# Homework 8 (ECE6255 Spring 2010) Grade=4/100

1. Consider the first order linear prediction $\tilde{x}(n) = \alpha x(n-1)$, with *x(n)* a stationary, zero-mean signal. Define the prediction error as $d(n) = x(n) - \tilde{x}(n)$.
   a) Show that *d(n)* has a variance, $\sigma_d^2 = \sigma_x^2 \bullet [1 + \alpha^2 - 2\alpha\phi_x(1)/\sigma_x^2]$;
   b) Show that $\sigma_d^2$ is minimized with $\hat{\alpha} = \phi_x(1)/\sigma_x^2$;
   c) Show that the minimum prediction error variance is $\sigma_d^2 = \sigma_x^2 \bullet (1 - \hat{\alpha}^2)$;
   d) Under what condition will it be true that $\sigma_d^2 < \sigma_x^2$ ?

2. In a differential coder, consider the first difference signal, $\hat{d}(n) = x(n) - \alpha_1 \hat{x}(n-1)$, where $\hat{x}(n)$ is the quantized signal, such that $\hat{x}(n-1) = x(n-1) + e(n-1)$, with $e(n)$ being the quantization error. Assume that *e(n)* and the original random signal *x(n)* are uncorrelated, i.e. $E[x(n)e(m)] = 0 \ \forall m, n$,

   a) Show that; $\sigma_{\hat{d}}^2 = \sigma_x^2 \bullet [1 + \alpha_1^2 - 2\alpha_1\phi_x(1)/\sigma_x^2] + \alpha_1^2\sigma_e^2$
   b) Using the results in part (a) to show that the gain due to differential coder is

$$G_p = \frac{\sigma_x^2}{\sigma_{\hat{d}}^2} = \frac{1 - \dfrac{\alpha_1^2}{SNR_Q}}{1 + \alpha_1^2 - 2\alpha_1\phi_x(1)/\sigma_x^2}$$

   where $SNR_Q = \sigma_{\hat{d}}^2 / \sigma_e^2$ is the signal-to-quantization noise ratio of the quantizer.

3. Work out Problem 12.4 of Quatieri [Hint: You need to understand the fundamental principle of the optimal Max quantizer illustrated in Section 12.3.3 of Quatieri (pp. 606-609) for quantizing random variables with any given pdf with the minimum mean-squared error (MMSE) criterion. Equation (12.8) shows the optimal boundary point between two successive quantization levels is the average of the two values. Equation (12.9) shows that the optimal quantized value for any given interval between two successive boundary points defined in Eq. (12.8) is the centroid over the interval. You then solve the problem in an iterative manner].

4. Work out Problem 12.9 of Quatieri [Hint: you need $\sigma_x^2$ and $\sigma_e^2$ in order to compute the quantization SNR, $\sigma_x^2$ can be obtained from Figure 12.32b using the equation $\sigma_x^2 = E[x^2] - (E[x])^2$; derive the quantization error functions in Part (a) and (b) first from Figures 12.32c and 12.32d, then compute $\sigma_e^2$ in each case].

(Although we did not go through the discussion on designing optimal quantizers this is a good example to compute SNR if we have to design them for some artificial signal situations in which we need to determine the best quantization level in the minimum mean-squared error, or MMSE, sense.)

5. Write a MATLAB program to quantize a speech file using a uniform quantizer with B-bits/sample, where $B$=12, 10, 8, 4, 2, 1. Determine the quantization error sequence for each value of $B$ (consider using any existing routine for the job, such as quantizer.m in Matlab), plot the quantization error histogram (similar to the one on Slide 44 of Lecture 30) of the quantization noise values (using the Matlab hist command—think carefully about how many bins should be used), and plot the spectrum of the original waveform and the quantization error at each of the bit rates on a single plot similar to the one on Slide 45 of Lecture 30. Listen to the waveform of both the quantized speech file and the quantization noise. At what bit rate can you still hear speech structure in the quantization noise? Determine the SNR of each quantized signal at each value of $B$. Use the speech file 1.wav and 5.wav to test your program.