# Solution for HW#4 (ECE6255 Spring 2010) – Total = 4/100

## Solution 1)

$\tau_1 = l_1 / c;\ \ \tau_2 = l_2 / c$

Assuming $r_G = r_L = 1;\ r_1 = \dfrac{A_2 - A_1}{A_2 + A_1}$

Then

$$V_a(s) = \frac{0.5(2)(2)(1+r_1)e^{-s(\tau_1+\tau_2)}}{1+r_1(e^{-s2\tau_1}+e^{-s2\tau_2})+e^{-s2(\tau_1+\tau_2)}}$$

a) The poles are values of $s$ such that

$$1+r_1(e^{-s2\tau_1}+e^{-s2\tau_2})+e^{-s2(\tau_1+\tau_2)} = 0$$

$$e^{-s(\tau_1+\tau_2)}\left[e^{s(\tau_1+\tau_2)}+r_1(e^{-s\tau_1+s\tau_2}+e^{-s\tau_2+s\tau_1})+e^{-s(\tau_1+\tau_2)}\right]=0$$

$$e^{-s(\tau_1+\tau_2)}\left[2\cosh\left[s\left(\tau_1+\tau_2\right)\right]+2r_1\cosh\left[s\left(\tau_2-\tau_1\right)\right]\right]=0$$

Since this is a lossless system, the poles must be on the $s = j\Omega$ axis.

Thus the hyperbolic functions reduce to circular functions, i.e., the poles satisfy

$$\cos\left[\Omega\left(\tau_1+\tau_2\right)\right]+r_1\cos\left[\Omega\left(\tau_2-\tau_1\right)\right]=0$$

To get the other form, substitute for $r_1$

$$\cos\left[\Omega\left(\tau_1+\tau_2\right)\right]+\frac{A_2-A_1}{A_2+A_1}\cos\left[\Omega\left(\tau_2-\tau_1\right)\right]=0$$

$$A_2\left(\cos\left[\Omega\left(\tau_1+\tau_2\right)+\cos\left[\Omega\left(\tau_2-\tau_1\right)\right]\right]\right)+A_1\left(\cos\left[\Omega\left(\tau_1+\tau_2\right)-\cos\left[\Omega\left(\tau_2-\tau_1\right)\right]\right]\right)=0$$

$$2A_2\cos(\Omega\tau_1)\cos(\Omega\tau_2)-2A_1\sin(\Omega\tau_1)\sin(\Omega\tau_2)=0$$

$$\frac{A_1}{A_2}\tan(\Omega\tau_2)=\cot(\Omega\tau_1)$$

b) Now substituting for $l_1, l_2, A_1, A_2$ we get a transcendental equation to solve either graphically or iteratively

| Vowel | F1 | F2 | F3 |
|---|---|---|---|
| /i/ | 256 | 1905 | 2917 |
| /ae/ | 646 | 1830 | 2358 |
| /a/ | 789 | 1276 | 2808 |
| /uh/ | 515 | 1544 | 2574 |

## Solution 2)

(a) Since the third peak (impulse) of the magnitude spectrum is located at π/4, the pitch frequency is simply π/12, or 250Hz because the sampling rate is 6000Hz. Therefore the pitch period is 4 msec. From the frequency response of the rectangular window (in HW#1) it can also be shown that the first zero is located at 2π/*L* Hz or π/24 in Figure 4.33 on p. 167 of Quartieri, or 125Hz or 8 msec (twice the pitch period of 4 msec).

(b) Using Eq. (4.18) the format frequencies, on the top of Page 125 in Quatieri, indicate that $2\pi * F_k = k*\pi*c/2*l$ so for the first format ($k=1$) $l$ = (35000 cm/sec)/(2*750Hz)=11.67 cm (this gives you some idea that it is not easy to get a first formant frequency of 750Hz because it implies a very short vocal tract length). It also show that the fundamental frequency of 250Hz in Part (a) is rather high as well;

(c) No change. The pitch controls the spacing of the window main lobe;

(d) The radiation load introduces a high frequency emphasis factor of about 6dB/octave. With it the speech magnitude spectrum will roll off faster at a rate of 6dB/octave in the high frequency region.

## Solution 3)

FBS in Eq. (7.13) and OLA in Eq. (7.20).
In the FBS the constraint sum is proportional to the value of the window at time zero, $w(0)$;
In the OLA the constraint sum is proportional to the value of the the Fourier transform at freq. 0, $W(0)$. In the FBS, we can reconstruct the signal if $N_W$ (length of the window in time) is less than the sampling period, $N$, in the frequency domain; In the OLA, we can reconstruct the signal if $B_W$ (bandwidth of the window in freq) is less than $2\pi / L$, with $L$ the temporal decimation factor. This is the important dual principle in both the time and freq domains/

## Solution 4)

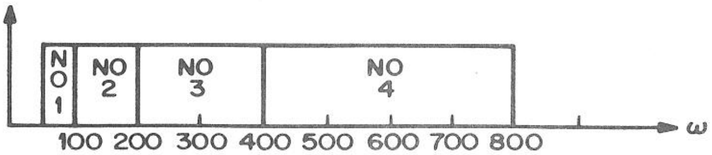
$F_{k-1} = 2^{k-1} F_1, \quad k = 1, 2, ..., M$ bandpass lower cutoff frequencies

$F_k = 2^k F_1, \quad k = 1, 2, ..., M$ bandpass upper cutoff frequencies

$a)$ given $50 < F_0 < 800$; let $F_1 = 50$, $M = 4$(bands), giving the following values

| $k$ | $F_{k-1}$ | $F_k$ |
|---|---|---|
| 1 | 50 | 100 |
| 2 | 100 | 200 |
| 3 | 200 | 400 |
| 4 | 400 | 800 |

$b)$ the ideal bandpass filters are shown below for positive frequency, where $M = 4$

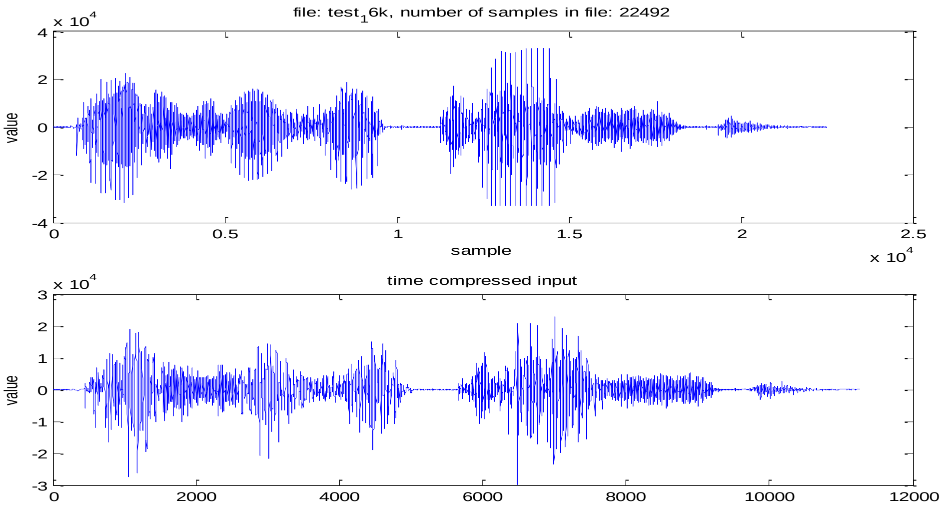



$c)$ Tone detection may be accomplished using a short-time zero crossing detector. If the zero-crossing count exceeds a given threshold for the filter, the tone is detected;

$d)$ In using non-ideal filters, the resulting overlap between bands will result in "cross-talk". If the energy due to cross-talk is sufficient, the zero-crossing detector may not produce the correct count.

$e)$ In this case fundamental frequencies of less than 300 Hz would be missing. One possibility is to pass the signal through a non-linearity in order to restore the fundamental. Another possibility is to determine the fundamental from differences between higher harmonics.

## Solution 6)




## Solution 4)

$$x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}[\sum_{k=-\infty}^{\infty} f(n,n-k)X(k,\omega)]e^{j\omega n}d\omega, \text{ with } X(k,\omega)=\sum_{m=-\infty}^{\infty} x(m)w(k-m)e^{-j\omega m}.$$

(a) clearly, $x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}[\sum_{k=-\infty}^{\infty} f(n,n-k)\sum_{m=-\infty}^{\infty} x(m)w(k-m)e^{-j\omega m}]e^{j\omega n}d\omega$

$$=\sum_{k=-\infty}^{\infty}\sum_{m=\infty}^{\infty} f(n,n-k)x(m)w(k-m)[\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{j\omega(n-m)}d\omega]$$

$$=\sum_{k=-\infty}^{\infty}\sum_{m=\infty}^{\infty} f(n,n-k)x(m)w(k-m)\delta(n-m)$$

$$=\sum_{k=-\infty}^{\infty} f(n,n-k)x(n)w(k-n)=x(n)[\sum_{k=-\infty}^{\infty} f(n,n-k)w(k-n)].$$

The contraint $\sum_{m=-\infty}^{\infty} f(n,-m)w(m)$ is obtained by setting $m=k-n$.

(b) Clearly Eq. (7.30) is reduced to the inverse Fourier transform to compute $x(n)$ if we set $f(n,m)=\delta(m)$.

(c) The OLA is a discrete version of Eq. (7.19) shown in Section 7.3.3 of Quatieri.

With the FBS constraint, we have $\sum_{p=-\infty}^{\infty} f(n,-p)w(p)=\sum_{p=-\infty}^{\infty} w(p)/W(0)=1,$

or $\sum_{p=-\infty}^{\infty} w(p-n)=W(0)$ which is equivalent to Eq. (7.20) for OLA with $L=1$.