

EE-2200

Lecture 2 Complex Exponentials 28-Sept-98

INFORMATION

- LABS start this week
 - Room 309 in CoC Building
 - Get you computer acct ASAP
 - Attend correct section
 - Verification must be signed during Lab
- RECITATIONS
 - Attend your assigned time

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Introduction to Discrete Systems

Autumn 1998

Lecture Time: M & F 11:05-11:55

Room: W200 Van Leer (Auditorium)

Instructor: [Dr. Jim McClellan](#)

Email: jim.mcclellan@ece.gatech.edu

Office: E475-C Van Leer, or 363 GCATT **Phone:** (404) 894-8325

Office Hours: Tu-Th 12:00-2:00p; F 12:00-1:00p, or by appointment

For Recitation instructors and TAs, please refer to the [Course Information and Help](#) page below.



information

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lab

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calendar

[Calendar of Course Events](#)

[Grader Student Management](#)



quiz

[Online Quizzes and Surveys](#)



tools

[Course Tools and Other Useful Links](#)

REMINDERS

- Web-CT Password:
 - SSN(4:8), 4th thru 8th digits of SSN
 - We are still updating the class lists
- Hard copy of Instructor Verification Sheet
 - Get PDF file of Lab#1 from WebCT
 - Lab #1 is different from the book
- Homework #1 will be posted on **Tuesday**

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 2, pp. 17–32
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chapter 2, pp. 31–43

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LECTURE OBJECTIVES

- Relate TIME–SHIFT to PHASE
- Introduce an ABSTRACTION:
 - Complex Numbers **represent** Sinusoids
 - Complex Exponential Signal

$$z(t) = Ze^{j\omega t}$$

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SINUSOIDAL SIGNAL

$$A \cos(\omega t + \varphi)$$

- FREQUENCY ω
 - Radians/sec
 - Hertz (cycles/sec)
 - $\omega = (2\pi)f$
- AMPLITUDE A
 - Magnitude
- PERIOD (in sec)
 - $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- PHASE φ

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TIME-SHIFT

- In a mathematical formula replace t with $t-t_1$
- For example, $x(t-t_1) = \cos(\omega(t-t_1))$
- Then the $t=0$ point moves to $t=t_1$
 - $x(t-t_1) = A \cos(\omega(t-t_1))$
- Peak value of $\cos(\omega(t-t_1))$ is at $t=t_1$

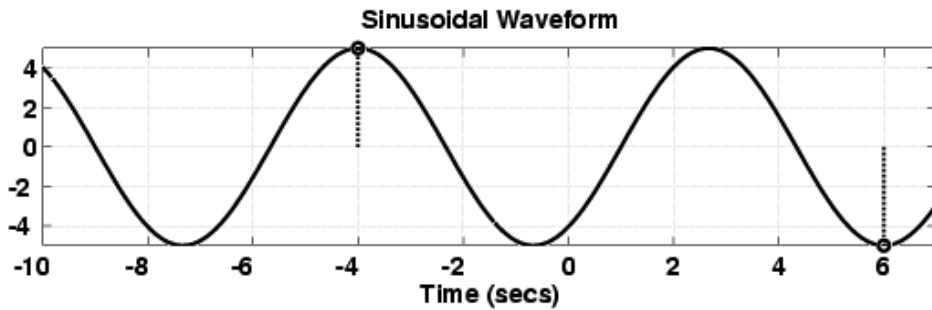
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TIME-SHIFTED SINUSOID

$$x(t) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t-(-4)))$$



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PHASE <--> TIME-SHIFT

- Equating the formulas:

$$A \cos(\omega(t - t_1)) = A \cos(\omega t + \phi)$$

- and we obtain: $-\omega t_1 = \phi$

- or,
$$t_1 = \frac{-\phi}{\omega}$$

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EXAMPLE: Phase from Time-Shift

- Frequency: $\omega = 30\pi$
- Phase: $\phi = -0.2\pi$
- What is the time shift?
 - Also called the “time delay”
 - $t_1 = -(-0.2\pi)/30\pi$
 - $t_1 = 1/150$ sec.
 - $T = 1/15$ sec. (period)

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PHASE is AMBIGUOUS

- The cosine signal is periodic

- Period is 2π

$$A \cos(\omega t + \phi + 2\pi) = A \cos(\omega t + \phi)$$

- Thus adding any multiple of 2π leaves $x(t)$ unchanged

- How much does t_1 change, when phase changes by 2π ?
$$t_1 = \frac{-\phi}{\omega} = \left(\frac{-\phi}{2\pi}\right)T$$

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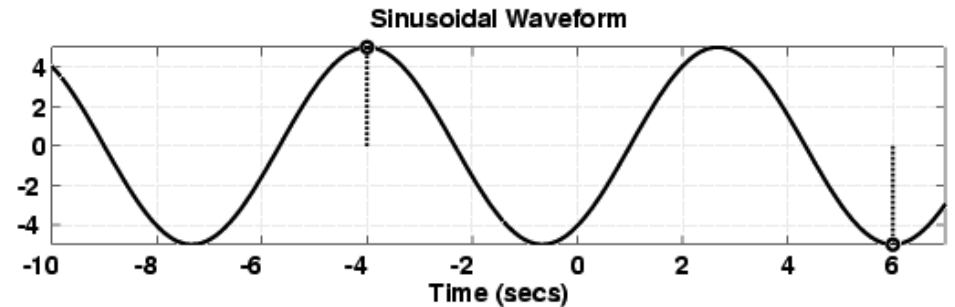
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SINUSOID from a PLOT

- Measure the period
 - Between peaks or zero crossings
 - Compute frequency: $2\pi/T$
- Measure time of peak: t_1
 - Compute phase: $\phi = -\omega t_1$
- Measure height of positive peak: A

(A, ω , ϕ) from a PLOT



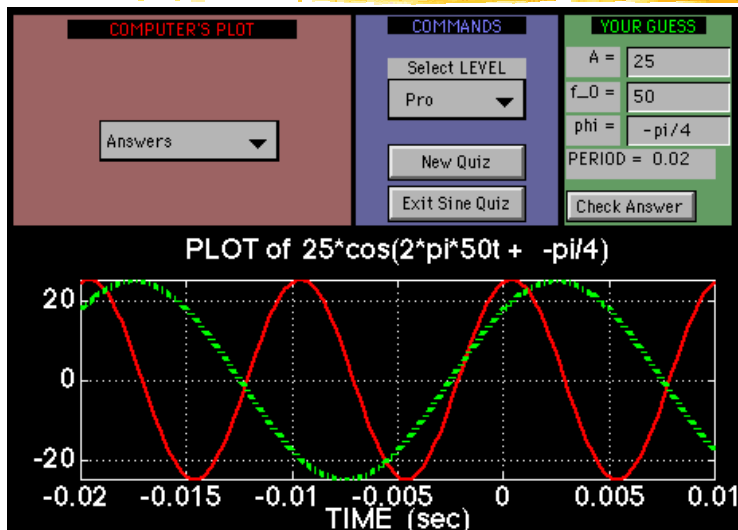
$$T = 10 / (1.5) = 20/3$$

$$t_1 = -4$$

$$\omega = 2\pi/T = 0.3\pi$$

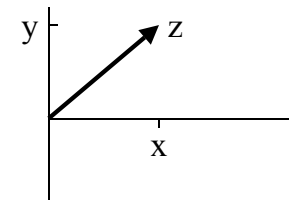
$$\phi = -(-4)(0.3\pi) = 1.2\pi$$

SINE DRILL



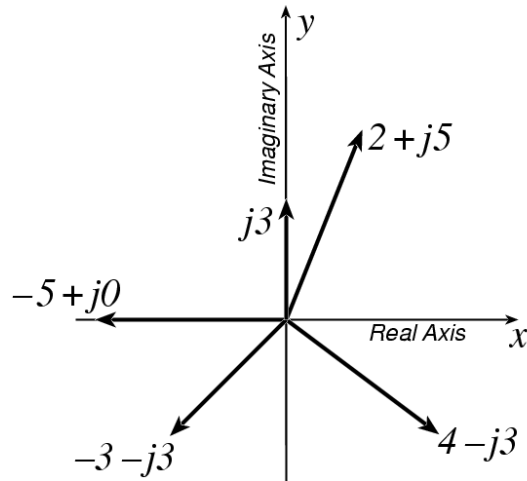
COMPLEX NUMBERS

- To solve: $z^2 = -1$
 - $z = j$
 - Math and Physics use $z = i$
- Complex number: $z = x + jy$



Cartesian
coordinate
system

EX: COMPLEX NUMBERS



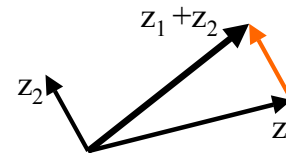
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ADD COMPLEX NUMBERS

- VECTOR Addition is necessary



- Example: $z = 3 + j2$, $w = -1 - j4$

- $z + w = (3 - 1) + j(2 - 4) = 2 - j2$

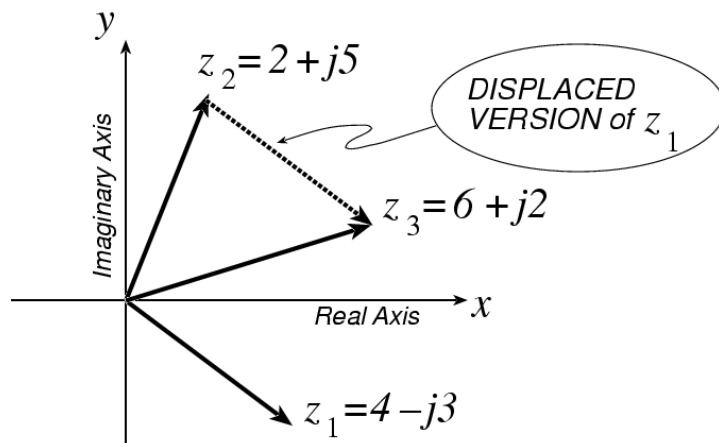
- Adding sinusoids = adding complex nos.

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EX: COMPLEX ADDITION



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POLAR FORM

- Vector Form

- Length = 1

- Angle = θ

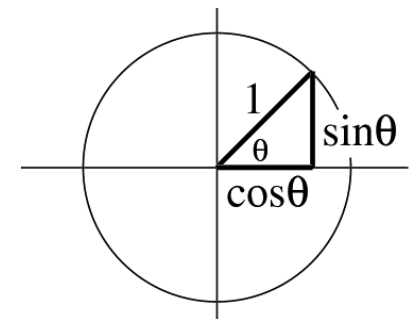
- Common Values

- j has angle of 0.5π

- -1 has angle of π

- $-j$ has angle of 1.5π

- or, the angle is -0.5π



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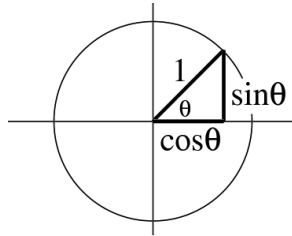
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Euler's FORMULA

Complex Exponential

- Real part is cosine
- Imaginary part is sine
- Magnitude is one



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

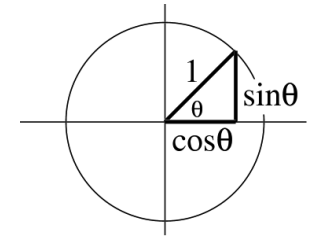
$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

Rotating Vector

- Angle changes vs. time
- $\theta = \omega t$
- ex: $\omega = 10\pi$
- Rotates 0.1π in **0.01** secs



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

Cos = REAL PART

Real Part of Euler's:

$$\cos(\omega t) = \Re\{e^{j\omega t}\}$$

General Sinusoid

$$x(t) = A \cos(\omega t + \varphi)$$

So,

$$A \cos(\omega t + \varphi) = \Re\{Ae^{j(\omega t + \varphi)}\}$$
$$= \Re\{Ae^{j\varphi} e^{j\omega t}\}$$

COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A \cos(\omega t + \varphi) = \Re\{Ae^{j\varphi} e^{j\omega t}\}$$

Complex Exponential

$$z(t) = Ze^{j\omega t} \quad Z = Ae^{j\varphi}$$

Sinusoid is REAL PART of $e^{j\omega t}$

$$x(t) = \Re\{z(t)\} = \Re\{Ze^{j\omega t}\}$$