EE-2200

Lecture 3

Phasor Addition Theorem

2-Oct-98

Web-CT Info

- Check the Bulletin Board for msgs
- Lectures are being posted
  - HTML format
  - PDF format (4 per page)
- Calendar has entries:
  - Quiz #1 on 16-Oct (Friday)
  - Quiz #2 on 13-Nov (Friday)

Homework Info

- Prob Set #1 due next Monday
  - In Lecture, before NOON
- HW will be posted on Tuesday
  - Due the following Monday
- Solutions will be posted on Tues
  - HW #0 is on the web site

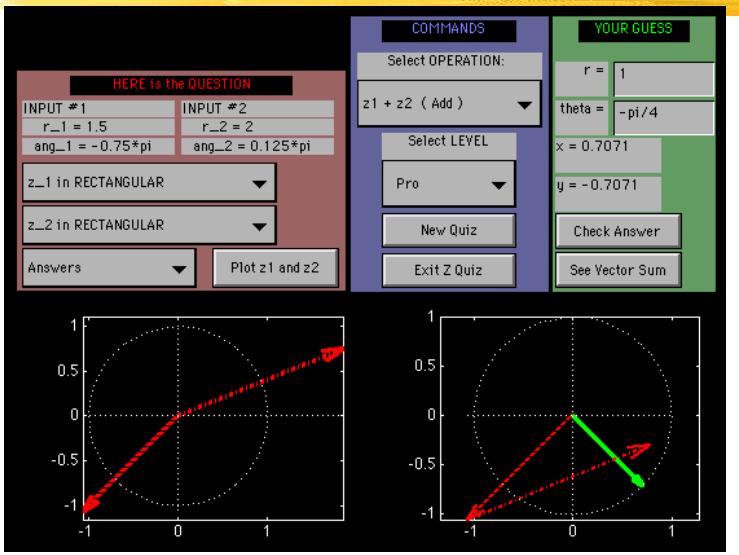
Lab Info

- Lab #1 Report
  - Turn in during your lab time
  - Write-up sections 2 and 3
  - Include INSTRUCTOR VERIFICATION
- Next Lab will be posted on Friday
  - Lab #2 later today

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 31–43
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: start Chapter 3

Z DRILL (Complex Arith)



LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Add Sinusoids = Complex Addition
  - PHASOR ADDITION THEOREM

$$z(t) = Ze^{j\omega t} = (Ae^{j\varphi})e^{j\omega t}$$

- Develop the ABSTRACTION:
  - Complex Numbers represent Sinusoids

AVOID Trigonometry

- Algebra, even complex, is easier
- Can you recall $\cos(\theta_1+\theta_2)$?
- Use the real part of $e^{j\theta_1}\, e^{j\theta_2}$

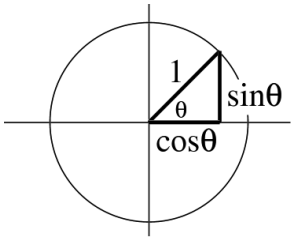
$$e^{j(\theta_1+\theta_2)} = e^{j\theta_1}e^{j\theta_2}$$

$$= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$

$$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + j(\ldots)$$

Euler's FORMULA

- Complex Exponential
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



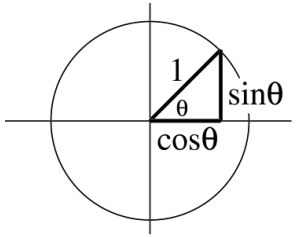
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\boxed{e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)}$$

COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

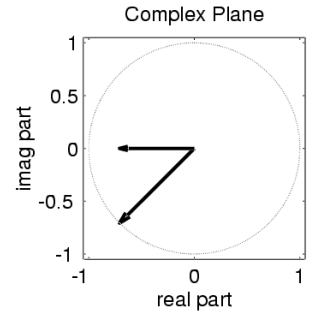
- Rotating Vector
  - Angle changes vs. time
  - $\theta = \omega t$
  - ex: $\omega = 10\pi$
  - Rotates 0.1π in 0.01 secs

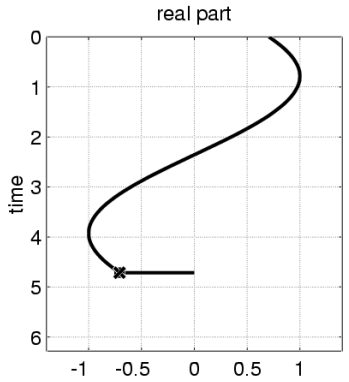


$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Rotating Phasor

See Demo on CD-ROM Chapter 2





Cos = REAL PART

- Real Part of Euler's:

$$\cos(\omega t) = \Re e\left\{e^{j\omega t}\right\}$$

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

- So,

$$A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j(\omega t + \varphi)}\right\}$$

$$= \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

COMPLEX AMPLITUDE

- **General Sinusoid**

$$x(t) = A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

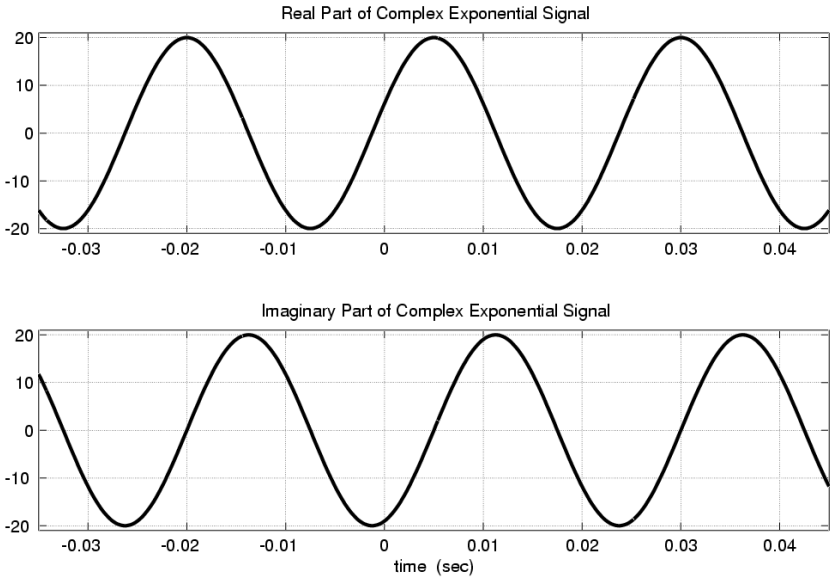
- **Complex Exponential**

$$z(t) = Ze^{j\omega t}$$

- **The COMPLEX AMPLITUDE is**

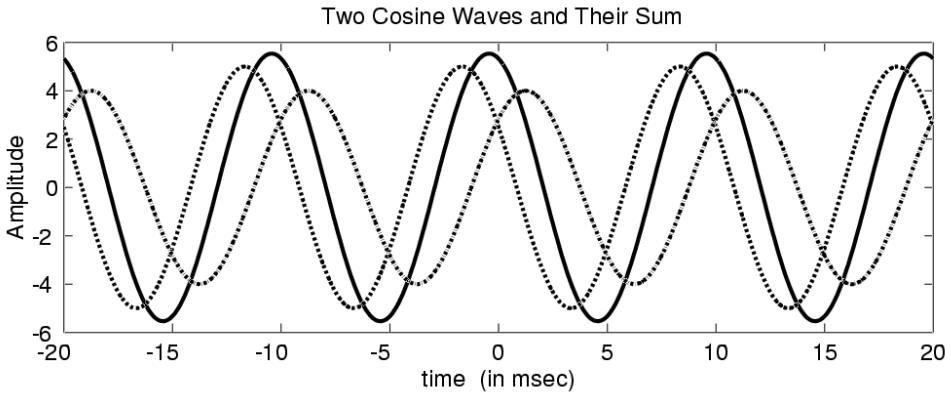
$$Z = Ae^{j\varphi}$$

Real & Imaginary Part Plots



ADD SINUSOIDS

- **Sum Sinusoid has same Frequency**



PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k)$$

$$= A\cos(\omega_0 t + \phi)$$

Get the new complex amplitude by addition

$$\sum_{k=1}^{N} A_k e^{j\phi_k} = Ae^{j\phi}$$

Phasor Addition Proof

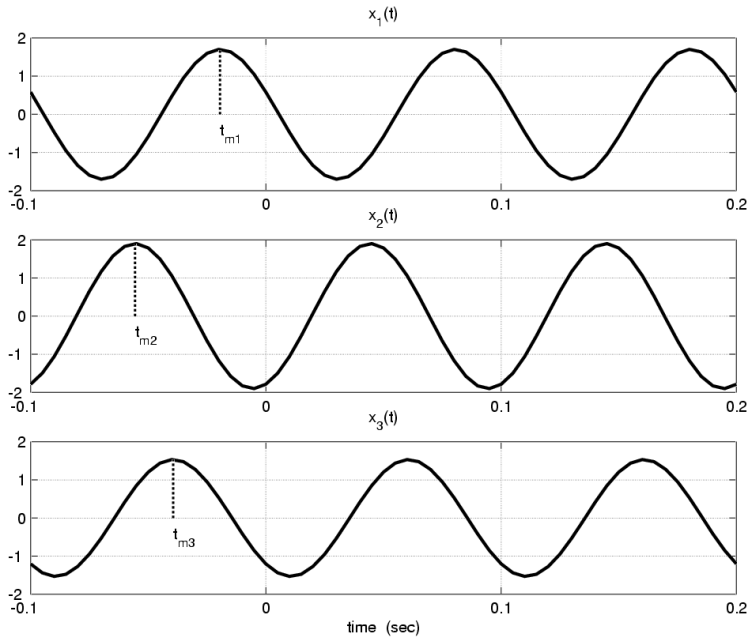
$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t+\phi_k)}\right\}$$

$$= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(A e^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)$$

ADD SINUSOIDS EXAMPLE

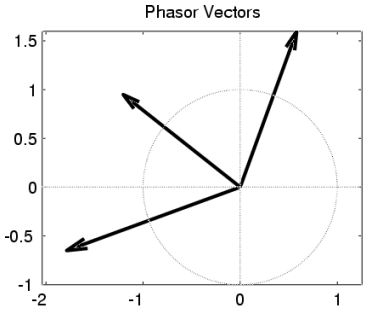


Convert Time-Shift to Phase

- Measure peak times:
  - t_{m1}=−0.0194, t_{m2}=−0.0556, t_{m3}=−0.0394
- Convert to phase (T=0.1)
  - ϕ_1=−2π(t_{m1} /T)=70π/180, ϕ_2= 200π/180
- Amplitudes
  - A_1=1.7, A_2=1.9, A_3=1.532

Phasor Add: Numerical

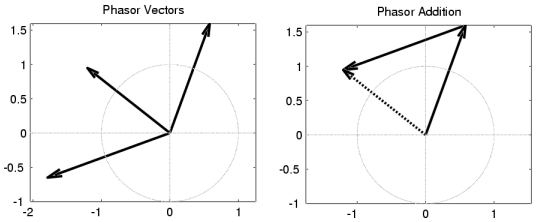
- Convert Polar to Cartesian
  - X_1 = 0.5814 + j1.597
  - X_2 = −1.785 − j0.6498
  - X_3 = −1.204 + j0.9476



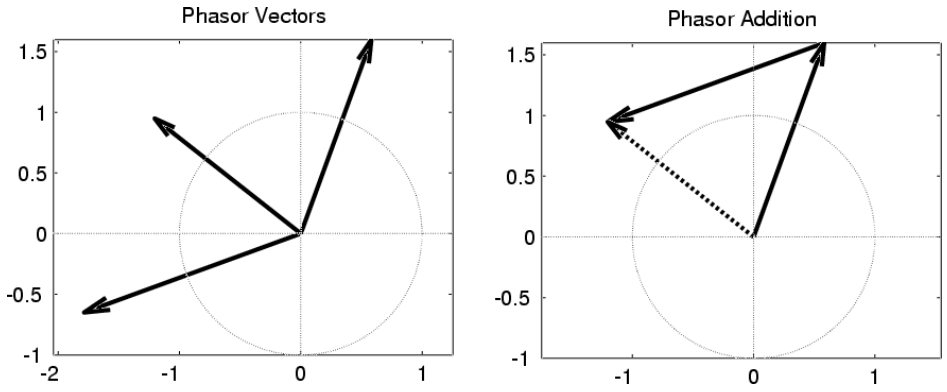
- Convert back to Polar
  - X_3 = 1.532 at angle 141.79π/180

ADD SINUSOIDS

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$
$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

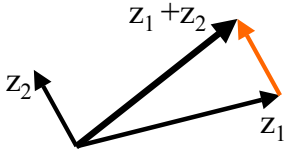


PHASOR (VECTOR) ADD



ADD COMPLEX NUMBERS

- **VECTOR Addition is necessary**



- **Example: z = 3+j2, w = −1−j4**
  - **z+w = (3−1) + j(2−4) = 2 −j2**
- ***Adding sinusoids = complex addition***

EX: COMPLEX ADDITION