EE-2200 Fall-98

Lecture 14

Relate H(z) to Frequency Response

16-Nov-98

Info: Web-CT, Lab, HW

- Calendar:
  - Final Exam is Period 15 (ugh!)
  - Quiz Solutions are posted
- Prob Set #6 posted by tomorrow
  - One more: Prob-Set #7 due on last day
- Lab #8 on Image Processing
  - Image Zooming as a Filter

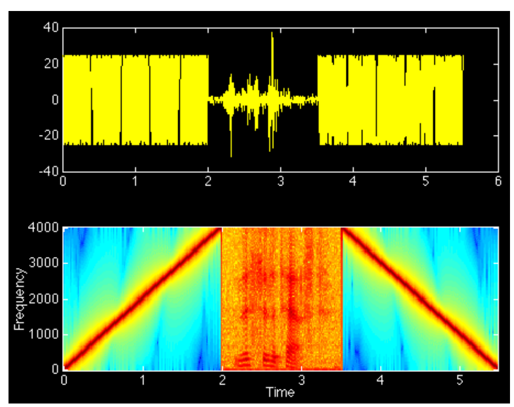
READING ASSIGNMENTS

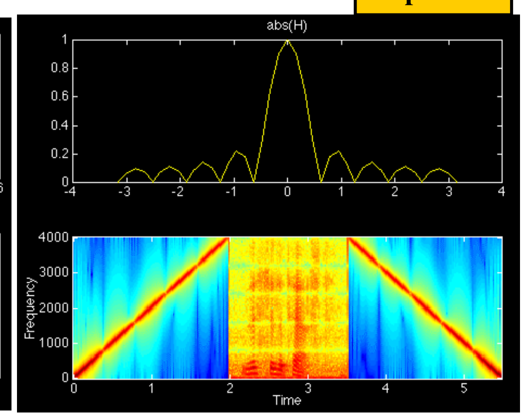
- This Lecture:
  - Chapter 7, pp. 220-230
- Other Reading:
  - Recitation: Ch. 7, pp. 217-220
    - CASCADING SYSTEMS
  - Next Lecture: Chapter 8, start

FIR Filtering Demo

- Chapter 6 Demos on CD-ROM

10-pt AVG





LECTURE OBJECTIVES

- Relate H(z) to FREQUENCY RESPONSE

$$H(\hat{\omega}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- Zeros of H(z)
- THREE DOMAINS:
  - Show Relationship for FIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

TRANSFORM CONCEPT

- Move to a new domain where
  - OPERATIONS are EASIER/FAMILIAR
  - Use POLYNOMIALS

$$H(z) = \sum_n h[n]z^{-n}$$

- TRANSFORM both ways
  - x[n] ---> X(z) (into the z domain)
  - x[n] <--- X(z) (back to the time domain)

Z-Transform DEFINITION

- POLYNOMIAL Representation

$$H(z) = \sum_n h[n]z^{-n}$$

APPLIES to Any SIGNAL

- EXAMPLE:

$$\{h[n]\} = \{2, 0, -3, 0, 2\}$$

$$H(z) = 2z^{-0} + 0z^{-1} - 3z^{-2} + 0z^{-3} + 2z^{-4}$$

$$= 2 - 3z^{-2} + 2z^{-4}$$

Z-Transform of FIR Filter

- h[n] is same as $\{b_k\}$

SYSTEM FUNCTION

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \sum_{k=0}^{M} h[k] z^{-k}$$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

FIR DIFFERENCE EQUATION

CONVOLUTION

DELAY PROPERTY

A delay of one sample multiplies the z-transform by z^{-1}.

$$x[n-1] \quad \Longleftrightarrow \quad z^{-1}X(z)$$

Time delay of n_0 *samples multiplies the z-transform by* z^{-n_0}

$$x[n-n_0] \quad \Longleftrightarrow \quad z^{-n_0}X(z)$$

CONVOLUTION PROPERTY

- Convolution in the n–domain
  - SAME AS
- Multiplication in the z–domain

$$y[n] = h[n] * x[n] \quad \Longleftrightarrow \quad Y(z) = H(z)X(z)$$

MULTIPLY Z–TRANSFORMS

$$y[n] = x[n] * h[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR Filter

CONVOLUTION PROPERTY

- PROOF:

$$y[n] = x[n] * h[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

$$Y(z) = \sum_{k=0}^{M} h[k]\left(z^{-k}X(z)\right)$$

$$= \left(\sum_{k=0}^{M} h[k]z^{-k}\right)X(z) = H(z)X(z).$$

MULTIPLY Z-TRANSFORMS

CONVOLUTION EXAMPLE

- Finite–Length input x[n]

Example 7.5

$$x[n] = \delta[n-1] - \delta[n-2] + \delta[n-3] - \delta[n-4]$$

and $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$

$$X(z) = 0 + 1z^{-1} - 1z^{-2} + 1z^{-3} - 1z^{-4}$$

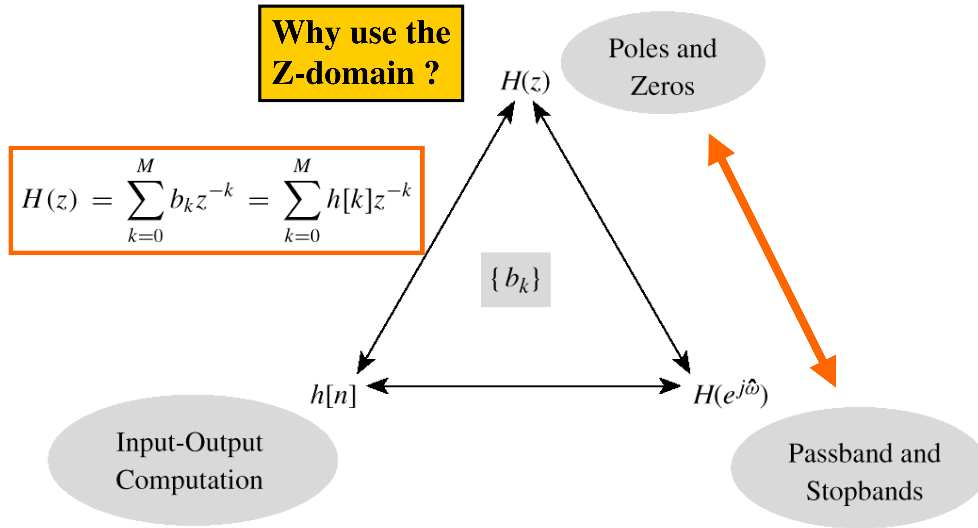
and $H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$

MULTIPLY Z-TRANSFORMS

ANOTHER ANALYSIS TOOL

- Why use the z–Transform ?
- The z–domain is **COMPLEX**
- H(z) is a **COMPLEX**–VALUED function of a **COMPLEX** VARIABLE z.

THREE DOMAINS



Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{b_k\}$ play a central role.

FREQUENCY RESPONSE ?

- Same Form: $z = e^{j\hat{\omega}}$

$\hat{\omega}$-Domain $\Longleftrightarrow$ z-Domain

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \Longleftrightarrow H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

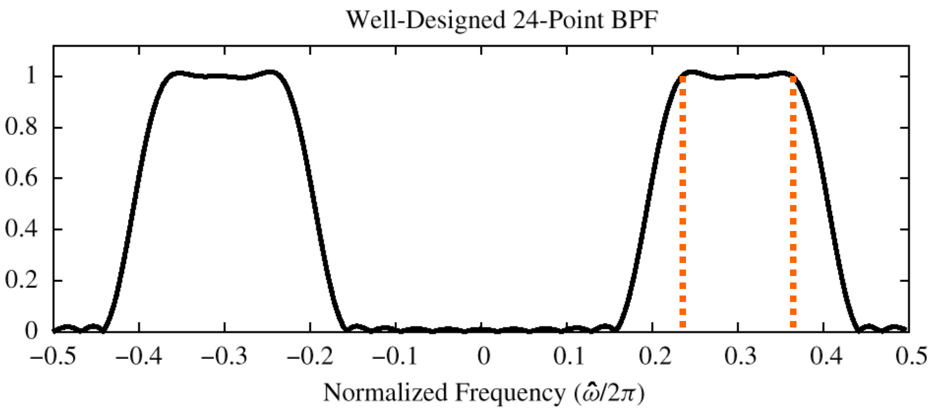
SAME COEFFICIENTS

WHY USE the z-DOMAIN ?

- FILTER DESIGN
  - Derive $\{b_k\}$ to **SYNTHESIZE** a good filter
  - The $\{b_k\}$ are **POLYNOMIAL** coefficients
  - Design via the Polynomial **ROOTS**
- EXAMPLE:
  - Design a Bandpass Filter with
  - Passband Spec: $0.48\pi \le \hat{\omega} \le 0.72\pi$

Filter Design SOFTWARE

- **FILTER DESIGN EXAMPLE (L=24):**
  - **BPF with Passband:** $0.48\pi \le \hat{\omega} \le 0.72\pi$



ROOTS of H(z)

$$H(z) = (1 - z^{-1})(1 - z^{-1} + z^{-2})$$

$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$, the roots are

$$z_1 = 1$$

$$z_2 = \tfrac{1}{2} + j\tfrac{1}{2}\sqrt{3} = 1e^{j\pi/3}$$

$$z_3 = \tfrac{1}{2} - j\tfrac{1}{2}\sqrt{3} = 1e^{-j\pi/3}$$

FACTORING H(z)

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$= (1 - z^{-1})(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})$$

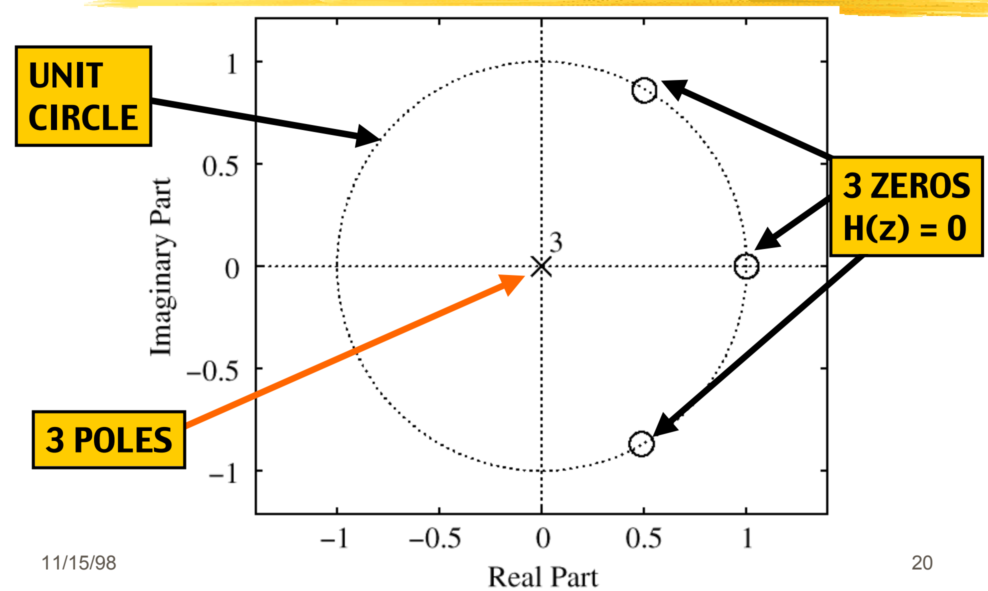
if we multiply $H(z)$ by z^3/z^3,

we obtain the following two equivalent forms:

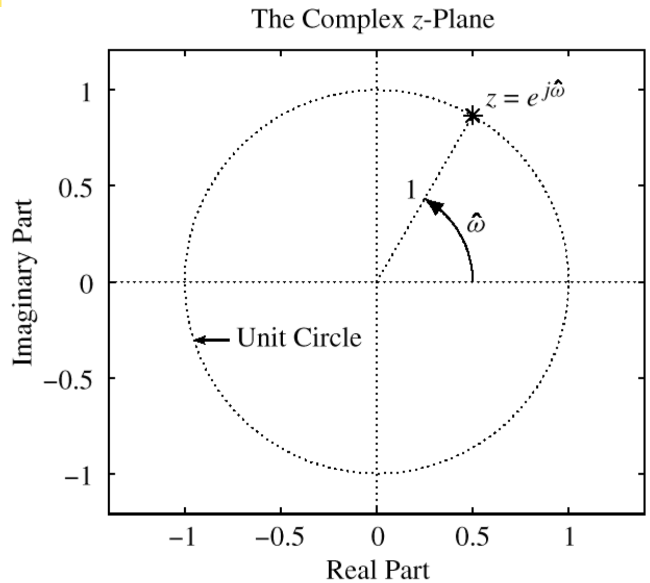
$$H(z) = \frac{z^3 - 2z^2 + 2z - 1}{z^3}$$

$$= \frac{(z - 1)(z - e^{j\pi/3})(z - e^{-j\pi/3})}{z^3}$$

PLOT ZEROS in z-DOMAIN



$$\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$



CHANGE in NOTATION

- **Relate H(z) to FREQUENCY RESPONSE**

$$H(\hat{\omega}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- **NOTATION for FREQUENCY RESPONSE**

$$H(\hat{\omega}) \leftrightarrow H(e^{j\hat{\omega}})$$

NULLING FILTER

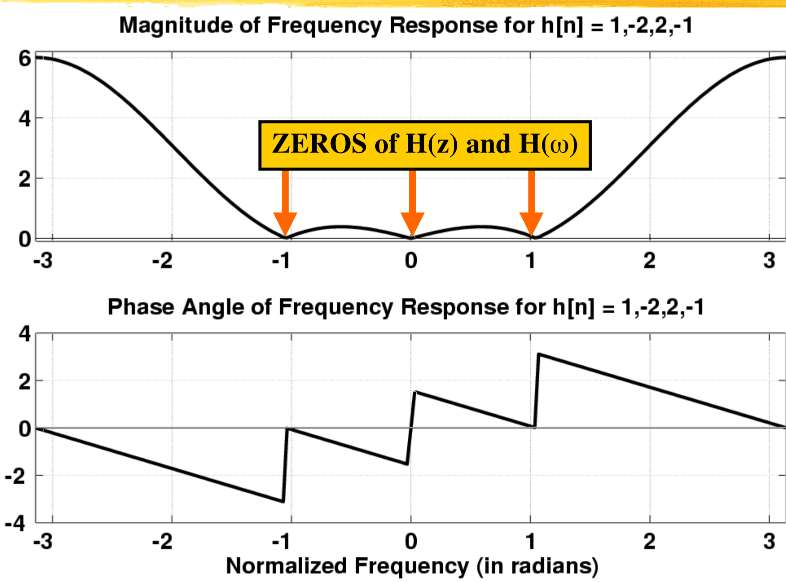
- **PLACE ZEROS to make y[n] = 0**

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$ ← **3 ZEROS H(z) = 0**

the output resulting from each of the following three signals will be zero:

$H(z_1) = 0$	$x_1[n] = (z_1)^n = 1$ →	$y_1[n] = 0$
$H(z_2) = 0$	$x_2[n] = (z_2)^n = e^{j\pi n/3}$	$y_2[n] = 0$
$H(z_3) = 0$	$x_3[n] = (z_3)^n = e^{-j\pi n/3}$	$y_3[n] = 0$

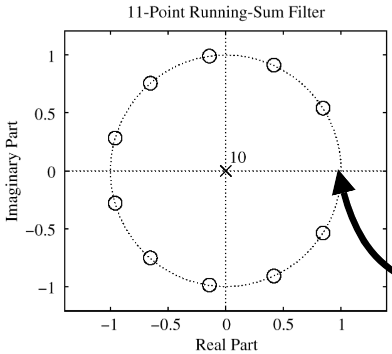
FIR Frequency Response

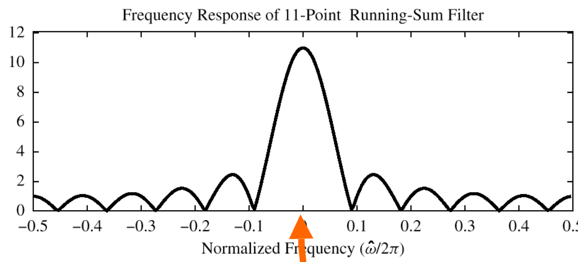


11-pt RUNNING SUM H(z)

$$H(z) = \sum_{k=0}^{10} z^{-k}$$

$$H(z) = (1 - e^{j2\pi/11}z^{-1})(1 - e^{j4\pi/11}z^{-1})\cdots(1 - e^{j20\pi/11}z^{-1})$$





NO zero at z=1

L-pt RUNNING SUM H(z)

$$H(z) = \sum_{k=0}^{L-1} z^{-k} = \frac{1 - z^{-L}}{1 - z^{-1}} = \frac{z^L - 1}{z^{L-1}(z-1)}$$

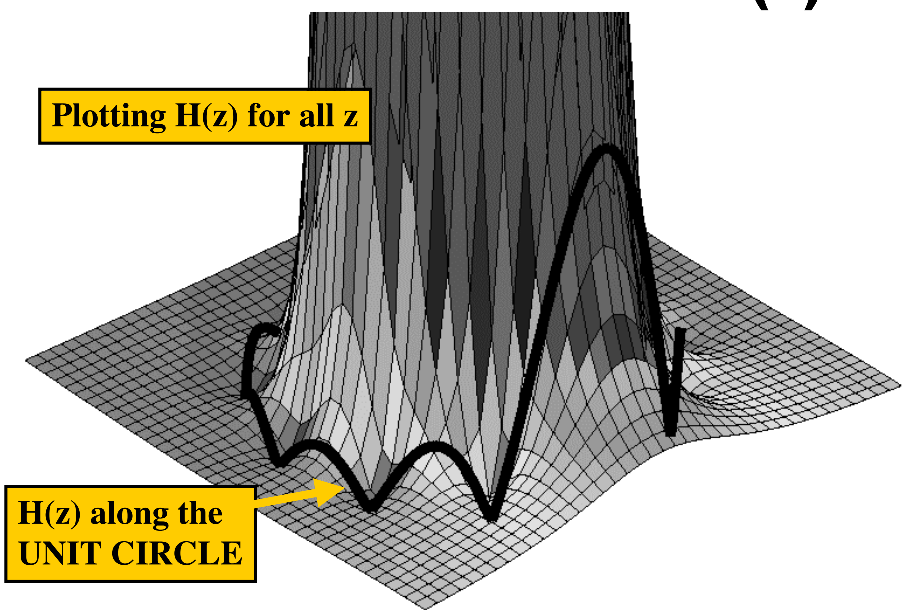
$$z^L - 1 = 0 \quad \Longrightarrow z^L = 1$$

$$z = e^{j2\pi k/L} \quad \text{for } k = 0, 1, 2, \ldots, L-1$$

ZEROS on UNIT CIRCLE

Numerator has (z−1) term when k=0

COMPLEX FUNCTION: H(z)



POP QUIZ

- Given: $H(\hat{\omega}) = e^{-j\hat{\omega}} \cos(\hat{\omega})$
- Find the Impulse Response, h[n]
- Find the output, y[n]
  - When $x[n] = \cos(0.25\pi n)$