Solution for HW#3 (ECE6255 Spring 2005) – Total = 4/100

Solution 2)

a) The only phoneme that never occurs in word initial position is /ng/ as in sing. The only other sound that almost never occurs naturally in English in initial word position is /zh/, except for some foreign words imported into English, such as gendarme.

b) Word initial consonant clusters of length three in English include:

/spl/-split /spr/-spring /spy/-spew /skw/-squirt

/sky/-skew /skr/-script /str/-string /skl/-sclerosis

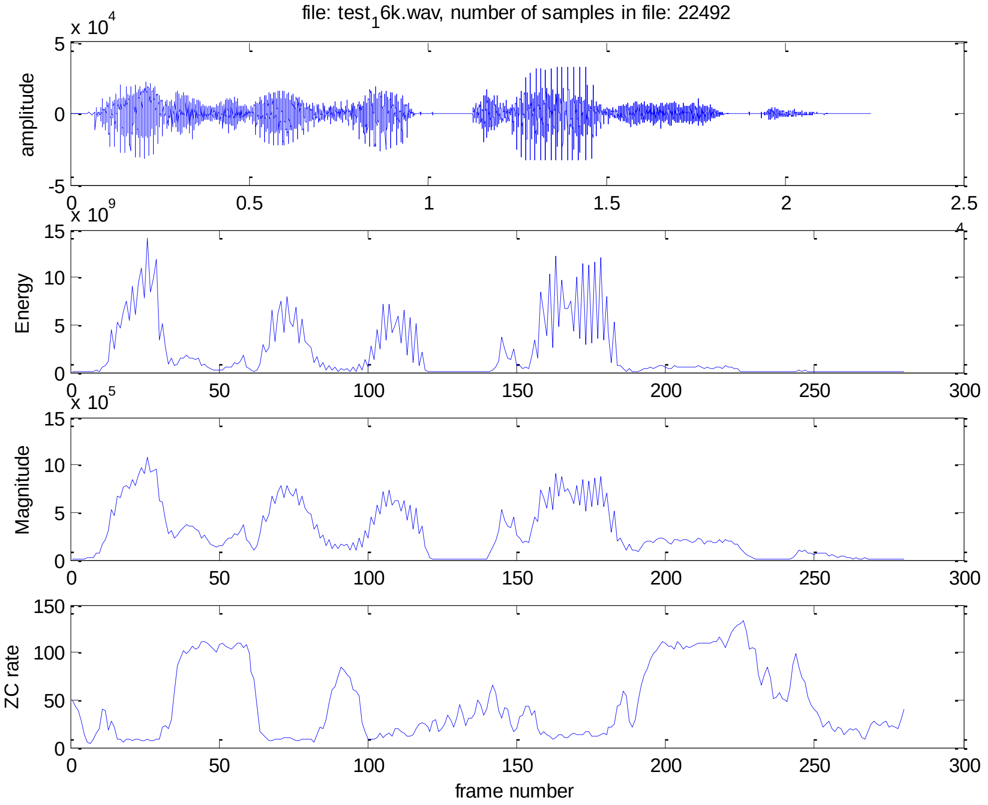
The general rule for such clusters is: /sound s/ --unvoiced stop-- /semi-vowel/

c) The general rule for a nasal-stop combination is that the nasal and stop have the same place of articulation; e.g., front/lips for /mp/, mid/dental for /nt/, back/velar for /ng k/. Exceptions occur in words like summed (/md/) or hanged (/ng d/) or dreamt (/mt/). There is often a tendency to insert an extra stop in such situations (e.g., dreamt -> /drempt/).

Solution 3) It is an easy exercise.

Solution 4)

The following MATLAB output example reads in a speech file, and computes the short-time energy, short-time magnitude, and short-time zero crossing rate using a 20 msec Hamming window, shifted by 5 msec (1/4 of the window). Below we plot results using a test utterance.



Solution 1)

$$\phi(k) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} x(m)x(m+k)$$

$$R_n(k) = \sum_{m=0}^{N-|k|-1} x(n+m)w'(m)x(n+m+k)w'(m+k)$$

$$\hat{R}_n(k) = \sum_{m=0}^{N-1} x(n+m)x(n+m+k)$$

$a)\ x(n) = x(n+P), \quad -\infty < n < \infty$

$$(i)\ \phi(k+P) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} x(m)x(m+k+P)$$

letting $m+k=n$ we see that $x(m+k+P) = x(m+k)$

$$\therefore\ \phi(k) = \phi(k+P); \quad -\infty < k < \infty$$

$$(ii)\ R_n(k+P) = \sum_{m=0}^{N-|k|-1} x(n+m)w'(m)x(n+m+k+P)w'(m+k+P)$$

$$= \sum_{m=0}^{N-|k|-1} x(n+m)w'(m)x(n+m+k)w'(m+k+P)$$

$\therefore R_n(k+P) \neq R_n(k)$ unless $w'(n+P) = w'(n)$

$(iii)\ \hat{R}_n(k) = \hat{R}_n(k+P)$ since $x(n+m+k+P) = x(n+m+k)$

$b)$

$$(i)\ \phi(-k) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} x(m)x(m-k)$$

let $m = m' + k,$ then

$m = -N$ as $N \to \infty \Rightarrow m' = -N$

$m = N$ as $N \to \infty \Rightarrow m' = N$

$$\phi(-k) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{m'=-N}^{N} x(m'+k)x(m') = \phi(k)$$

$$\therefore\ \phi(-k) = \phi(k)$$

$$(ii)\ R_n(-k) = \sum_{m=-\infty}^{\infty} x(n+m)w'(m)x(n+m-k)w'(m-k)$$

again let $m = m' + k$

$$R_n(-k) = \sum_{m'=-\infty}^{\infty} x(n+m'+k)w'(m'+k)x(n+m')w'(m')$$

for a causal window of length $N-1,$ the product is zero outside the range

$0 \le m \le N$ - $|k| - 1$

$$\therefore\ R_n(-k) = R_n(k)$$

$$(iii)\ \hat{R}_n(-k) = \sum_{m=0}^{N-1} x(n+m)x(n+m-k)$$

let $m = m' + k,$ then

$m = 0 \quad \Rightarrow \quad m' = -k$

$m = N-2 \quad \Rightarrow \quad m' = N-1-k$

$$\hat{R}_n(-k) = \sum_{m'=-k}^{N-1-k} x(n+m')x(n+m'+k)$$

the product term is the same form as for $\hat{R}_n(k)$, but the range of summation is different, therefore $\hat{R}_n(-k) \ne \hat{R}_n(k)$.

$c)$ (i) examine $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} \left\{ \left[x(m+k) \pm x(m) \right]^2 \right\} \geq 0$

$$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} \left\{ \left[x(m+k)x(m+k) \pm 2x(m+k)x(m) + x(m)x(m) \right] \right\}$$

$$= 2\left[\phi(0) \pm \phi(k) \right] \geq 0$$

therefore we have $-\phi(0) \leq \phi(k) \leq \phi(0)$ and $\phi(k)$ is maximum at the origin;

$$(ii) \quad \sum_{m=-\infty}^{\infty} \left[x(n+m+k)w'(m+k) \pm x(n+m)w'(m) \right]^2 \geq 0$$

$$2\left[R_n(0) \pm R_n(k) \right] \geq 0$$

as before, this relation implies $R_n(k) \leq R_n(0)$;

(iii) $\hat{R}_n(k)$ is not $\leq \hat{R}_n(0)$ in general. This can be seen by considering the sequence

$$x(n) = \delta(n) + 2\delta(n-N)$$

then we get $\hat{R}_0(0) = \sum_{m=0}^{N-1} x^2(m) = 1,$

$$\hat{R}_0(N) = \sum_{m=0}^{N-1} x(m)x(m+N) = 2,$$

$$\hat{R}_0(N) > \hat{R}_0(0) \Rightarrow \hat{R}_n(k) \text{ is not } \leq \hat{R}_n(0) \ \forall \ n,k;$$

$d)$ (i) $\phi(0) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} x^2(m)$

$$\lim_{N\to\infty} \sum_{m=-N}^{N} x^2(m) = \text{total energy of the signal}$$

$$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} x^2(m) = \frac{\text{average energy}}{\text{unit time}} = \text{power (in signal);}$$

$$(ii) \quad R_n(0) = \sum_{m=0}^{N-1} \left[x(n+m)w'(m) \right]^2 = \sum_{m=-\infty}^{\infty} \left[x(n+m)w'(m) \right]^2$$

since $w'(m)$ is a causal window of length N. Now substitute $w'(m) = w(-m)$

$$R_n(0) = \sum_{m=-\infty}^{\infty} \left[x(n+m)w(-m) \right]^2 \quad \text{(let } m = m' - n\text{)}$$

$$R_n(0) = \sum_{m'=-\infty}^{\infty} \left[x(m')w(n-m') \right]^2 = \text{short time energy;}$$

$$(iii) \quad \hat{R}_n(0) = \sum_{m=0}^{N-1} \left[x(n+m) \right]^2$$

let $m = m' - n$

$$\hat{R}_n(0) = \sum_{m'=n}^{N-1+n} \left[x(m') \right]^2 = \sum_{m'=n}^{N-1+n} \left[x(m')w(n-m') \right]^2$$

where $w(n) = 1 \quad 0 \leq n \leq N-1$

$= 0 \quad otherwise$

we see that even if we restrict the window to be rectangular, $\hat{R}_n(0)$ does not represent the short-time energy, since the range of summation is different.