EE-2200

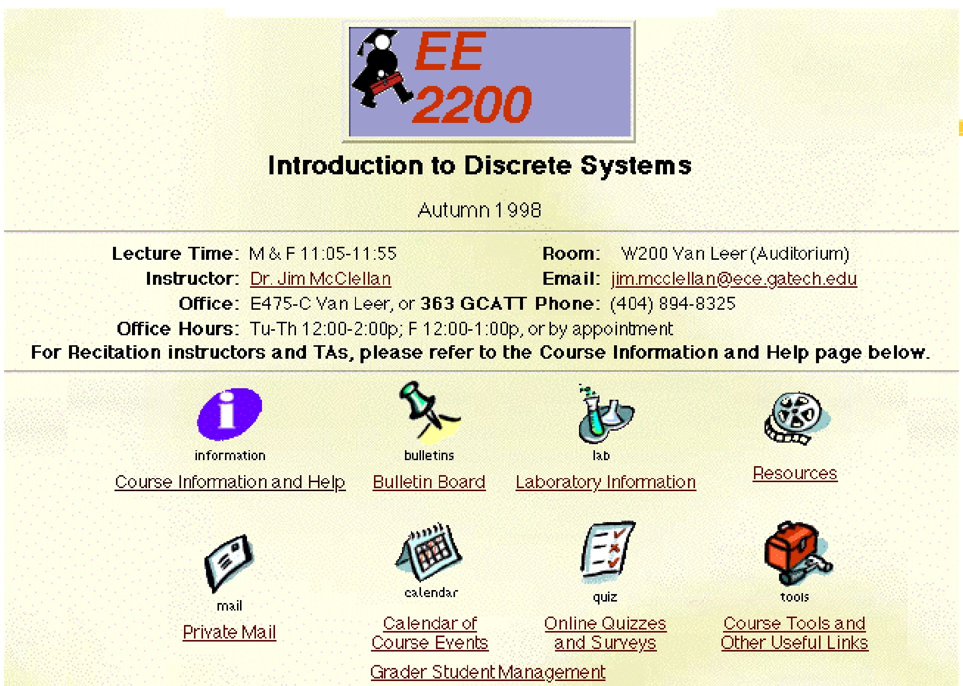
Lecture 2

Complex Exponentials

28-Sept-98

INFORMATION

- LABS start this week
  - Room 309 in CoC Building
  - Get you computer acct ASAP
  - Attend correct section
  - Verification must be signed during Lab
- RECITATIONS
  - Attend your assigned time



REMINDERS

- Web-CT Password:
  - SSN(4:8), 4th thru 8th digits of SSN
  - We are still updating the class lists
- Hard copy of Instructor Verification Sheet
  - Get PDF file of Lab#1 from WebCT
  - Lab #1 is different from the book
- Homework #1 will be posted on Tuesday

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 17–32
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chapter 2, pp. 31–43

LECTURE OBJECTIVES

- Relate TIME–SHIFT to PHASE
- Introduce an ABSTRACTION:
  - Complex Numbers represent Sinusoids
  - Complex Exponential Signal

$$z(t) = Ze^{j\omega t}$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t + \varphi)$$

- FREQUENCY ω
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega = (2\pi)f$$

- PERIOD (in sec)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

- AMPLITUDE A
  - Magnitude
- PHASE φ

TIME-SHIFT

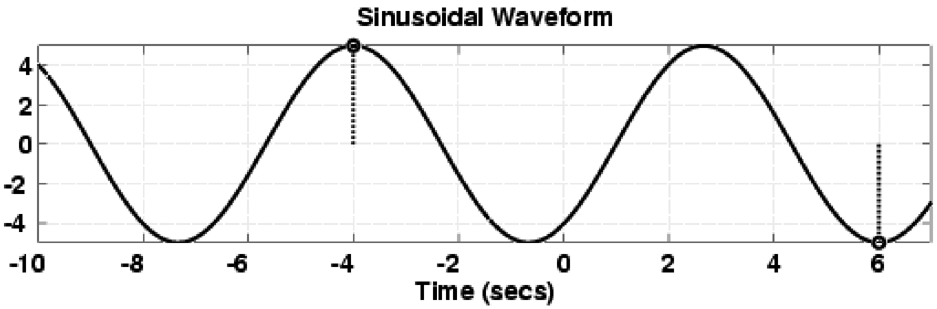
- In a mathematical formula replace t with $t-t_1$
- For example, $x(t-t_1) = \cos(\omega(t-t_1))$
- Then the t=0 point moves to $t=t_1$

$$x(t-t_1) = A\cos(\omega(t-t_1))$$

- Peak value of $\cos(\omega(t-t_1))$ is at $t=t_1$

TIME-SHIFTED SINUSOID

$x(t) = 5\cos(0.3\pi(t{+}4)) = 5\cos(0.3\pi(t{-}({-}4)))$



PHASE <--> TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t - t_1)) = A\cos(\omega t + \varphi)$$

- and we obtain: $-\omega t_1 = \varphi$
- or, $$t_1 = \frac{-\varphi}{\omega}$$

EXAMPLE: Phase from Time-Shift

- Frequency : $\omega = 30\pi$
- Phase: $\phi = -0.2\pi$
- What is the time shift?
  - Also called the **"time delay"**
  - $t_1 = -(-0.2\pi)/30\pi$
  - $t_1 = 1/150$ sec.
  - T = 1/15 sec. (period)

PHASE is AMBIGUOUS

- The cosine signal is periodic
  - Period is 2π

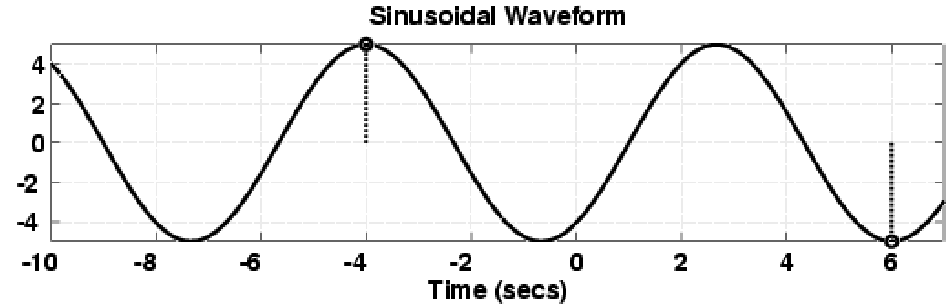
$$A\cos(\omega t + \varphi + 2\pi) = A\cos(\omega t + \varphi)$$

  - Thus adding any multiple of 2π leaves x(t) unchanged
  - How much does t_1 change, when phase changes by 2π ? $t_1 = \frac{-\varphi}{\omega} = \left(\frac{-\varphi}{2\pi}\right)T$

SINUSOID from a PLOT

- Measure the period
  - Between peaks or zero crossings
  - Compute frequency: $2\pi/T$
- Measure time of peak: t_1
  - Compute phase: $\phi = -\omega\, t_1$
- Measure height of positive peak: A

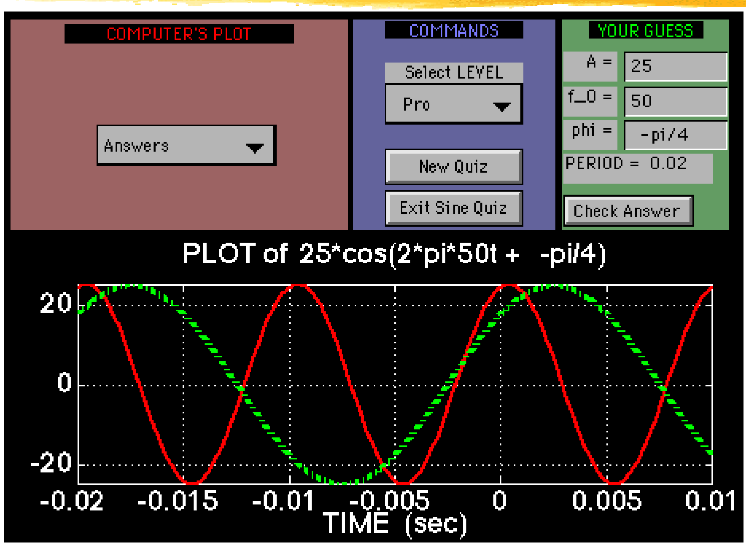
(A, ω, φ) from a PLOT



$T = 10/(1.5) = 20/3$
$t_1 = -4$

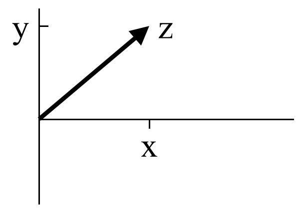
$\omega = 2\pi/T = 0.3\pi$
$\phi = -(-4)(0.3\pi) = 1.2\pi$

SINE DRILL



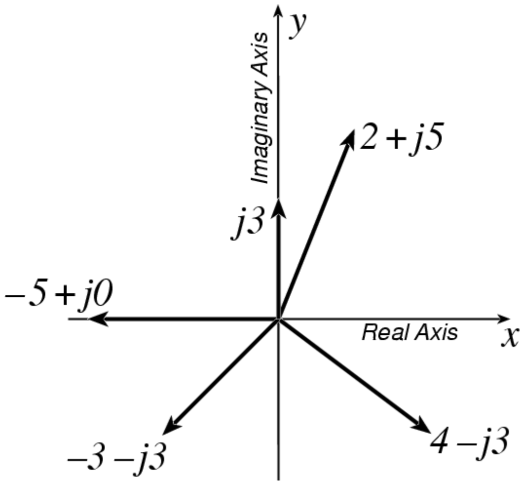
COMPLEX NUMBERS

- To solve: $z^2 = -1$
  - $z = j$
  - Math and Physics use $z = i$
- Complex number: $z = x + jy$



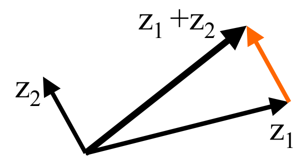
Cartesian coordinate system

EX: COMPLEX NUMBERS



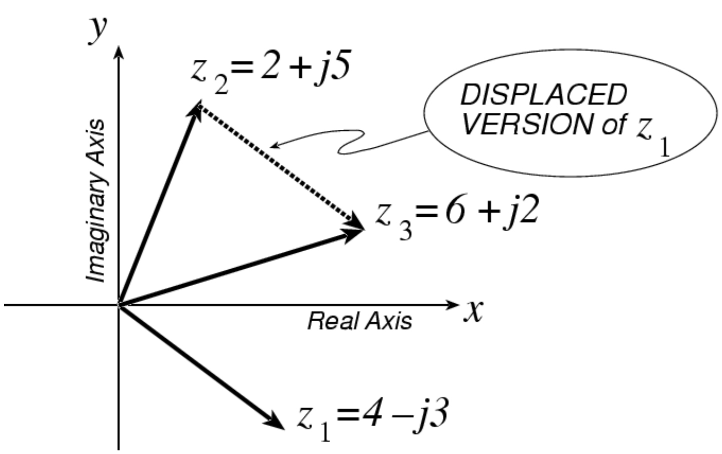
ADD COMPLEX NUMBERS

- **VECTOR Addition is necessary**



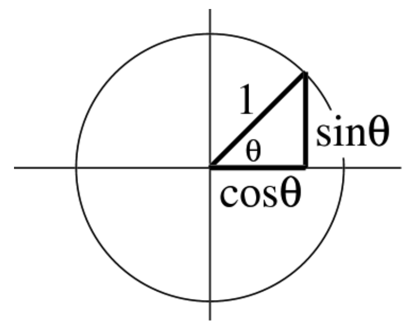
- **Example: z = 3+j2, w = −1−j4**
  - **z+w = (3−1) + j(2−4) = 2 −j2**
- **Adding sinusoids = adding complex nos.**

EX: COMPLEX ADDITION



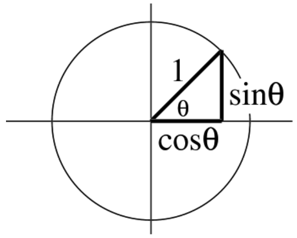
POLAR FORM

- **Vector Form**
  - **Length =1**
  - **Angle =** θ
- **Common Values**
  - j **has angle of** 0.5π
  - -1 **has angle of** π
  - -j **has angle of** 1.5π
  - **or, the angle is** -0.5π



Euler's FORMULA

- Complex Exponential
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



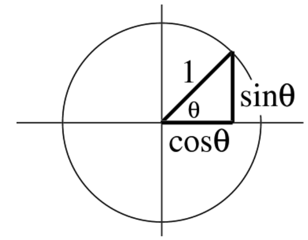
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Rotating Vector
  - Angle changes vs. time
  - $\theta = \omega t$
  - ex: $\omega = 10\pi$
  - Rotates 0.1π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Cos = REAL PART

- Real Part of Euler's:

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

- So,

$$A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t + \varphi)}\}$$

$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

COMPLEX AMPLITUDE

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

- Complex Exponential

$$z(t) = Ze^{j\omega t} \qquad Z = Ae^{j\varphi}$$

- Sinusoid is REAL PART of $e^{j\omega t}$

$$x(t) = \Re e\{z(t)\} = \Re e\{Ze^{j\omega t}\}$$