

AP–Stat Competition Round 2

Full Name:

Please read the instructions carefully.

- Closed notes.
- Calculator is allowed but not needed.
- **You can leave your answers involving terms like $n!$, $\binom{n}{k}$ etc., if you like.**
- No collaboration.
- No electronic device.
- Internet connection is not allowed.

Best of Luck!!

★ Intentionally left blank. Turn the page for more problems. ★

You can use the following formulas if you need:

(1) Binomial theorem:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.$$

(2) Bayes's theorem:

$$\mathbb{P}(F_j|E) = \frac{\mathbb{P}(E|F_j)\mathbb{P}(F_j)}{\sum_{i=1}^n \mathbb{P}(E|F_i)\mathbb{P}(F_i)}.$$

(3) Variance expressions:

$$\begin{aligned} \text{Var}(X) &= \mathbb{E}[(X - \mu)^2], \quad \text{where } \mu = \mathbb{E}(X) \\ &= \mathbb{E}(X^2) - (\mathbb{E}(X))^2. \end{aligned}$$

(4) Binomial distribution:

$$\mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}. \quad \mathbb{E}(X) = np \quad \text{Var}(X) = np(1-p).$$

(5) Poisson distribution:

$$\mathbb{P}(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}. \quad \mathbb{E}(X) = \lambda \quad \text{Var}(X) = \lambda.$$

(6) Geometric distribution:

$$\mathbb{P}(X = k) = p(1-p)^{k-1}. \quad \mathbb{E}(X) = \frac{1}{p} \quad \text{Var}(X) = \frac{1-p}{p^2}.$$

(7) Hypergeometric distribution:

$$\mathbb{P}(X = k) = \frac{\binom{m}{k} \binom{N-m}{n-k}}{\binom{N}{n}}. \quad \mathbb{E}(X) = \frac{nm}{N} \quad \text{Var}(X) = n \frac{m}{N} \left(1 - \frac{m}{N}\right) \left(1 - \frac{n-1}{N-1}\right).$$

(8) Uniform distribution:

$$f(x) = \frac{1}{\beta - \alpha}, \quad \alpha < x < \beta. \quad \mathbb{E}(X) = \frac{\beta + \alpha}{2} \quad \text{Var}(X) = \frac{(\beta - \alpha)^2}{12}.$$

(9) Exponential distribution:

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0. \quad \mathbb{E}(X) = \frac{1}{\lambda} \quad \text{Var}(X) = \frac{1}{\lambda^2}.$$

(10) Normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty. \quad \mathbb{E}(X) = \mu \quad \text{Var}(X) = \sigma^2.$$

(11) A table for standard normal CDF is given at the end.

1. Mark True/False and pick the correct answer. **No explanation is needed.**

(a) If A and B are two independent events, then $\mathbb{P}(A \cap B) = \mathbb{P}(A) \times \mathbb{P}(B)$. [TRUE / FALSE]

(b) Jane tosses an unbiased coin with $\mathbb{P}(\text{Head}) = 0.6$ until she gets a tail. What is the expected number of times she tosses a coin?

- (i) 6 (ii) 4 (iii) $5/3$ (iv) $5/2$

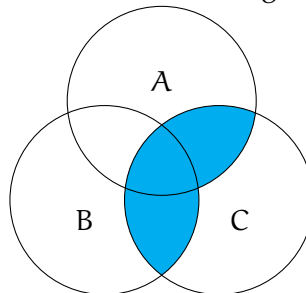
(c) How many different letter arrangements can be made from the letters of 'BOOKKEEPER'?

- (a) $10!$ (b) $\frac{10!}{(3!)^3}$ (c) $\frac{10!}{3!(2!)^2}$ (d) none of these

(d) The probability that a certain medical treatment will cause allergic reaction is 5%. If 20 patients went through that medical treatment, the number of patients having an allergic reaction can be modeled as the following distribution:

- (a) Bernoulli (b) Binomial (c) Poisson (d) Geometric

(e) What is the set theoretic expression for the Venn diagram below?



- (a) $(A \cap C) \cup (B \cap C)$ (b) $(A \cap B) \cup C$ (c) $(A \cup C) \cap (B \cup C)$ (d) none of these

2. A postman has to deliver four letters to four different houses in a street. Unfortunately, the rain has erased the address, so he just distributed them randomly, one letter per house. What is the probability that every house gets the right letter?

3. In a Biology class, the scores on the final exam were normally distributed, with a mean of 85 and a variance of 25. Susan got a final exam score of 95. Express her exam result as a Z-score and interpret its meaning.

4. Each of the members of a 7-judge panel independently makes a correct decision with probability 0.7. If the panel's decision is made by majority rule, what is the probability that the panel makes the correct decision?

5. An exam consists of many multiple-choice questions. Each question has four possible answers, among which only one is correct. Therefore, for any particular question, if someone picks one answer randomly, then the probability that the answer is correct is 0.25. For a correct answer 6 points will be given. How many points should be deducted for a wrong answer in order to make the expectation of total points received this way equal to 0?

6. A blood test for a certain disease has a false positive rate of 0.01 and a false negative rate of 0.05. Suppose that 2% of a certain population has the disease. If a random individual from this population tests positive, what is the probability that this person actually has the disease?

7. A radio talk show host is interested in the proportion p of adults in his listening area who think that the drinking age should be lowered to 18. To find this proportion, he poses the following question to his listeners: "Do you think that the drinking age should be reduced to 18?" He asked listeners to phone in and vote "yes" or "no" depending on their opinions. Of 200 people who phone in, 140 answer "yes". What is the standard error of the sample proportion \hat{p} of "yes" votes among those who phone in?

8. A random sample of 85 sixth-graders in a large city take a course designed to improve scores on a reading comprehension test. Based on this sample, a 90% confidence interval for the mean improvement in test scores for all sixth-graders in the city taking this course is found to be (12.6, 14.8). What is the sample mean and margin of error on which this interval is based?

9. There are four rounds in an exam. Each round has one question, which the student must answer correctly in order to go to the next round; otherwise she fails. Assume that the probabilities of answering correctly are $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{5}$ for the 1st, 2nd, 3rd, and 4th rounds respectively. In addition, the performance in each round does not impact any others. What is the probability that a student fails in the 3rd round?

10. (**Bonus question***) What is the expected number of times a fair coins needs to be tossed to get 3 consecutive heads?

★ Left blank for rough work. ★

★ Left blank for rough work. ★

* Left blank for rough work. *

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

★ Intentionally left blank. ★
