CEE 6460. Theoretical Geomechanics
Georgia Institute Of Technology

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CEE 6460: Theoretical Geomechanics – Spring 2021

Mode of instruction: Hybrid – Note: Students interested in taking the class fully remotely are allowed to do so. No formal justification is needed: an email to the instructor requesting permission to attend the class remotely is enough.
Meeting Times: Mondays and Wednesdays, 9.30am-10.45am
Classroom: Mason 3132 for in-person lectures
Office hours: Tue. & Wed. 2-3pm on Blue Jeans (https://bluejeans.com/114493918). Any time on Piazza.

Prerequisites: COE undergraduate coursework.

Catalog description: Field equations of linear elasticity, solutions of boundary value problems, steady/transient flow in porous media. Yielding and failure of soils; plasticity theory and limit analyses. Constitutive soil models. Introduction to finite elements with geotechnical engineering applications.

Course contents: This course provides an introduction to the theories of elasticity, plasticity, poro-elasticity and fluid flow in porous media. The course covers analytical and numerical methods to solve and analyze boundary value problems in geomechanics. In particular, the course explains the fundamental principles of the Finite Element Method (FEM), with an emphasis on plane elasticity, fluid flow, heat transfer, and coupled hydro-mechanical problems. The material presented is mostly common to the general study of solid mechanics and groundwater hydraulics, and can be of general interest to students who are not majoring in geotechnical engineering. Topics include general theorems of elasticity, fundamental principles of perfect plasticity, basic elastoplastic constitutive models, limit analysis for slope and foundation stability, theory of consolidation, variational formulations, numerical integration schemes and Finite Element stability criteria.

Learning Objectives:

1. Calculate the state of stress in a solid, the principal stresses and the stress invariants
2. Calculate small and large deformations in a solid
3. Solve analytically basic boundary problems of elasticity, such as circular cavity expansion
4. Solve analytically seepage problems in soils with a rigid skeleton
5. Predict stresses and pore pressures in porous media filled with one or more fluid phases
6. Predict the occurrence of plasticity and calculate plastic strains in an elastic-perfectly plastic medium
7. Recommend constitutive models for civil engineering materials
8. Calculate the bounds of the critical load that are applied on geotechnical structures in undrained and drained conditions
9. Design slopes and assess the stability of foundations in undrained and drained conditions
10. Approximate the solution of Partial Differential Equations (PDEs) by using variational methods.
11. Build a Finite Element Model (weak form, interpolation functions, element governing equation, global stiffness matrix) for singe-variable problems - including eigenvalue and time-dependent problems
12. Analyze a geotechnical system with the Finite Element Method

Textbook: You will have access to my own notebook. Below are references that I recommend for further study.

- Continuum mechanics, elasticity, plasticity, elasto-plastic models for soils and rocks, limit analysis:
  - Atkinson, J. (1981). *An introduction to the mechanics of soils and foundations: through critical state soil mechanics*. John Wiley & Sons, New-York.
  - Jaeger, J.C., Cook, N.G., & Zimmerman, R. (2009). *Fundamentals of rock mechanics*. Wiley.
  - Mase, G.T., Smelser, R.E., & Rossmann, J.S. (2020). *Continuum mechanics for engineers*. CRC Press.
  - Yu, H. S. (2007). *Plasticity and geotechnics* (Vol. 13). Springer Science & Business Media.
- Poromechanics and fluid flow in porous media:
  - Coussy, O. (2011). *Mechanics and physics of porous solids*. John Wiley & Sons.
  - Dormieux, L., Kondo, D., & Ulm, F. J. (2006). *Microporomechanics*. John Wiley & Sons.

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- Finite Element Method:
  - Reddy, J.N. *An Introduction to the Finite Element Method,* 3rd edition (2006), McGraw-Hill
  - Reddy, J.N. *An Introduction to the Nonlinear Finite Element Analysis,* (2004), Oxford University Press
  - Zienkiewicz, O. C., Chan, A. H. C., Pastor, M., Schrefler, B. A., & Shiomi, T. (1999). *Computational geomechanics*. John Wiley.

Tentative schedule:

Week	Deadlines	Topics
1 (01/18)		**Mathematical introduction.** Review of tensor algebra; properties and operations of vectors and tensors, tensor calculus, Gauss theorem, review of matrix algebra.
2 (01/25)		
3 (02/01)	HW 1	**Elements of continuum mechanics.** Stress at a point; Mohr circle representation; principal stresses; stress invariants and stress decomposition; equilibrium equations and conservation of momentum; measures of deformations and strains, strain compatibility; stress-strain relations; boundary value problem representation.
4 (02/08)	HW 2	
5 (02/15)	HW 3	**Linear elasticity.** Formulation of field equations; two-dimensional problems in rectangular and polar coordinates (e.g., cavities expansion problems); method of displacements; superposition; principle of virtual work; uniqueness of solution; reciprocal theorem.
6 (02/22)	HW 4	
7 (03/01)	HW 5	**Introduction to the FEM Method in 1D and 2D.** Variational approach, interpolation functions, stiffness matrix assembly, boundary conditions, resolution methods and post-processing techniques. Triangular and rectangular linear elements, higher order elements, serendipity elements, master elements, coordinate transformation. Numerical integration in 1D and 2D: Newton Cotes quadrature, Gauss quadrature. Applications: bar and Euler-Bernouilli beam elements, 1D fluid flow, 1D heat transfer, plane elasticity.
8 (03/08)	HW 6	
9 (03/15)	HW 7	**Elements of poro-elasticity.** Continuity equation and conservation of mass, linear flow regime: Darcy's law and permeability tensor transient flow, free surface movement and unconfined flow; Formulation of uncoupled diffusion equation for 1D consolidation, solution of simple problems; Introduction to poro-elasticity, Biot's equations for coupled consolidation in 2D & 3D, introduction to unsaturated soil mechanics.
10 (03/22)	Project Flyer	
11 (03/29)		**Introduction to the FEM for diffusion problems.** Stability conditions. Application to consolidation problems.
12 (04/05)	HW 8	**Fundamental principles of plasticity.** Yield function, flow rule, normality rule, perfect plasticity, isotropic hardening, kinematic hardening.
		Perfect plasticity. Associated flow rule for cohesive soils (Tresca and von Mises models); non-associated flow rule for frictional soils (Mohr-Coulomb and Drucker-Prager models); Hoek and Brown's model for rocks.
13 (04/12)	Project Report	
14 (04/19)	HW 9	**Introduction to limit analysis.** Upper bound and lower bound theorems, resolution method, stresses on slip fans. If time allows: undrained stability of soil structures (vertical cut, retaining wall, foundation), drained stability of soil structures (infinite slope, smooth retaining wall, foundation).
15 (04/26)	HW 10	

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Course Assessment: There will not be any exam. Homework (HW) will cover calculus, mechanical analysis, and finite element modeling. Each homework will comprise three problems in average, for an estimated work load of 2 to 6 hours per assignment. A penalty of 10% per day will be applied for late homework submission. The project is an individual assignment aimed to analyze and/or design a geotechnical structure, a geotechnical system or a composite porous medium. Students will be given a list of 3 problems that they will be able to choose from for their project. They will also be able to propose their own topic of study if they prefer. First, students will be required to prepare a two-page flyer to explain the state of the art, key questions, objectives, theoretical background and numerical methods of their project. Based on the instructor's feedback, students will then conduct their analysis with the computational software of their choice (e.g. MATLAB, ABAQUS, ANSYS, PLAXIS, COMSOL). Students will be required to write a report to summarize their work. A dedicated project syllabus will be posted later during the semester. The estimated workload for the project is 20 to 40 hours (depending on how much you want to learn), including the literature review, the theoretical modeling, the numerical analysis and the writing assignments.

Example Projects: Finite Element simulation of tunnel excavation; Finite Element of the tensile behavior of bio-inspired geogrids; Analytical and numerical solutions of 1D consolidation in saturated and unsaturated soil; 1D Finite Element of seismic site responses; Yield Criterion for "Off Brand" Supplementary Cementitious Materials; Finite Element model of nuclear waste disposal; Finite Element model of heat exchanger piles

Grading: *Final grad*e: F<60%≤D<70%≤C<80%≤B<90%≤A≤100%
Score: 10 HW @ 8% each = 80%. Project flyer: 5%. Project Report: 15%.

Academic Integrity: Working in group on homework and projects is allowed (and encouraged). However, each student must write up and turn in his/her own solutions. In-class exams are strictly individual. Any student suspected of cheating or plagiarizing on a quiz, exam, or assignment, will be reported to the Office of Student Integrity, who will investigate the incident and identify the appropriate penalty for violations. Georgia Tech aims to cultivate a community based on trust, academic integrity, and honor. Students are expected to act according to the highest ethical standards. For information on Georgia Tech's Academic Honor Code, please visit http://www.catalog.gatech.edu/policies/honor-code/ or http://www.catalog.gatech.edu/rules/18/.

Accommodations for Students with Disabilities: If you are a student with learning needs that require special accommodation, please contact the Office of Disability Services at (404)-894-2563 or http://disabilityservices.gatech.edu/, as soon as possible, to make an appointment to discuss your special needs and to obtain an accommodations letter. Please also e-mail me as soon as possible in order to set up a time to discuss your learning needs.

Diversity Statement: I consider the classroom (physical or virtual) to be a place where you will be treated with respect, and I welcome individuals of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, ability – and other visible and nonvisible differences. All members of this class are expected to contribute to a respectful, welcoming and inclusive environment for every other member of the class.

Safe Zone Statement: I am a member of a Safe Zone Ally community network, and I am available to listen and support you in a safe and confidential manner. As a Safe Zone Ally, I can help you connect with resources on campus to address problems you may face that interfere with your academic and social success on campus as it relates to issues surrounding sexual orientation and gender identity. I will gladly honor your request to address you by an alternate name or gender pronoun. Please advise me of this preference early in the semester so that I may make appropriate changes to my records. My goal is to help you be successful and to maintain a safe and equitable campus.