Solution for HW#1 (ECE6255 Spring 2010) – Total = 3/100

Solution 7)

a) $H(e^{j\omega}) = 1 \qquad \omega_1 < |\omega| < \omega_2$

$\qquad = 0 \qquad |\omega| < \omega_1;\ \omega_2 < |\omega| \le \pi$

-- simple direct solution:

$$h_{BPF} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{BPF}(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\omega_2}^{-\omega_1}(1)\,e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{\omega_1}^{\omega_2}(1)\,e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\cdot\frac{1}{jn}\left[e^{-j\omega_1 n} - e^{-j\omega_2 n} + e^{j\omega_2 n} - e^{j\omega_1 n}\right]$$

$$= \frac{1}{2\pi jn}\left[2j\sin(\omega_2 n) - 2j\sin(\omega_1 n)\right] = \frac{\left[\sin(\omega_2 n) - \sin(\omega_1 n)\right]}{\pi n}.$$

-- Alternate Solution:

□ we can express a bandpass filter as a difference between 1 and an appropriate lowpass filter and an appropriate highpass filter

$$H_{BP}(e^{j\omega}) = 1 - H_{LP} - H_{HP}$$

$$H_{LP}(e^{j\omega}) = 1 \qquad |\omega| < \omega_1$$

$$H_{HP}(e^{j\omega}) = 1 \qquad |\omega| > \omega_2$$

□ can design the highpass by inverting a lowpass using the relation

$$H_{HP}(e^{j\omega}) = H_{LP}(e^{j(\omega-\pi)}) \Rightarrow h_{HP}(n) = (-1)^n\, h_{LP}(n)$$

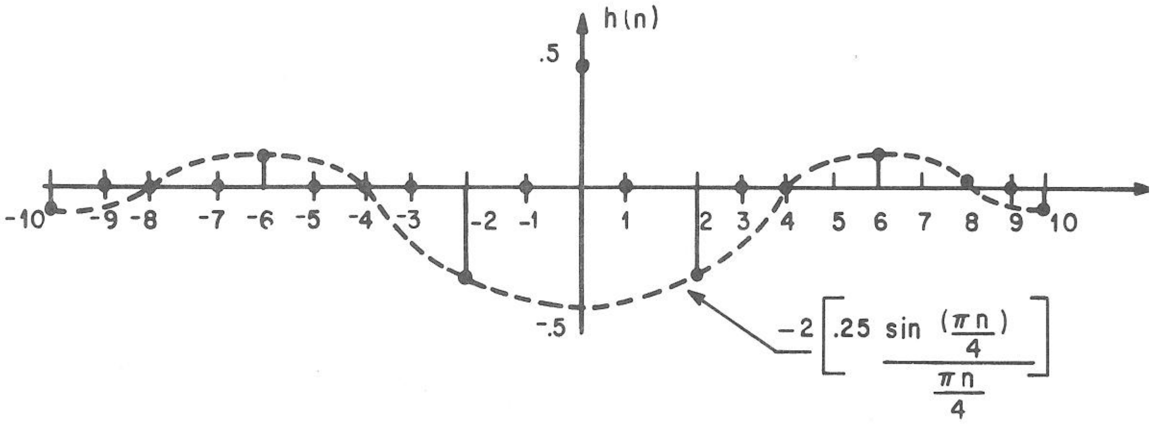
$$\Rightarrow h(n) = \delta(n) - \frac{\sin(\omega_1 n)}{\pi n} - (-1)^n\,\frac{\sin((\pi - \omega_2)n)}{\pi n}.$$

$$a)\quad h(n) = \delta(n) - \frac{1}{4}\cdot\frac{\sin(\pi n/4)}{\pi n/4} - (-1)^n\,\frac{\sin((\pi - 3\pi/4)n)}{\pi n}$$

$$= \delta(n) - \frac{1}{4}\cdot\frac{\sin(\pi n/4)}{\pi n/4} - (-1)^n\,\frac{1}{4}\cdot\frac{\sin(\pi n/4)}{\pi n/4}$$

$$= \delta(n) - \frac{1}{4}\cdot\frac{\sin(\pi n/4)}{\pi n/4}\left[1 + (-1)^n\right].$$

b)



Solution 5)

$$c_{xx}(n) = \sum_{k=-\infty}^{\infty} x(k) \bullet x(n+k)$$

a) $C_{xx}(z) = \sum_{n=-\infty}^{\infty} c_{xx}(n) z^{-n} = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(k) \bullet x(n+k) z^{-n}$

$$= \sum_{k=-\infty}^{\infty} x(k) \bullet [\sum_{n=-\infty}^{\infty} x(n+k) z^{-n}] = \sum_{k=-\infty}^{\infty} x(k) \bullet [\sum_{m=-\infty}^{\infty} x(m) z^{k-m}]$$

$$= \sum_{k=-\infty}^{\infty} x(k)[z^{-1}]^k \bullet [\sum_{m=-\infty}^{\infty} x(m) z^{-m}] = X(1/z) \bullet X(z)$$

b) $x(n) = a^n u(n) \quad \Rightarrow \; X(z) = \frac{1}{1-az^{-1}}$ ROC: $|z| > |a|$

$x(-n) = a^{-n} u(-n) \quad \Rightarrow \; X(z^{-1}) = \frac{1}{1-az}$, ROC: $|z| < \frac{1}{|a|}$

$C_{xx}(z) = X(z) X(z^{-1}) = \frac{1}{(1-az^{-1})(1-az)}$, ROC: $|a| < |z| < \frac{1}{|a|}$

for a stable sequence $x(n)$, $|a| < 1$. By decomposing $C_{xx}(z)$ into two terms with first order poles:

$C_{xx}(z) = \frac{1}{1-a^2}[\frac{1}{(1-az^{-1})} - \frac{1}{(1-\frac{1}{a} z^{-1})}]$, ROC: $|a| < |z| < \frac{1}{|a|}$,

therefore we have $\quad c_{xx}(n) = \frac{1}{1-a^2}[a^n u(n) + a^{-n} u(-n-1)]$.

The first term on the RHS is a causal sequence and the second is non-causal.

c) Since $c_{xx}(n) = x(n) * x(-n) \Rightarrow$ clearly, the signal, $y(n) = x(-n)$, i.e. the time reversed version of $x(n)$ will also yield the same $c_{xx}(n)$.

Solution 8)

$(a) \quad w_2(n) = 1 \qquad 0 \le n \le N-1$

$\qquad\qquad = 0 \qquad$ otherwise

$$W_2(z) = \sum_{n=0}^{N-1} (1) z^{-n} = \frac{1-z^{-N}}{1-z^{-1}}$$

$$W_2(e^{j\omega}) = \frac{1-e^{-j\omega N}}{1-e^{-j\omega}} = e^{-j\omega(N-1)/2} \frac{\sin(\omega N/2)}{\sin(\omega/2)};$$

$(b) \quad w_3(n) = 0.54 - 0.46\cos(2\pi n/(N-1) \qquad 0 \le n \le N-1$

$\qquad\qquad = 0 \qquad$ otherwise

∴ we can write $w_3(n)$ as a combination of shifted and scaled versions of $w_2(n)$

$$w_3(n) = 0.54 w_2(n) - 0.23 w_2(n) e^{j2\pi n/(N-1)} - 0.23 w_2(n) e^{-j2\pi n/(N-1)}$$

$$W_3(z) = 0.54 W_2(z) - 0.23 W_2(z e^{-j2\pi/(N-1)}) - 0.23 W_2(z e^{j2\pi/(N-1)})$$

$$W_3(e^{j\omega}) = e^{-j\omega(N-1)/2}\left[0.54\frac{\sin(\omega N/2)}{\sin(\omega/2)} + 0.23\frac{\sin\left[\left(\omega - 2\pi/(N-1)\right)N/2\right]}{\sin\left[\left(\omega - 2\pi/(N-1)\right)1/2\right]} + 0.23\frac{\sin\left[\left(\omega + 2\pi/(N-1)\right)N/2\right]}{\sin\left[\left(\omega + 2\pi/(N-1)\right)1/2\right]}\right].$$