# EE-2200 Fall-98

**Lecture 12**

**Digital Filtering of Analog Signals**

**6-Nov-98**

## Info: Web-CT, Lab, HW

- Calendar:
  - Quiz #2 on 13–Nov (Friday)
  - REVIEW next WEDNESDAY
- Prob Set #5 due MONDAY
  - Solutions for #4 are posted (complete)
- Lab #7 on DTMF
  - Touch–Tone Phone



## READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, pp. 188–194
- Other Reading:
  - Recitation: Ch. 6, pp. 176–188
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7, start

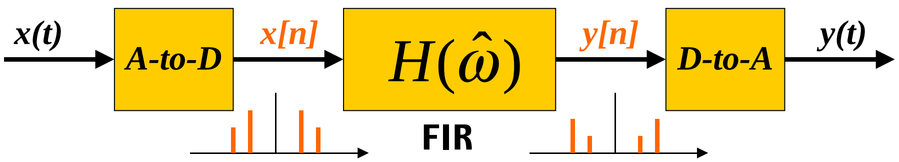


## COURSE OBJECTIVE

- Students will be able to:
- Understand mathematical descriptions of signal processing algorithms and express those algorithms as computer implementations (MATLAB)
- Where do you stand?

# LECTURE OBJECTIVES

- Track the spectrum of x[n] thru an FIR Filter.
- UNIFICATION:
  - How does Frequency Response affect x(t) to produce y(t) ?






# LECTURE #11 REVIEW

- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING:

MAG

PHASE

$$H(\hat{\omega}) = \left|H(\hat{\omega})\right| e^{j\varphi(\hat{\omega})}$$

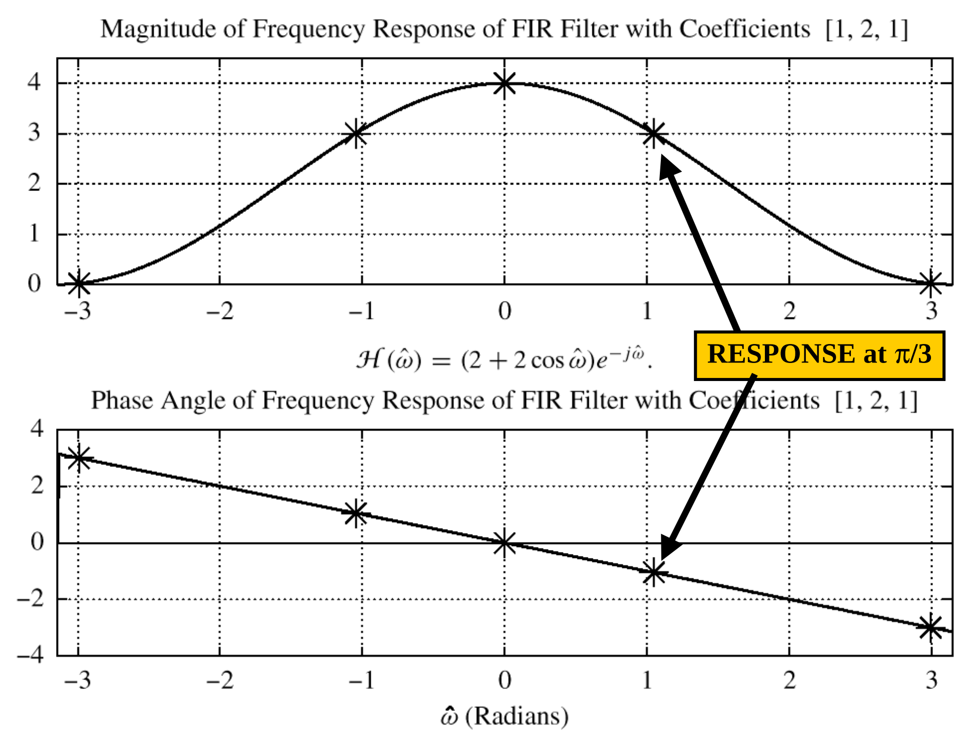


# FREQUENCY RESPONSE

- `yy = freqz(bb,1,ww)`
  - VECTOR bb contains Filter Coefficients
  - DSP-First: `yy = freekz(bb,1,ww)`
- FILTER COEFFICIENTS $\{b_k\}$

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$





# LTI SYSTEMS

- LTI:
  - Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - FREQUENCY RESPONSE
  - IMPULSE RESPONSE h[n]
- Two DOMAINS: time & frequency
  - Go back and forth



# TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

*The frequency response of an LTI system*

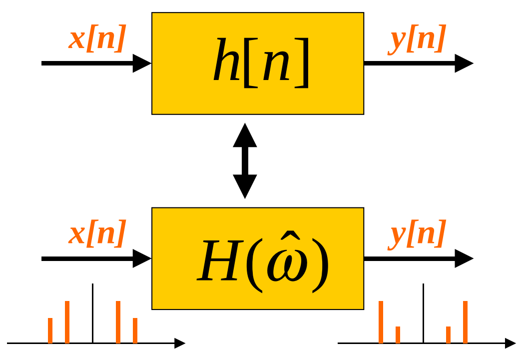
$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k} \qquad (6.1.4)$$

IMPULSE RESPONSE



# BLOCK DIAGRAMS

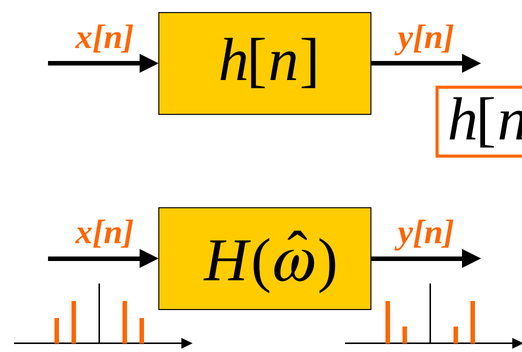
- Equivalent Representations






# DELAY SYSTEM ??

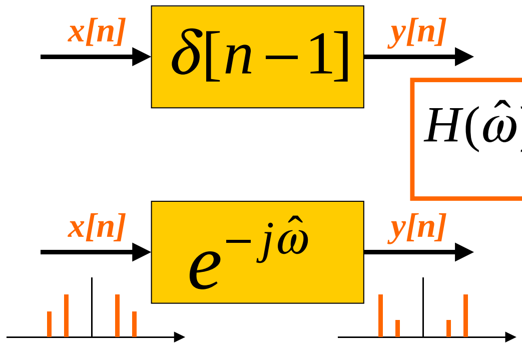
- UNIT DELAY: Find $h[n]$ and $H(\hat{\omega})$




$$h[n] = \delta[n-1]$$

# DELAY SYSTEM

- **UNIT DELAY:** Find $h[n]$ and $H(\hat{\omega})$



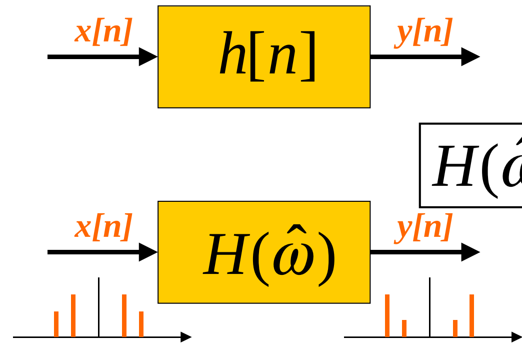

$$H(\hat{\omega}) = \sum_k \delta[k-1] e^{-j\hat{\omega}k}$$

**k=1 ONLY**



# FREQ DOMAIN --> TIME ??

- **START with** $H(\hat{\omega})$ and find $h[n]$ or $b_k$




$$h[n] = ?$$

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$



# FREQ DOMAIN --> TIME

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$

**EULER's Formula**

$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

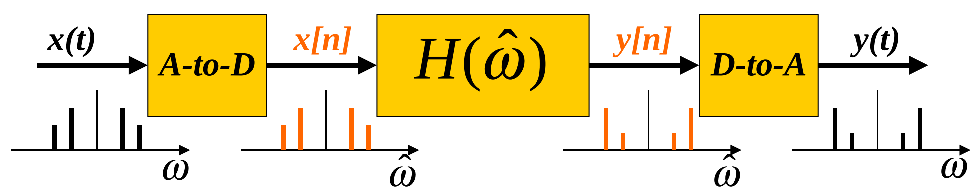
$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0, 3.5, 0, 3.5\}$$

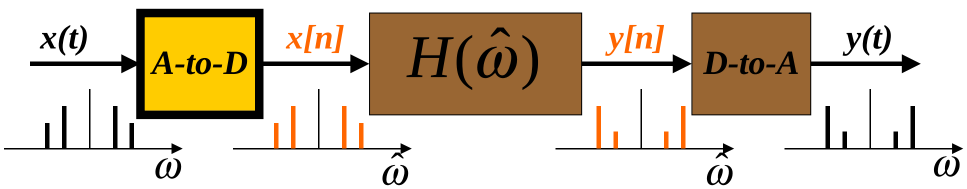


# DIGITAL "FILTERING"




- **SPECTRUM of x(t) (SUM of SINUSOIDS)**
- **SPECTRUM of x[n]**
  - **Is ALIASING a PROBLEM ?**
- **SPECTRUM y[n] (FIR Gain or Nulls)**
- **Then, OUTPUT y(t) = SUM of SINUSOIDS**

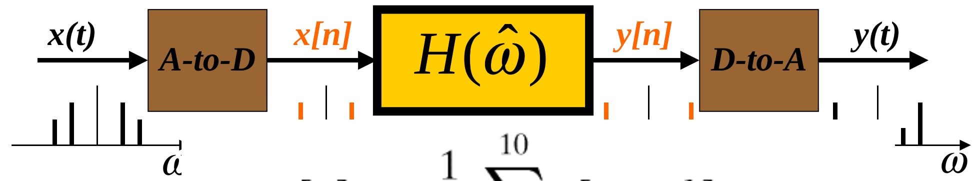
# FREQUENCY SCALING




- **TIME SAMPLING:** $t = nT_s$
  - **POSSIBLE ALIASING**
  - **FREQUENCY SCALING** $\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$



# 11-pt AVERAGER Example



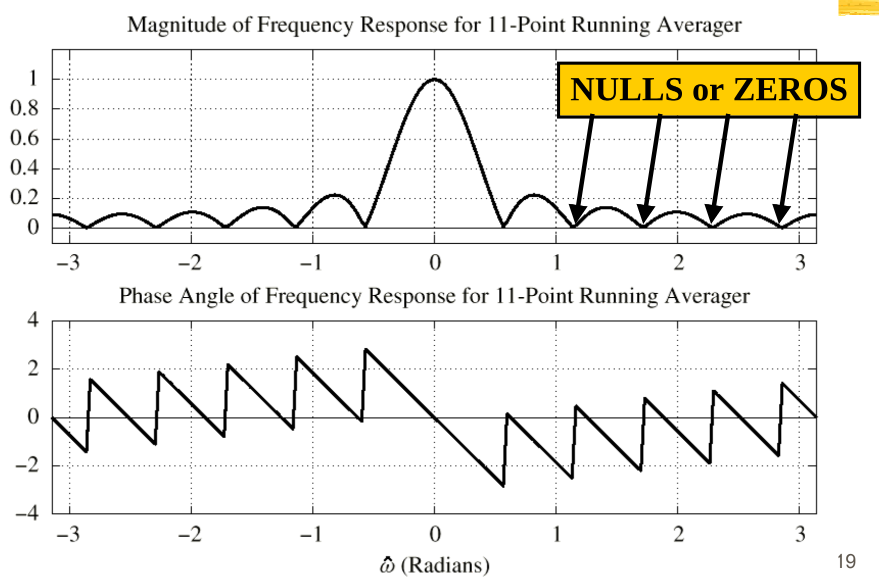

$$y[n] = \frac{1}{11}\sum_{k=0}^{10} x[n-k]$$

$$\mathcal{H}(\hat{\omega}) = \frac{\sin(\hat{\omega}11/2)}{11\sin(\hat{\omega}/2)} e^{-j\hat{\omega}5}$$

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$



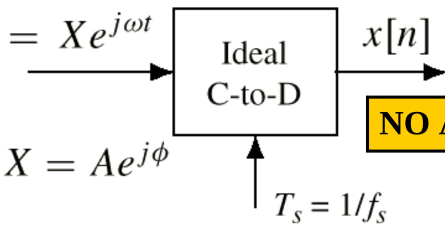
# 11-pt AVERAGER






# 11-pt AVG Example (2)

$x(t) = Xe^{j\omega t}$




$x[n] = x(nT_s) = Xe^{j\omega nT_s}$

$= Xe^{j\hat{\omega}n}$

$X = Ae^{j\phi}$

**NO Aliasing**

$$y[n] = \mathcal{H}(\hat{\omega})Xe^{j\hat{\omega}n}$$

$$\hat{\omega} = \omega T_s$$

$$y[n] = \mathcal{H}(\omega T_s)Xe^{j\omega T_s n}$$

$$y(t) = \mathcal{H}(\omega T_s)Xe^{j\omega t}$$

# SINUSOID thru FIR

$$x[n] = X_0 + \sum_{k=1}^{N} \left( \frac{X_k}{2} e^{j\hat{\omega}_k n} + \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right)$$

$$= X_0 + \sum_{k=1}^{N} |X_k| \cos(\hat{\omega}_k n + \angle X_k)$$

if $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$, the corresponding output is

$$y[n] = \mathcal{H}(0) X_0 + \sum_{k=1}^{N} \left( \mathcal{H}(\hat{\omega}_k) \frac{X_k}{2} e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k) \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right)$$

$$= \mathcal{H}(0) X_0 + \sum_{k=1}^{N} |\mathcal{H}(\hat{\omega}_k)| |X_k| \cos\left(\hat{\omega}_k n + \angle X_k + \angle \mathcal{H}(\hat{\omega}_k)\right)$$

# EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$f_s = 1000$

$$\mathcal{H}(2\pi(25)/1000) = \frac{\sin(\pi(25)(11)/1000)}{11 \sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

$$= 0.8811 e^{-j\pi/4}$$

MAG SCALE

PHASE CHANGE

$$\mathcal{H}(2\pi(250)/1000) = \frac{\sin(\pi(250)(11)/1000)}{11 \sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

$$= 0.0909 e^{-j\pi/2}$$

$$y(t) = 0.8811 \cos(2\pi(25)t - \pi/4) + 0.0909 \sin(2\pi(250)t - \pi/2)$$

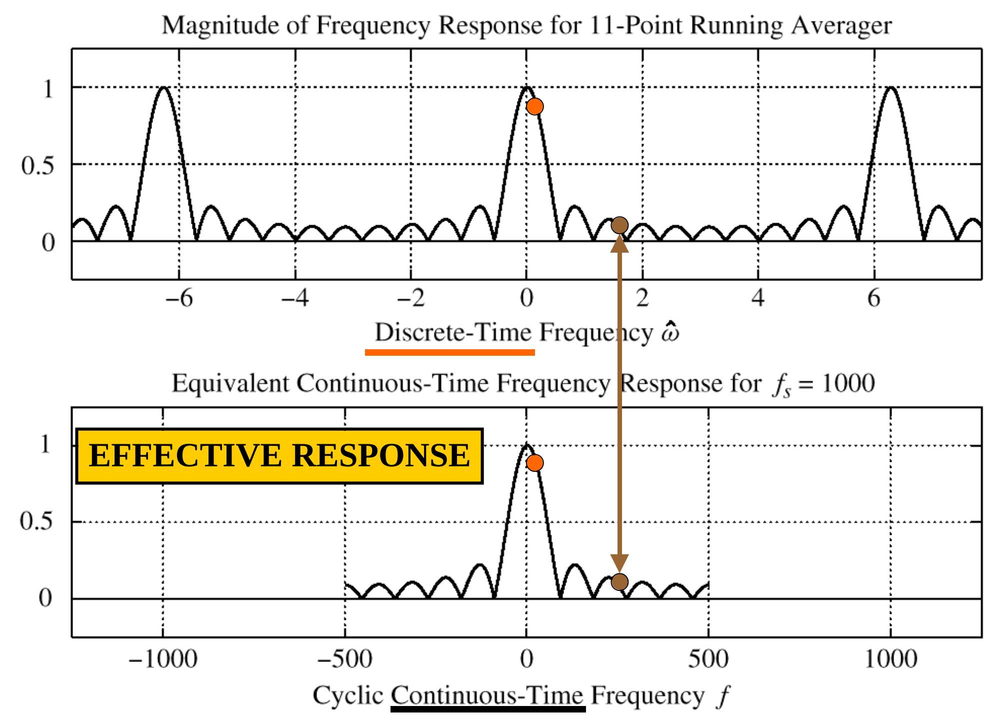
# EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <---> DIGITAL FREQ

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

  - So, we can plot:
  - Scaled Freq. Axis

$$H(\omega T_s) \text{ vs. } \omega$$

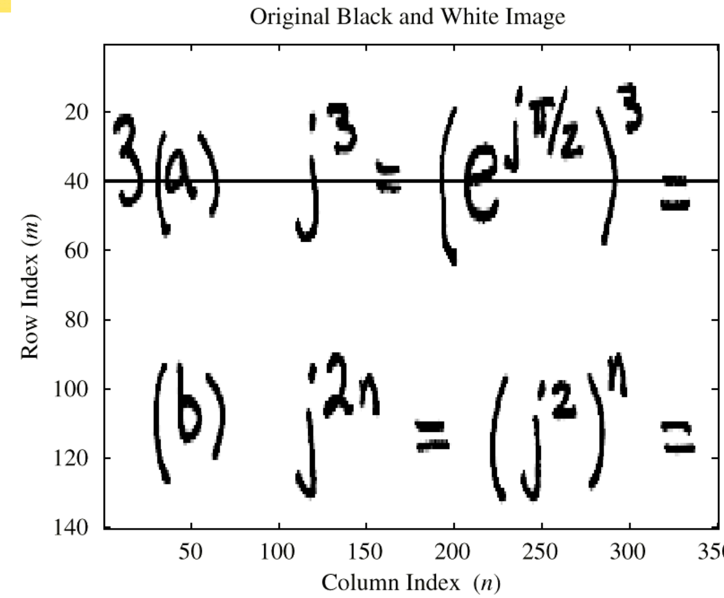


# FILTER TYPES

- LOW–PASS FILTER (LPF)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH–PASS FILTER (HPF)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND–PASS FILTER (BPF)



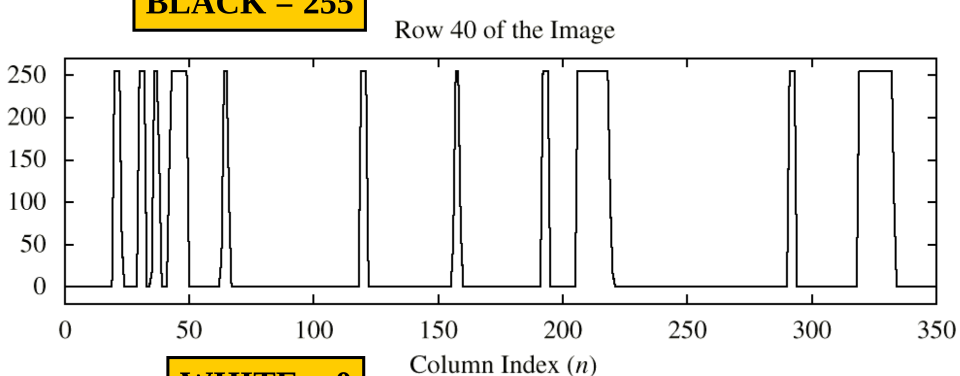
# B & W IMAGE






# ROW of B&W IMAGE

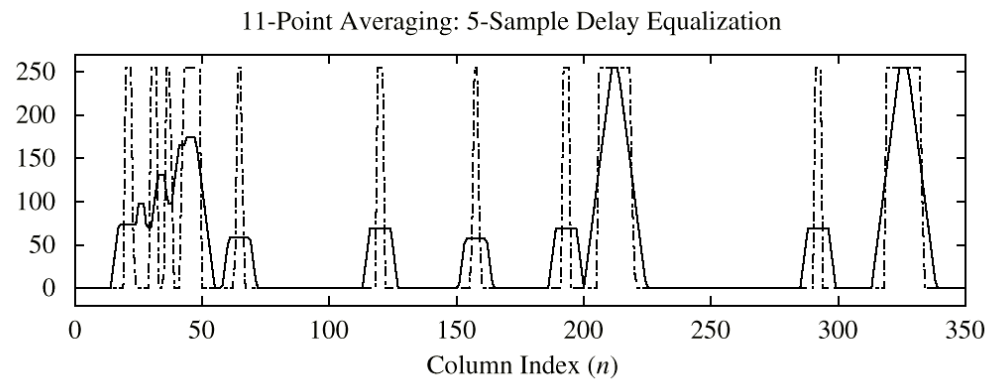
**BLACK = 255**




**WHITE = 0**

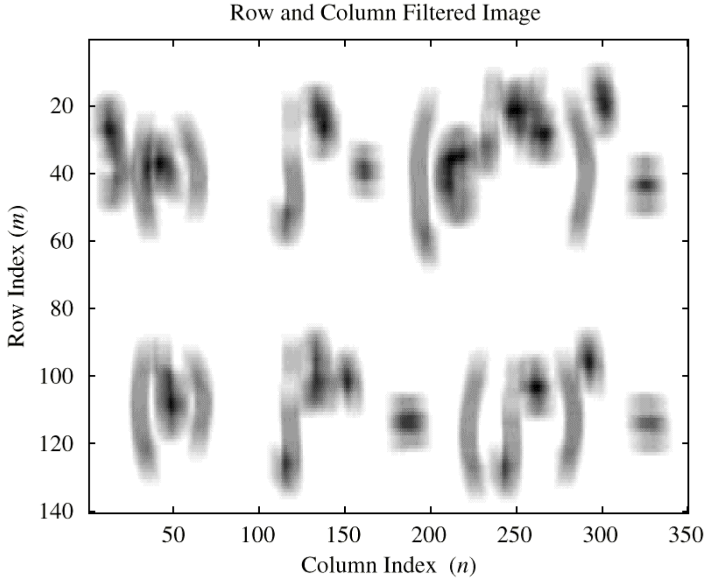


# FILTERED ROW of IMAGE




**ADJUST DELAY by 5 samples**

# FILTERED B&W IMAGE




**LPF: BLUR**



# B&W IMAGE with COSINE

**FILTERED: 11-pt AVG**