

EE-2200

Fall-98

Lecture 13

Z Transforms: Introduction

9-Nov-98

Info: Web-CT, Lab, HW

■ **Calendar:**

- **Quiz #2 on 13-Nov (Friday)**
- **REVIEW this WEDNESDAY (ECE Aud)**

■ **Prob Set #5 due TODAY**

- **Solutions for #5 will be posted ASAP**

■ **Lab #7 on DTMF**

- **Touch-Tone Phone**

READING ASSIGNMENTS

■ **This Lecture:**

- **Chapter 7, pp. 202–216**

■ **Other Reading:**

- **Recitation: Ch. 7, pp. 217–220**
 - **CASCADING SYSTEMS**
- **Next Lecture: Chapter 7, more**

LECTURE OBJECTIVES

■ **INTRODUCE the Z–TRANSFORM**

- **Give Mathematical Definition**
- **Show how $H(z)$ POLYNOMIAL simplifies analysis**
 - **CONVOLUTION EXAMPLE**

■ **Z–Transform can be applied to**

- **FIR Filter: $h[n] \rightarrow H(z)$**
- **Signals: $x[n] \rightarrow X(z)$**

$$H(z) = \sum_n h[n]z^{-n}$$

TRANSFORM CONCEPT

- Move to a new domain where
 - OPERATIONS are EASIER/FAMILIAR
 - Use **POLYNOMIALS**
- TRANSFORM both ways
 - $x[n] \rightarrow X(z)$ (into the z domain)
 - $X(z) \rightarrow x[n]$ (back to the time domain)

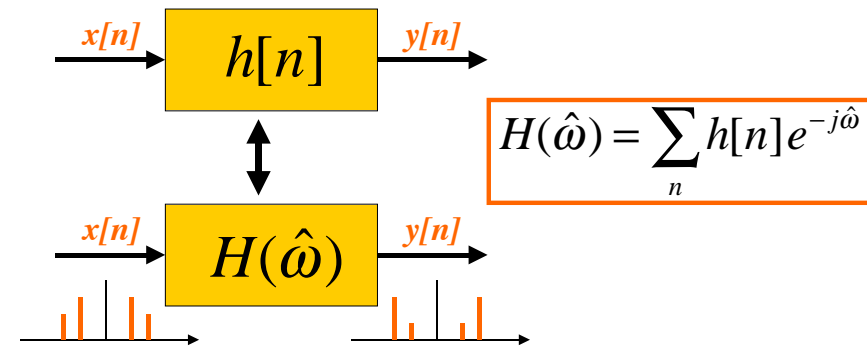
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TRANSFORM EXAMPLE

- Equivalent Representations



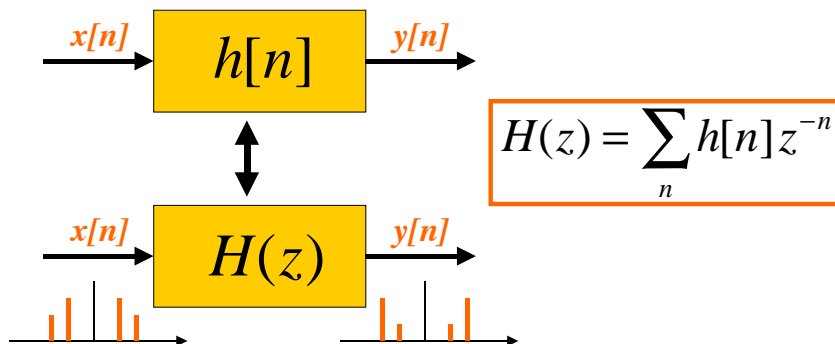
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Z-TRANSFORM IDEA

- POLYNOMIAL Representation



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Z-Transform DEFINITION

- POLYNOMIAL Representation

$$H(z) = \sum_n h[n] z^{-n}$$

- EXAMPLE:

APPLIES to Any SIGNAL

$$\{h[n]\} = \{2, 0, -3, 0, 2\}$$

$$\begin{aligned} H(z) &= 2z^{-0} + 0z^{-1} - 3z^{-2} + 0z^{-3} + 2z^{-4} \\ &= 2 - 3z^{-2} + 2z^{-4} \end{aligned}$$

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Z-Transform POLYNOMIAL

$$x[n] = \sum_{k=0}^N x[k] \delta[n - k]$$

APPLIES to Any SIGNAL

$$X(z) = \sum_{k=0}^N x[k] z^{-k}$$

$$X(z) = \sum_{k=0}^N x[k] (z^{-1})^k$$

POLYNOMIAL in z^{-1}

Example 7.2

$$X(z) = 1 - 2z^{-1} + 3z^{-3} - z^{-5}$$

EXPONENT GIVES TIME LOCATION

$$x[n] = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ -2 & n = 1 \\ 0 & n = 2 \\ 3 & n = 3 \\ 0 & n = 4 \\ -1 & n = 5 \\ 0 & n > 5 \end{cases}$$

$$x[n] = \delta[n] - 2\delta[n - 1] + 3\delta[n - 3] - \delta[n - 5]$$

Z-Transform of FIR Filter

h[n] is same as {b_k}

$$H(z) = \sum_{k=0}^M b_k z^{-k} = \sum_{k=0}^M h[k] z^{-k}$$

$$y[n] = \sum_{k=0}^M b_k x[n - k] = \sum_{k=0}^M h[k] x[n - k]$$

FIR DIFFERENCE EQUATION

CONVOLUTION

Z-Transform of FIR Filter

Get H(z) DIRECTLY from the {b_k}

Example 7.3

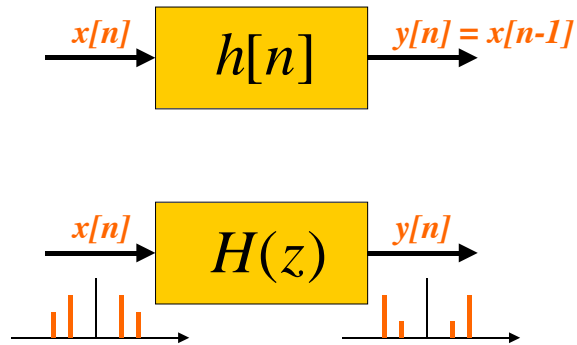
$$y[n] = 6x[n] - 5x[n - 1] + x[n - 2]$$

$$H(z) = 6 - 5z^{-1} + z^{-2} = (3 - z^{-1})(2 - z^{-1})$$

$$= 6 \frac{(z - \frac{1}{3})(z - \frac{1}{2})}{z^2}$$

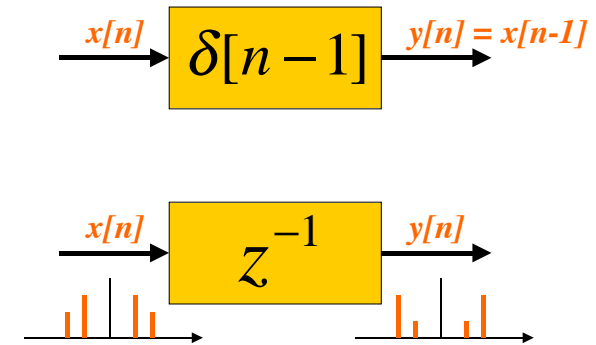
DELAY SYSTEM ??

UNIT DELAY: find $h[n]$ and $H(z)$



DELAY SYSTEM

UNIT DELAY: find $h[n]$ and $H(z)$



DELAY EXAMPLE

UNIT DELAY: find $y[n]$ via polynomials

$x[n] = \{3, 1, 4, 1, 5, 9, 0, 0, 0, \dots\}$

$$\begin{aligned}
 Y(z) &= z^{-1}X(z) \\
 &= z^{-1}(3 + z^{-1} + 4z^{-2} + z^{-3} + 5z^{-4} + 9z^{-5}) \\
 &= 0z^0 + 3z^{-1} + z^{-2} + 4z^{-3} + z^{-4} + 5z^{-5} + 9z^{-6}
 \end{aligned}$$

n	$n < 0$	0	1	2	3	4	5	6	$n > 6$
$y[n]$	0	0	3	1	4	1	5	9	0

DELAY SYSTEM

POLYNOMIAL MULTIPLICATION

$$Y(z) = z^{-1}X(z)$$

$$H(z) = z^{-1}$$

$$Y(z) = H(z)X(z)$$



DELAY PROPERTY

A delay of one sample multiplies the z -transform by z^{-1} .

$$x[n - 1] \iff z^{-1}X(z)$$

Time delay of n_0 samples multiplies the z -transform by z^{-n_0}

$$x[n - n_0] \iff z^{-n_0}X(z)$$

GENERAL I/O PROBLEM

How to combine $X(z)$ and $H(z)$?

Consider Finite-Length Inputs $x[n]$

Example 7.5

$$x[n] = \delta[n - 1] - \delta[n - 2] + \delta[n - 3] - \delta[n - 4]$$

$$\text{and } h[n] = \delta[n] + 2\delta[n - 1] + 3\delta[n - 2] + 4\delta[n - 3]$$

$$X(z) = 0 + 1z^{-1} - 1z^{-2} + 1z^{-3} - 1z^{-4}$$

$$\text{and } H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

FIR Filter = CONVOLUTION

$x[n], X(z)$	0	+1	-1	+1	-1			
$h[n], H(z)$	1	2	3	4				

	0	+1	-1	+1	-1			
		0	+2	-2	+2	-2		
			0	+3	-3	+3	-3	
				0	+4	-4	+4	-4

$y[n], Y(z)$	0	+1	+1	+2	+2	-3	+1	-4

$$y[n] = \sum_{k=0}^M b_k x[n - k] = \sum_{k=0}^M h[k] x[n - k]$$

CONVOLUTION PROPERTY

PROOF:

$$y[n] = x[n] * h[n] = \sum_{k=0}^M h[k] x[n - k]$$

$$Y(z) = \sum_{k=0}^M h[k] (z^{-k} X(z))$$

$$= \left(\sum_{k=0}^M h[k] z^{-k} \right) X(z) = H(z) X(z).$$

MULTIPLY
Z-TRANSFORMS



CONVOLUTION EXAMPLE

Finite-Length input $x[n]$

Example 7.5

$$x[n] = \delta[n - 1] - \delta[n - 2] + \delta[n - 3] - \delta[n - 4]$$

and $h[n] = \delta[n] + 2\delta[n - 1] + 3\delta[n - 2] + 4\delta[n - 3]$

$$X(z) = 0 + 1z^{-1} - 1z^{-2} + 1z^{-3} - 1z^{-4}$$

and $H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$

**MULTIPLY
Z-TRANSFORMS**

CONVOLUTION EXAMPLE

Finite-Length input $x[n]$

FIR Filter (L=4)

**MULTIPLY
Z-TRANSFORMS**

$$Y(z) = H(z)X(z)$$

$$= (1 + 2z^{-1} + 3z^{-2} + 4z^{-3})(z^{-1} - z^{-2} + z^{-3} - z^{-4})$$

$$= z^{-1} + (-1 + 2)z^{-2} + (1 - 2 + 3)z^{-3} + (-1 + 2 - 3 + 4)z^{-4}$$

$$+ (-2 + 3 - 4)z^{-5} + (-3 + 4)z^{-6} + (-4)z^{-7}$$

$$= z^{-1} + z^{-2} + 2z^{-3} + 2z^{-4} - 3z^{-5} + z^{-6} - 4z^{-7}$$

$y[n] = ?$

CASCADE SYSTEMS

Does the order of S_1 & S_2 matter?

NO, LTI SYSTEMS can be rearranged !!!

WHAT ARE THE FILTER COEFFS? $\{b_k\}$

WHAT is the FREQUENCY RESPONSE ?

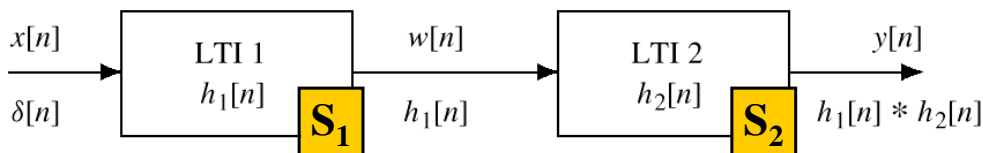
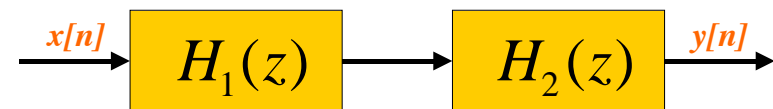


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

Multiply the System Functions



**EQUIVALENT
SYSTEM**



$$H(z) = H_1(z)H_2(z)$$