM [1], [4]. For each consumer, the utility function represents the amount of satisfaction they receive from using a certain amount of power. It is in the self interest of demand-responsive consumers to attempt to maximize their respective utility for a given electricity price and budget. As a result of privacy concerns, consumers are unwilling to release their local consumption behavior. DSM designers typically make the following assumptions [1], [2], [5]: the utility function of consumers is known, and all consumers are utility maximizers. A challenge for DSM designers is making accurate predictions of the utility function of consumers. If the observed price-demand data is inconsistent with the constructed demand model, it is impossible to determine if the failure is a result of the selected parametric form of the utility or the consumer not satisfying utility maximization [6]. To overcome these difficulties, we provide a DSM strategy that detects utility maximization (i.e., demand-responsive) consumers that does not require parametric assumptions of the consumer's utility function.

A. Main Results

The main results of this paper are a feasibility test and forecasting algorithm that can be used for DSM as illustrated in Fig. 1. The feasibility test is used to detect if a consumer is a utility maximizer (i.e., demand responsive). The test is nonparametric—that is, explicit knowledge of the consumers utility function is not required. The nonparametric feasibility test to detect for utility maximization is constructed based on a remarkable result from the economics literature known as Afriat's theorem [7]–[9]. The consumption forecasting algorithm is nonparametric and does not require the utility function of the consumer be known *a priori*. The only requirement is that if the consumer is a utility maximizer, then their utility function is monotone and Lipschitz continuous. There are three main steps to the forecasting algorithm. First, the power budget of the consumer is independently forecasted using a least squares support vector machine (LS-SVM) learning algorithm. Second, a nonparametric learning algorithm, based on the results in [8] and [9] is used to estimate the utility function of responsive consumers. The third step uses the forecasted power budget and utility function to forecast power consumption behavior for unobserved electricity tariffs. The key advantages

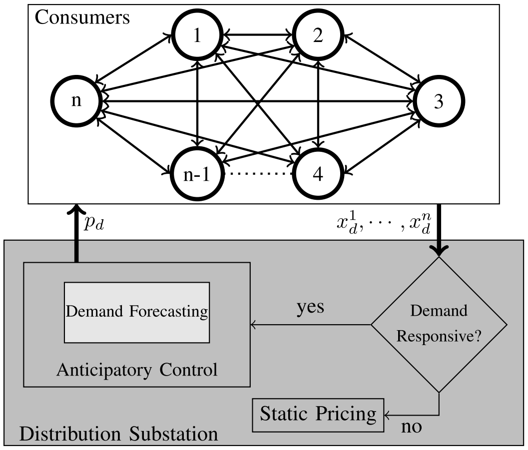


Fig. 1. Schematic of the proposed anticipatory control scheme in the smart grid as presented in Section III. $p_d \in \mathbb{R}^T$ is the electrical tariff on day d with each day containing T periods, x_d^i is the power demand of consumer $i \in \{1, 2, \ldots, n\}$ on day d for the given electricity tariffs p_d. For consumers that are responsive to time-of-day prices (i.e., responsive consumers), a demand forecasting algorithm, presented in Section V is used for anticipatory-based DSM. For nonresponsive consumers, a static pricing scheme is used. The distribution substation's DSM strategy is contained in the gray box and the consumer's in the white box. Note that the dotted line denotes consumers $4, \ldots, n-1$.

of the feasibility test and forecasting algorithm presented in this paper are as follows.

1) The feasibility test and forecasting algorithm are nonparametric.
2) The LS-SVM can model the nonlinear relationship between environmental parameters and the total aggregate power consumption of consumers.
3) The detection of utility maximization only requires observed electricity tariffs and power consumption.
4) The utility function of consumers can be constructed from observed electricity tariffs and power consumption.

Using real-world data from Ontario's power grid, we apply the nonparametric test for utility maximization and find that the aggregate demand of the zones satisfies the test. Using the observed electricity tariffs and power consumption data, the utility function of the zones is constructed. As we show from real temperature and associated power consumption in the Ontario power grid, the LS-SVM can be used to forecast the aggregate power budget of consumers. To validate the LS-SVM, we compare the results with a multiple linear regression (MLR) model constructed from the results in [10]. Numerical examples are also included to illustrate the efficiency and performance of the consumption forecasting algorithm compared with the artificial neural network (ANN) algorithm presented in [11] and the LS-SVM in this paper.

Though this paper focuses on smart pricing for DSM, the results of the nonparametric feasibility test and forecasting algorithm can be used in voluntary control power management strategies. For example in [12], an auctioning mechanism is used which requires consumers to voluntarily declare their suspected electrical energy consumption and preferences for consumption. This information is utilized to select the electricity prices for each consumer. If the consumer used more energy then declared, then their cost for electricity increases in the next operating period. Note that using the *a priori* price and consumption history of the consumer, the nonparametric learning algorithm presented in this paper can be used to estimate the consumers preferences (i.e., utility function of the consumer). The estimated utility function can then be used directly in the voluntary auctioning mechanism in [12].

This paper is organized as follows. Section II presents the background and motivation for the DSM strategy presented in this paper. Section III introduces the DSM strategy, illustrated in Fig. 1, and describes the behavior of consumers. The feasibility test for utility maximization consumers based on Afriat's theorem is given in Section IV. Algorithms to perform demand forecasting of utility maximization consumers are given in Section V. Numerical examples are presented in Section VI.

II. Background and Motivation

In the smart grid, consumers may have a preference for using power at a particular time of the day. By observing how much power is consumed in certain portions of the day as result of different electricity tariffs, the preferences of the consumer are revealed. Major contributions to the area of revealed preferences have been given by Afriat [7], Varian [8] (currently chief economist at Google), and Diewert [9] in the microeconomic literature. How consumers respond to changes in electricity prices has been studied extensively by Ida *et al*. [14] and Ito [13]. Using randomized field experiments, it was found that consumers provided with information provisions significantly reduce consumption in response to changes in hourly marginal prices [14]. In [13], it was suggested that consumers respond to average prices rather then marginal prices as the consumes are not provided with necessary information provisions (i.e., prices, consumption rate, etc.), to make informed decisions. The results of Ito and Ida *et al.* suggest that informed consumers satisfy utility maximization, a necessary assumption for the DSM strategies presented in [1]–[5]. In [1]–[5], the utility function of consumers is also assumed to be known. The forecasting algorithm presented in this paper can detect utility maximization and construct the utility function of consumers using only observed electricity tariffs and power consumption. The constructed utility can then be used in the DSM strategies presented in [1]–[5].

Several techniques for forecasting consumer demand can be found throughout the literature. Classical white-box methods for forecasting demand include linear regression [15] and ARIMA [16] models. As mentioned in [17] and [18] these classical methods provide poor estimates if the consumption behavior is nonlinear, nonstationary, and not known prior to model construction. As a result black-box machine learning algorithms such as ANNs and support vector machines (SVMs) are of growing interest for forecasting the power demand of consumers [19]. A weakness with the ANN is that the structure of the network (e.g., number of hidden layers and transfer function of neurons) must be defined in advance [20]. SVMs are based on the structural risk minimization principle which allows them to achieve optimal network structure [21], [22]. A difficulty with

SVMs is the determination of accurate model parameters from training data [20]. For example, in [23], the proposed method does not include exogenous parameters such as temperature which could impact the predictive accuracy of the method. A weakness with the ANN and SVM black-box modeling approaches, as compared with white-box models, is that the model structure and parameters usually give no explicit information about the system behavior.

In the nonparametric forecasting algorithm, presented in this paper, we utilize the LS-SVM which requires the solution of a system of linear equations as compared to standard SVM which requires quadratic programming techniques allowing model parameters to be straightforwardly computed. The forecasting algorithm accounts for environmental parameters (i.e., temperature) and the power usage preference of consumers from the estimated utility function. Though the forecasting algorithm is a black-box model, the estimated utility function provides an explicit explanation about the consumption behavior of consumers. Remarkably, though the forecasting algorithm is nonparametric, predictions can be computed using standard linear programming techniques such as the interior-point method [24].

III. System Model and Problem Formulation

This section defines the DSM strategy and sets the stage for the nonparametric test for utility maximization and the nonparametric learning algorithm for demand forecasting.

A. DSM Strategy

In Fig. 1, we assume there is a single distributed substation that provides power to consumers. The distributed substation can differentiate power consumption in time and provide electrical pricing tariffs to multiple consumers using the "smart grid" infrastructure. On each day, the distribution substation provides the consumers with an electricity tariff. The electricity tariff informs the consumer of the cost of using electricity in prespecified time slots throughout the day. The substation records the aggregate power consumption for each consumer for each respective time slot. Generally, the number of time slots is based on the demand behavior of consumers such that time-of-day prices can be applied [1]. We assume that the time-of-day prices are constrained such that every consumer in the network is able to operate essential devices (e.g., refrigerators, security systems, and medical devices). The electricity tariffs only effect the consumers controllable loads which can be adjusted to operate in certain portions of time throughout the day (i.e., washing machines, dryers, charging electric vehicles, etc.). The parameters in the DSM strategy are defined in Table I. Using the collected data $\mathcal{D} = \{(p_d, x_d^1, x_d^2, \ldots, x_d^n) : d \in \{1, 2, \ldots, N\}\}$ from the distribution substation, the goals are as follows:

1) detect if the data is consistent with demand-responsive consumers;
2) learn the power budget and aggregate consumption behavior of the consumers allowing the design of p_d to achieve power consumption objectives.

TABLE I
DSM Parameters

Symbol	Description	Unit
d	Day	1
T	Number of time slots in d	1
N	Total days	1
n	Number of consumers	1
M	Training examples used for budget estimation	1
x_d^i	Aggregate power consumption of consumer i in each time slot in d with $x_d^i \in \mathbb{R}^T$	kWh
p_d^i	Electricity tariff for each time slot in d with $p_d \in \mathbb{R}^T$	\$/kWh
I_d^i	Total electricity budget of consumer i on d with $I_d^i \in \mathbb{R}$	\$
w_d	Temperature on day d	^{o}C

1) Processing of Consumer Data: An important step in our DSM strategy is the detection of demand-responsive consumers using the price-demand data $\mathcal{D}$—that is, does the data $\mathcal{D}$ satisfy utility maximization defined below.

Definition 1 [7], [9], [25]: A consumer is a utility maximizer if for every electricity tariff $p_d \in \mathbb{R}^T$ and power budget I_d, the chosen power demand $x_d \in \mathbb{R}^T$ satisfies

$$x_d = x^*(p_d, I_d) \in \operatorname*{arg\,max}_{\{p_d' x \leq I_d\}} u(x) \tag{1}$$

where $u(x)$ is a nonsatiated utility function. Nonsatiated means that for any $\eta > 0$, there exists a x with $\|x - x_d\|_2 < \eta$ such that $u(x) > u(x_d)$—for any selected x_d, there exists a x that is arbitrarily close that would provide a higher utility.

Note that nonsatiated simply implies that the utility function is nondecreasing—that is, consumers always strive to consume more power [1], [2], [5]. For demand-responsive consumers, we estimate the utility function of the consumer $u(x)$ using the data $\mathcal{D}$. The estimated utility function is substituted into (1) to forecast future demands of the consumer for given electricity prices.

With the estimated demand function constructed from $u(x)$ and (1), the DSM designer can select electricity prices to achieve power usage objectives such as reducing peak load demand. For consumers that fail (1), a "static pricing" scheme is used as the response of the consumer is not dependent on the price of electricity. The DSM strategy for the distributed substation is illustrated in Fig. 1. The algorithm to detect if a consumer is a utility maximizer (1) and a nonparametric learning algorithm to estimate $u(x)$ for demand forecasting is presented in Sections IV and V, and are the key components of the DSM strategy in Fig. 1.

B. Consumer Preference and Utility Function

Consumer power demand is dependent on several parameters including the price of electricity, the consumers power budget, and everyday usage for lighting, heating, cooking, etc. Each consumer powered by the substation, refer to Fig. 1, is independent with independent consumption behavior. In [1] and [4], the consumption behavior of consumers is modeled using utility functions. We denote the utility function of a consumer by $u(x)$, with x defined in Section III-A.

Note that a consumers utility is only dependent on their personal power consumption, x^i for consumer i, and that no other externalities effect the utility. An example utility function for a consumer is the Cobb–Douglas utility function given by

$$u^i(x^i) = \sum_{t=1}^{T} \alpha_t^i \ln\left[x^i(t)\right] \tag{2}$$

where α_t^i represents the preference of consumer i to use power in time slot t. For a consumer with utility function (2), and power budget I_d^i on day d, the power demand of the consumer x_d^i is computed by substituting (2) into (1) and is given by

$$x_d^i(t) = \alpha_t^i \frac{I_d^i}{p_d(t)} \qquad \forall t \in T. \tag{3}$$

Note that the utility function (2) and associated demand function (3) are only introduced as examples and for numerically testing the forecasting algorithms in Section VI. The nonparametric utility maximization test (5) in Section IV and the forecasting algorithm (11) do not require the utility function of the consumer to be defined.

To forecast demand of a residential consumer i for an unobserved electricity tariff, we must estimate the power budget I_d^i of the consumers. I_d^i may be strongly dependent on environmental, economic, and demographic parameters. As noted in [26]–[29], the power consumption of consumers typically increases as a result of air-conditioners being activated; and a similar result is seen when the temperature decreases and electric-heaters are switched on. Since utility maximization consumers satisfy $I_d^i = p_d' x_d^i$, this temperature dependent trend must impact the power budget of consumers. In this paper, we use a LS-SVM to independently forecast the power budget of consumers to perform demand forecasting, refer to Section V-A. The forecasting of I_d^i is computed independently of the nonparametric test (5) and nonparametric learning algorithm (11).

IV. Afriat's Test for Utility Maximization

In this section, we provide a nonparametric test to detect if a set of power demand observations is consistent with utility maximization (1) behavior with no *a priori* knowledge of the utility function being maximized. As discussed in Section I, the utility maximization requirement is necessary for forecasting power demand of consumers to unobserved electricity prices and is a vital component for DSM strategies [1], [4].

Testing the utility maximization requirement is nontrivial as we are not given any *a priori* information about the utility function of the consumer. The celebrated Afriat's theorem provides a necessary and sufficient condition for a finite dataset D to have originated from an utility maximizer.

Theorem 1 (Afriat's Theorem): Given a dataset

$$D = \{(p_d, x_d) : d \in \{1, 2, \ldots, N\}\} \tag{4}$$

the following statements are equivalent.

1) The agent is a utility maximizer and there exists a nonsatiated and concave utility function that satisfies (1).
2) For scalars u_d and $\lambda_d > 0$ the following set of inequalities has a feasible solution:
$$u_r - u_d - \lambda_d p_d'(x_r - x_d) \leq 0 \quad \text{for } d, r \in \{1, 2, \ldots, N\}. \tag{5}$$
3) A nonsatiated and concave utility function that satisfies (1) is given by
$$u(x) = \min_{d \in N} \left\{u_d + \lambda_d p_d'(x - x_d)\right\}. \tag{6}$$
4) The dataset D satisfies the generalized axiom of revealed preference (GARP), namely for any $k \leq N$, $p_d' x_d \geq p_d' x_{d+1} \quad \forall d \leq k-1 \implies p_k' x_k \leq p_k' x_1$.

As pointed out in Varian's [30] influential paper, a remarkable feature of Afriat's theorem is that if the dataset can be rationalized by a nontrivial utility function, then it can be rationalized by a continuous, concave, monotonic utility function. "Put another way, violations of continuity, concavity, or monotonicity cannot be detected with only a finite number of demand observations."

Verifying GARP (statement 4 of Theorem 1) on a dataset D comprising N points can be done using Warshall's algorithm with $O(N^3)$ [30] computations. Alternatively, determining if Afriat's inequalities (5) are feasible can be done via a linear program feasibility test (using for example interior point methods [24]). Note that the utility (6) is not unique and is ordinal by construction. Ordinal means that any monotone increasing transformation of the utility function will also satisfy Afriat's theorem. Therefore, the utility mimics the ordinal behavior of humans—that is, humans make ordinal decisions since humans tend to think in symbolic ordinal terms. The monotone requirement follows naturally for consumers in the smart grid as the satisfaction consumers receive from using more power generally increases [1], [4]. Geometrically the estimated utility (6) is the lower envelop of a finite number of hyperplanes that is consistent with the dataset D.

If the data D (4) fails the nonstochastic test for utility maximization (5), could this be a result of measurement noise? If the data is believed to have failed (5) as a result of measurement noise then the feasibility test in [25] can be applied to detect for utility maximization. The associated utility function can be constructed using (6) with the parameters u_d and λ_d computed using the nonlinear mathematical program in [31].

V. Forecasting Power Consumption of Utility Maximization Consumers

In this section, the nonparametric consumption forecasting algorithm for utility maximization consumers is presented. Recall from Section I that the forecasting algorithm contains three steps: power budget prediction, utility function estimation, and using the power budget and utility function for forecasting consumption behavior. In Section V-A, the first step of the forecasting algorithm is presented—that is a LS-SVM for predicting the aggregate consumption budget as a function of temperature. Section V-B provides the last two steps of the forecasting algorithm which show how the utility function of consumers can be learned and used for forecasting future consumption behavior. To compare the performance of the LS-SVM for power budget prediction, a MLR method is presented in Section V-C. Section V-D provides the ANN method presented in [11] and a LS-SVM method for forecasting the consumption behavior of consumers and is used to compare to the nonparametric method presented in Section V-B.

A. Least-Squares SVMs for Power Budget Estimation

Recall from Section I that the nonparametric learning algorithm for demand forecasting includes three forecasting stages. First, the aggregate daily power budget of a consumer is estimated. Second, the utility function is estimated, and third the estimated power budget and utility function with the electricity tariffs are used to forecast the intraday power demand. In this section, we define the nonparametric LS-SVM algorithm used to forecast the power budget I_o of a consumer.

The goal of the LS-SVM predictor is to estimate I_o based on historically observed data which includes power consumption, electricity tariffs, and temperature. It has been shown that eliminating trends and noise in the data prior to training can improve the forecasting accuracy of learning algorithms [18]. In this paper, we use smoothing splines to remove trends and noise from data prior to training. Given a set of data $\{(t_d, I_d) : d \in \{1, 2, \dots, M\}\}$ with t_d the time index (e.g., day the of the year) and I_d the associated total electricity budget, we utilize De Boor's algorithm to solve

$$\min_{\mu_d} \left[\gamma \sum_{d=1}^{M} \left(I_d - \hat{\mu}_d\right)^2 + (1-\gamma) \int_{t_d}^{t_{d+1}} \left(\frac{d^2 \hat{\mu}(t)}{dt^2} \right)^2 dt \right]$$
$$\hat{\mu}(t) = \sum_{d=1}^{M} \mu_d f_d(t) \tag{7}$$

where $\hat{\mu}(t)$ is the estimated smooth spline function, $\mu_d = \hat{\mu}(t_d)$, f_d is the cubic spline basis function, and $\gamma \in [0, 1]$ is the smoothing parameter. The de-trended and de-noised data after applying the smoothing splines are given by $\bar{I}_d = I_d - \hat{\mu}(t_d)$.

Consider a set of de-noised and de-trended observation data $\{(\bar{w}_d, \bar{I}_d) : d \in \{1, 2, \dots, M\}\}$, where $\bar{w}_d$ is the temperature on day d and $\bar{I}_d$ is the associated total electricity budget. It is desirable to have an estimate of the functional dependence of $\bar{I}_d$ on $\bar{w}_d$—this can be done using the LS-SVM classifier

$$\bar{I}(\bar{w}) = \operatorname{sign}\left[\sum_{d=1}^{M} \alpha_d \bar{I}_d K(\bar{w}, \bar{w}_d) + b \right] \tag{8}$$

where $\alpha_d \in \mathbb{R}_+$ are Lagrange multipliers, $b \in \mathbb{R}$ is a constant, and $K(\bar{w}, \bar{w}_d) = \phi(\bar{w})^T(\bar{w}_d)$ is the kernel function. Details on the LS-SVM can be found in [32]. In this paper, we use the Gaussian radial basis function kernel $K(\bar{w}, \bar{w}_d) = \exp(\beta\|\bar{w} - \bar{w}_d\|_2^2)$ with β a free parameter. This kernel function was selected as it allows the LS-SVM to learn nonlinear relationship mappings and is guaranteed to satisfy Mercer's condition [32], [33]. The parameters α_d and b are computed by solving the least-squares optimization problem

$$\min_{\omega,b,e} \left[\frac{1}{2}\omega^T \omega + \frac{\gamma}{2} \sum_{d=1}^{M} e_d \right] \tag{9}$$
$$\text{s.t.} \quad \bar{I}_d \left[\omega^T \phi(\bar{w}_d) + b\right] = 1 - e_d \qquad d = 1, 2, \dots, M$$
$$\text{with } \omega = \sum_{d=1}^{M} \alpha_d \bar{I}_d \phi(\bar{w}_d)$$

using the Karush–Kuhn–Tucker conditions [32]. The parameter γ in (9) controls the amount of regularization versus the sum squared error. The quality of the estimated classifier (8) from the LS-SVM (9) is estimated by computing the 95% confidence interval using the algorithm defined in [34].

B. Nonparametric Learning Algorithm for Demand Forecasting

The nonparametric learning algorithm relies on using observed electricity prices and power demands D (4), recorded by the distribution substation (refer to Fig. 1) to estimate the utility function of a consumer satisfying (1). To estimate the power demand function $x^i(p, I)$ of each consumer i, refer to Fig. 1, the estimated utility function and forecasted power budget, refer to Section V, is plugged into (1). Note that the learning algorithm presented here is in the supervised learning class of machine learning in which we are attempting to infer a functional form to training data consisting of electricity tariffs and power demands.

To begin, we consider the full set of feasible demands that a consumer can have using the following definition.

Definition 2: Given the power budget of consumer i, denoted as I_0^i, electricity tariff $p_o \in \mathbb{R}_+^T$, and data D (4), a power demand $x_o^i \in \mathbb{R}_+^T$ is a feasible demand if D and (p_0, x_0^i) satisfies utility maximization (1) and $p_0 \cdot x_0^i = I_0^i$.

Definition 2 provides conditions under which forecasted demands must obey to be consistent with utility maximization (1); however, for anticipatory control it is desirable to have a specific forecasted demand given the electricity price and the consumers power budget. To forecast a specific demand requires an estimated of the consumers utility function. The consumer's utility function can be estimated using (6). Utilizing (6), the forecasted demand of a utility maximization consumer is computed by substituting (6) into (1). Given the dataset D (4), power budget I_o^i, and electricity tariff p_o the forecasted demand is computed from the following optimization problem:

$$x_0^i = x^i(p_0, I_0) \in \arg\max \left[\min_{d \in N} \{u_d + \lambda_d p_d'(x^i - x_d^i)\} \right]$$
$$\text{s.t.} \quad p_0' x^i \le I_0^i, \quad x_0^i(t) \ge 0 \qquad \forall t \in T. \tag{10}$$

The optimization problem (10) is a linear program with a piecewise linear objective and can be solved in polynomial time complexity. Note that if the forecasted power budget I_0 has an error ε from the actual power budget, then the error in the forecasted demand, denoted by Δx_0^i, satisfies $\Delta x_o^i(t) \le \varepsilon / p_0(t)$ with the total error satisfying $\varepsilon = p_o \cdot \Delta x_0^i$. The algorithm used to compute the forecasted demand x_0^i (10)

for anticipatory control is given as follows.

Step 0: Select unobserved electricity tariff $p_0 \in \mathbb{R}_+^T$ for the estimation of optimal demand $x_0(p_0, I_0^i)^i \in \mathbb{R}^T$ of consumer i.

Step 1: Using historical data, estimate the power budget of the consumer I_0^i using the LS-SVM algorithm presented in Section V-A.

Step 2: For dataset D (4), compute the parameters u_d and λ_d using (5).

Step 3: Solve the following linear programming problem given D, p_0, I_0^i and computed u_d and λ_d from Step 2:

$$\begin{aligned}&\max z\\ &\text{s.t.}\quad z \le u_d + \lambda_d p_d'(x_0^i - x_d^i)\ \text{for} d = 1, \ldots, N\\ &p_0' x_0^i \le I_0^i\\ &x_0^i(t) \ge 0 \quad \forall t \in T\end{aligned} \tag{11}$$

to obtain the forecasted demand $x_0^i(p_o, I_o^i)$ (10) for consumer i.

A key question is when does the nonparametric learning algorithm (11) admit a sufficiently accurate estimate of the consumer's demand function? Equivalently, what demand functions $x(p, I)$ are probably approximately correct (PAC) learnable using the algorithm (11). If we denote $\mathcal{C}$ as the set of possible demand functions under consideration, then demand functions in the set $\mathcal{C}$ are efficiently PAC-learnable if $\mathcal{C}$ satisfies the following definition.

Definition 3 [35]: A set of demand functions $\mathcal{C}$ is efficiently PAC-learnable if for any $\epsilon, \delta > 0$, $h \in \mathcal{C}$ and probability distribution $\mathbb{P}$ on the electricity tariffs and demands, there exists a polynomial time algorithm for a set of observations to find a demand function h such that

$$\mathbb{E}_{(p,x)}\left(\|h(p, I) - x(p, I)\|_\infty^2\right) < \epsilon$$

with probability $1 - \delta$. $\mathbb{E}$ is the expectation with respect to (x, p), and $x(p, I)$ is the actual demand.

As shown in [6] for the utility function of a consumer, the set of all demand functions induced by monotone concave utility functions is too rich to be efficiently PAC-learnable. Further assumptions are necessary to ensure PAC-learnability of the demand function $x(p, I)$. As shown in [6] and [36], the class of income-Lipschitz demand functions is efficiently PAC-learnable.

Definition 4 [36]: A demand function $x(p, I) \in \mathbb{R}^T$ is income-Lipschitz if for every strictly positive vector $p \in \mathbb{R}_+^T$, and scalar $I \in \mathbb{R}_+$, there exists positive reals $L > 0$ and $\eta > 0$ such that if $(p, p') \in P \subset \mathbb{R}_+^T$, $\|p - p'\| < \eta$, and $|I - I'| < \eta$, then

$$\frac{\|x(p', I) - x(p', I')\|}{|I - I'|} \le L. \tag{12}$$

The requirement of the demand function to have the income-Lipschitz property does not impose any parametric assumptions on the utility function. The income-Lipschitz (4) property merely imposes a stability assumption on the demands which rules out dramatically different demands for similar electricity tariffs.

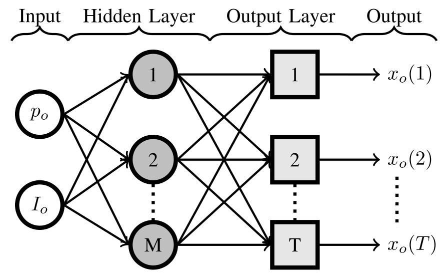


Fig. 2. ANN architecture used to forecast consumer demand. The input to the network is composed of the electricity tariff $p_d \in \mathbb{R}^T$, and consumer income I_o. The hidden layer is composed of a total of M log-sigmoid neurons, and the output layer is composed of T linear neurons. The output of the network $x_o = [x_o(1), x_o(2), \ldots, x_o(T)]$ is the forecasted demand of the consumer in each time slot.

C. MLR for Power Budget Estimation

Hong *et al.* [10] utilized MLR methods to relate the aggregate power consumption with the temperature. Estimating the parameters of the MLR is straightforward using such methods as maximum likelihood estimators. However, choosing the parametric form *a priori* and the model dimensions is not straightforward. Hong *et al.* [10] analyzed the aggregate power consumption and associated temperature profiles for a four year period from a U.S. utility company. It was found that a piecewise function with linear, quadratic, and cubic terms can be used to relate the temperature to the aggregate consumption. In this paper, we consider the MLR model given by

$$\bar{I}(\bar{\omega}, m) = \left(\alpha_{m0} + \alpha_{m1}\bar{\omega} + \alpha_{m2}\bar{\omega}^2 + \alpha_{m3}\bar{\omega}^3\right) \tag{13}$$

with $\bar{\omega}$ the temperature, $m \in \{1, 2, \ldots, 12\}$ the month, and $\alpha_{m0}, \alpha_{m1}, \alpha_{m2}, \alpha_{m3}$ the parameters to be estimated for each month m. The model parameters in (13) are estimated using least-squares.

D. ANN and Least-Squares SVM for Demand Forecasting

In this section, we introduce the LS-SVM and ANN consumption forecasting algorithms, which we compare with the nonparametric learning algorithm presented in Section V-B. Note that the LS-SVM and ANN demand forecasting algorithms are trained using the dataset D (4) with $I_d = p_d \cdot x_d$.

The LS-SVM forecasting algorithm is given by (8) and (9) with the electricity tariffs p_o and power budget I_o as inputs, and intraday power demands x_o as the output. The same input and outputs are used for the ANN, where the ANN is composed of a two layer feed-forward neural network architecture with the hidden layer neurons governed by a log-sigmoid function, and the output layer neurons governed by a linear function. It is known that this type of network architecture can fit multidimensional data arbitrarily well [21]. The input data is composed of electricity tariffs and consumers income, and the output is the forecasted power demands of each consumer. The structure of the network is illustrated in Fig. 2.

The number of neurons in the hidden layer was set to $M = 16$, as done in [11]. If we denote w as the weight vector of the neuron, b is the scalar bias, the neuronal input as a with the same dimensions as w, the output by y, and the neural transfer function by f, then the neuron is mathematically described by $y = f(w \cdot a + b)$. Note that $\cdot$ denotes the dot product. The weights w and biases b in the neural network are computed using the Bayesian regularization algorithm [37]. The Bayesian regularization method is based on the Levenberg–Marquardt training method with regularization that allows the algorithm to produce networks with good generalization capabilities to novel inputs not included in the training dataset. In [11], the Bayesian regularization method is shown to have excellent power demand forecasting capabilities.

VI. Utility Maximization Test and Power Demand Forecasting: Real Data and Numerical Examples

In this section, the nonparametric test for utility maximization (5) and the consumption forecasting algorithm (11) are applied to real-world data from the Ontario power grid. To compare the performance of the nonparametric learning algorithm with the ANN and LS-SVM learning algorithms presented in Section V-D, we conduct numerical experiments with power consumption generated from a consumer maximizing a Cobb-Douglas utility function (2).

A. Detection of Utility Maximization and Nonparametric Learning of Utility Function

As a result of the "Freedom of Information and Protection of Privacy Act" and other similar acts, specific consumer power demand usage is not publicly available. Here, we apply the nonparametric test (5) and the nonparametric learning algorithm (11) to the publicly available total aggregate power demand in the Ontario power gird. The data is obtained from the Independent Electricity System Operator[1] (IESO).

To test for utility maximization, we use the power consumption and electricity tariffs from the Bruce zone in the IESO starting from Feb. 2013 for a period of $T = 46$ days. Applying the feasibility test (5), the data is consistent with utility maximization. Since the data satisfies the feasibility test the associated utility function of the Bruce zone can be constructed using (6). Fig. 3 presents the electricity tariff, power budget, and estimated utility function for the Bruce zone. Several features are of interest. First, since the electricity tariffs [Fig. 3(a)] are not designed for learning the utility function, the consumption characteristics of the consumer can only be estimated in a narrow range as seen in Fig. 3(c). There does not appear to be a significant preference in power usage between $x_d(1)$ and $x_d(2)$. This could be a result of consumers not being aware of the current prices. As found in the research by Ida *et al.* [14], consumers significantly reduce their power consumption in response to changes in electricity tariffs. Note that in [14], the data is obtained on a household resolution with randomized field experiments with consumers equipped with power usage displays and current temperature and electricity tariffs. Although we do not detect a significant preference between the power usage slots, as seen in Fig. 3, there is a significant effect in satisfaction with an electricity tariff of $p_{37} = [8.20, 15.52]$ and power budget $I_{37} = 0.02$ as compared with other electricity and budget data. Therefore, we conclude that the existence of utility maximization consumers is true, and the power usage behavior of consumers is effected by the electricity tariffs and power budget.

[1] http://ieso-public.sharepoint.com/

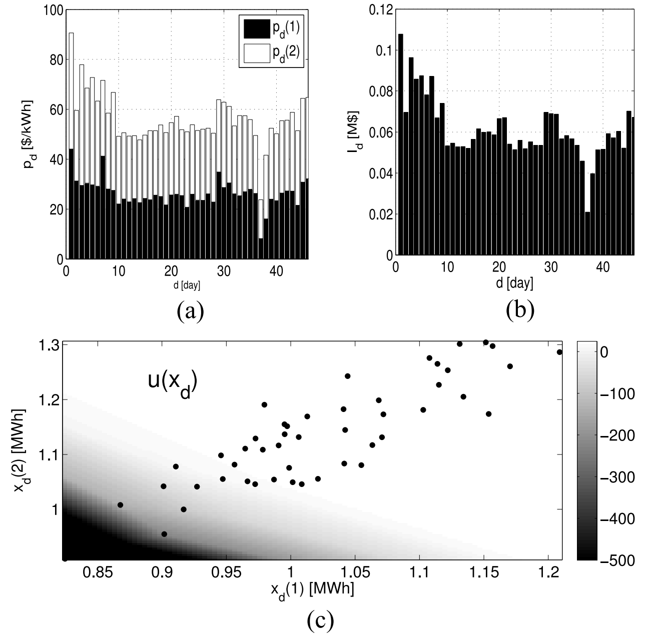


Fig. 3. Electricity tariffs, power budget, and estimated utility function of the Ontario power grid computed using data obtained from the IESO database starting from Feb. 2013. (a) Electricity tariffs p_d. (b) Power budget $I_d = p_d \cdot x_d$. (c) Estimated utility function $u(x_d)$ for the given electricity tariffs p_d from (a) and power budget Id from (b). The utility function is estimated using (5) and (6).

Is the data presented in Fig. 3 consistent with a Cobb–Douglas utility function (2)? Note that for the Cobb–Douglas utility function the power consumption x_d is proportional to the ratio of the power budget I_d and electricity tariff p_d via α_t, refer to (3). Therefore, this can be viewed as a MLR model, where the preference of the consumer α_t is to be estimated. If the data originated from a Cobb–Douglas utility function then $\alpha_t(d)$ must be equal for all d. From (3), the preference for power consumption in each time associated with $x_d(t)$ can be estimated using $\alpha_t(d) = (x_d(t)p_d(t))/I_d$. The preferences of the consumers are given by $\alpha_1 = 0.44 \pm 0.05$ and $\alpha_2 = 0.55 \pm 0.05$ where the errors denote the standard deviation. This suggests there is a preference for using power in the time slot associated with $p_d(2)$. However, the preference $\alpha_1 \in [0.32, 0.55]$ and $\alpha_2 \in [0.45, 0.68]$ which shows that the data is not consistent with a Cobb–Douglas utility function as α_1 and α_2 are dependent on d.

B. Power Budget Prediction

To perform power consumption forecasting using the nonparametric learning algorithm (11) requires the power budget of the consumer. Here, we apply the LS-SVM algorithm presented in Section V-A and MLR model presented in Section V-C to forecast the aggregate power budget of consumers in the Ontario power grid. The power consumption,

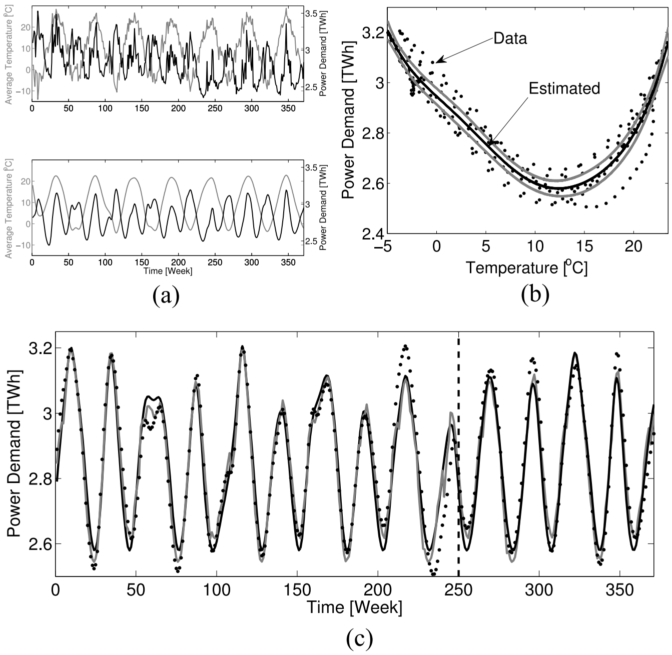


Fig. 4. Real and estimated power demand with average weekly temperature as the dependent variable using the LS-SVM defined in Section V-A and MLR defined in Section V-C. The data is obtained from the IESO database starting from 2004. (a) Original and smoothed data using smoothing spline method defined in Section V-A. (b) 95% confidence interval (gray) computed using the algorithm defined in Section V-A. (c) Real and estimated total aggregate power demand of consumers. The training period is the first 250 weeks indicated by the dashed vertical line. The black line is the LS-SVM and the gray line is the MLR. Note that the power budget I_d can be computed by multiplying the power demand by the average weekly electricity tariff.

average temperature, and average weekly electricity tariffs for the years 2004–2010 are obtained from the IESO database. To estimate if aggregate weekly demand is strongly dependent on temperature and price, or just temperature, we trained the LS-SVM and MLR models using both sets of data and compared the results. In Fig. 4, the temperature, power consumption, confidence interval of training for the LS-SVM, and the fitted results are presented. The relationship between temperature and the power consumption in Fig. 4(a) is expected from the results in [26]–[29]; for high and low temperatures climate control equipment (e.g., air-conditioners, heaters) are switched on. The quality of the fit is provided in Fig. 4(b) and (c). The mean absolute percentage error (MAPE) of the real and forecasted consumption in Fig. 4(c) is 1.58% for the LS-SVM, and 1.52% for the MLR. This shows that both the LS-SVM and MLR can be used to predict the power budget of consumers. The small MAPE suggests that the total aggregate power consumption of consumers is dependent on the temperature, in agreement with the results found in [16] and [38]. To test this result, we apply the LS-SVM algorithm defined in Section V-A with the dependent variables being price and temperature. The resulting MAPE is 1.78%, which is comparable to the MAPE when only temperature is the dependent parameter. Since the MAPE value is approximately unchanged when the electricity tariff is included, this suggests that the total aggregate weekly power consumption is only dependent on the temperature and not the electricity tariff.

C. Numerical Example: 2-Period Pricing Scheme

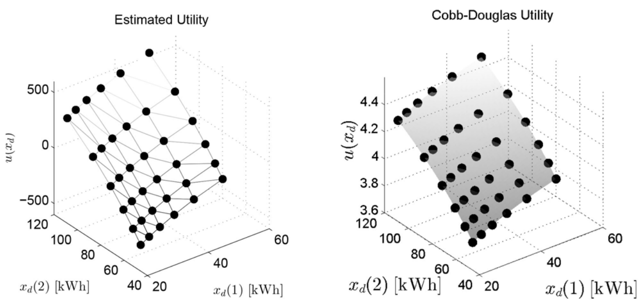
To numerically compare the accuracy of the nonparametric learning algorithm (11) with the ANN and LS-SVM in Section V-D, we have selected a two tier electricity tariff system for analysis as several electricity utilities have implemented this pricing scheme including: Pacific Power,[2] and Ausgrid.[3] To simplify the numerical analysis, we take the forecasted power budget I_o to be given. Note that for the nonparametric learning algorithm the maximum error on the forecasted consumption resulting from errors in I_o is bounded, as discussed in Section V-B.

1) Training Data: The time-of-day pricing scheme for the Ausgrid utility comprises a weekend and holiday schedule containing $T = 2$ time-slots: off-peak and shoulder (i.e., mid-peak). Consider a consumer being powered by a distributed substation operated by the Ausgrid utility that records the daily power consumption of a consumer for the two time slots: off-peak and mid-peak on weekends and holidays. The consumer's behavior, unknown to the distribution substation, is described by the utility function (2) with power consumption preferences $\alpha = [0.3, 0.7]$ for off-peak and mid-peak power usage respectively. Let us consider that the off-peak and mid-peak prices satisfy $p_d(t) \in [0.06, 0.13]$\$/kWh and that the consumers power budget is constant with $I_d = \$10.00$. We construct the price-consumption training dataset D_{t2} (4) by sampling p_d uniformly over the price range $[0.06, 0.13]$\$/kWh for $N = 36$ days. The consumer's consumption x_d are computed by substituting the consumer income I_d, preferences α, and prices p_d into (3). A noise parameter ε with normal distribution is added to the consumption x_d prior to training to simulate the noise in a real measurement. Here, $\varepsilon \sim \mathcal{N}(0, 1)$ such that the measured consumption in D_{t2} is given by $y_d = x_d + \varepsilon_d$ with ε_d generated from the normal distribution.

The accuracy of the nonparametric learning algorithm (6) is illustrated in Fig. 5. The nonparametric learning algorithm (6) was trained using the dataset D_{t2}. The actual Cobb–Douglas utility of the consumer (2), given in Fig. 5(b), and the estimated utility of the consumer using the learning algorithm (6) in Fig. 5(a) are in excellent agreement. Note that the estimated utility approximates the Cobb–Douglas utility by a finite number of hyperplanes as expected from the form of (6) and Theorem 1. If we were to use the estimated utility function illustrated in Fig. 5(a) in the constrained optimization (1) for given electricity tariffs $p_d \in \mathbb{R}^2$ and consumer income I_d, we would expect the estimated consumption to be in good agreement with the actual consumption (3) from our discussion in Section V-B.

To test the forecasting accuracy of the nonparametric learning algorithm, we generate 40 electricity tariffs from the uniform distribution $p_d \sim \mathcal{U}(0.06, 0.13)$\$/kWh, and compute the estimated power consumption by substituting the estimated utility function in Fig. 5(a) into the constrained optimization problem (11). To compare the performance of the nonparametric learning algorithm (11) to the ANN and LS-SVM presented in Section V-D, these algorithms were

[2]https://www.pacificpower.net/
[3]https://www.ausgrid.com.au/



(a) (b)

Fig. 5. Estimated and actual utility function of a consumer defined by a Cobb–Douglas utility function with preferences $\alpha = [0.3, 0.7]$, a constant power budget $I_d = \$10.00$. The estimate is computed over $N = 36$ days with electricity tariffs equally spaces over $p_d(t) \in [0.06, 0.13]$\$/kWh. The black dots represent the consumer's power demand and associated utility level for the given electricity tariffs. (a) Estimated utility function of the consumer with data collected over $N = 36$ days computed using (5) and (6). (b) Cobb–Douglas utility function of the consumer. The black dots indicate the computed power demands given electricity tariffs over the $N = 36$ days.

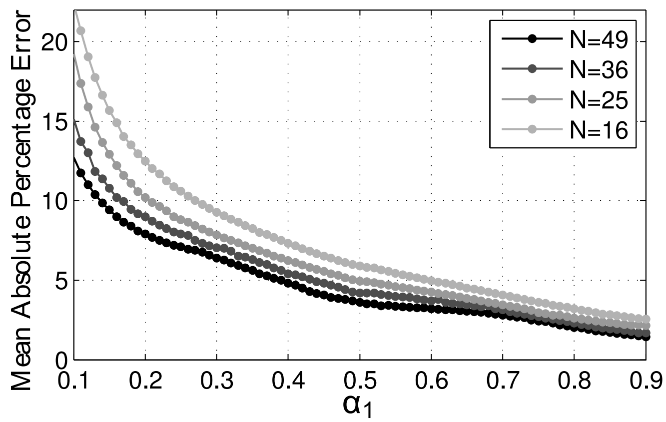


Fig. 6. MAPE for the nonparametric learning algorithm (11) for estimating the aggregate consumption of a Cobb–Douglas utility (3) as described in Section VI-C.

TABLE II
MAPE

Learning Algorithm	$x_d(1)$	$x_d(2)$
Non-parametric:	7.03	3.01
ANN:	1.02	0.64
LS-SVM:	2.37	0.56

trained using the dataset D_{t2}. The performance of the forecasting algorithms is computed using the MAPE. This metric of performance was selected as it is a classical tool in time series and is particularly relevant here as the aggregate power consumption is always positive. The MAPE associated with the estimated power consumption from the nonparametric, ANN, and LS-SVM learning algorithms is provided in Table II. As seen the consumption of the consumer is well approximated by the nonparametric learning algorithm (11), the ANN, and the LS-SVM forecasting algorithms. As expected, the ANN was able to give excellent predictive results when trained using the Bayesian regularization method, refer to Section V-D for details. The LS-SVM has comparable forecasting accuracy to the ANN learning algorithm. For detailed analysis of the performance of the ANN and SVN, the reader is referred to [19]. The largest deviation between the expected and actual consumption for the nonparametric algorithm occurs at $d = 9$ with $\Delta x_9 = [-9.39, 11.6]$kWh. The reason this takes place is that the electricity tariffs used to generate the estimated utility for the forecasting algorithm did not sufficiently sample the region near $p_9 = [0.089, 0.069]$\$; if this region of pricing was of critical importance then nonuniform electricity tariffs could be used to train the nonparametric learning algorithm to increase the accuracy of the estimated consumption.

To gain further insight into the predictive accuracy of the learning algorithm (11), the MAPE is computed for different preferences α and training sample sizes N. The training data is constructed by uniformly sampling $p_d(t)$ over the price range of $[0.06, 0.13]$\$/kWh for N days. The associated aggregate consumption x_d is computed using the Cobb–Douglas utility function (3) with $I_d = \$10.00$. Fig. 6 provides the computed MAPE between the actual and predicted aggregate power consumption for $x_d(1)$. Recall from (3) that the MAPE for $x_d(2)$ is identical to the MAPE for $x_d(1)$ if the associated preference $\alpha_2 = \alpha_1$ in Fig. 6. For example the MAPE for $x_d(2)$ for $N = 49$ and $\alpha_2 = 0.8$ is 2.42%. As expected, as the number of training samples N increases, the associated error between the actual and predicted consumption decreases. From Fig. 6, the accuracy of the predicted power consumption from the learning algorithm (11) increases as the preference for the associated consumption time slot increases. This is a beneficial feature as the consumer prefers to use the largest amount of power during the time slot associated with the largest preference value.

VII. Conclusion

The main result of this paper was the development of a nonparametric test for the detection of consumers that participate in time-of-use electricity pricing initiatives that does not require the utility function of the consumers to be known. For utility maximization consumers, a nonparametric learning algorithm was used to forecast the power demand of these consumers for unobserved electricity tariffs. Real-world and numerical examples illustrate the efficacy of the nonparametric test and accuracy of the nonparametric learning algorithm compared with a conventional ANN and support vector methods. The common theme of the paper is that it bridges the gap of a social network composed of energy consumers, and a technological network composed of energy distribution agents (e.g. smart meters). A standing assumption in our development is that consumers do not interact. In a more elaborate social network with interacting consumers, the formulation of utility maximization consumers and the forecasting of demands can be accomplished by testing if the consumers are playing a zero sum game. This is the subject of our ongoing research.

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