EE-2200 Fall-98

Lecture 6

Time-Varying Frequency

12-Oct-98

Web-CT Info

- Check the Bulletin Board for msgs
- Quiz Review on Wed (EE Aud @ 11am)
- Calendar has entries:
  - Quiz #1 on 16-Oct (Friday)
  - Quiz #2 on 13-Nov (Friday)

Homework Info

- Prob Set #2 due TODAY
  - In Lecture, before NOON
- Solutions will be posted on Tues
  - HW #1 solutions are on the web site

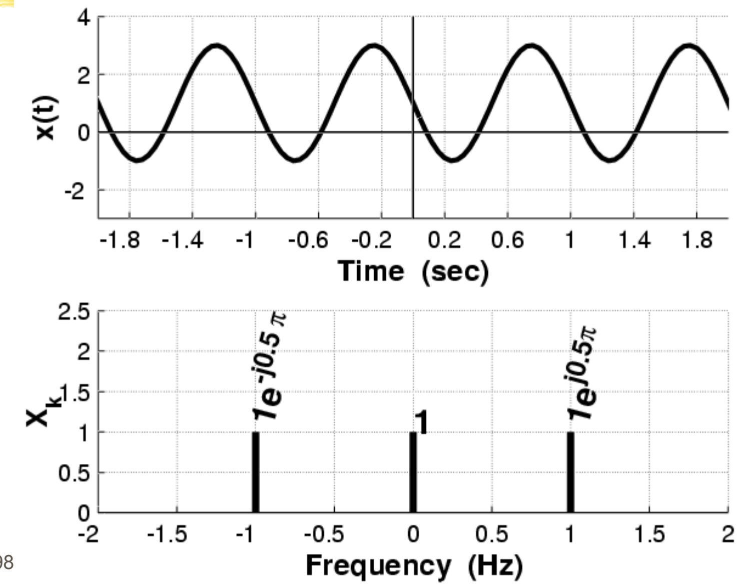
Lab Info

- Lab #2 Report
  - Turn in during your lab time
  - Write-up sections 4 and 5
  - Include INSTRUCTOR VERIFICATION
- Lab #3 was posted on Friday
  - Music Synthesis Lab (learn notation)

READING ASSIGNMENTS

- This Lecture:
  - Chapter 3, pp. 68–77
- Other Reading:
  - Notes on Fourier Series
    - (3 pages posted to WebCT)
  - Next Lecture: start Chapter 4

X(t) <---> X_k ?



LECTURE OBJECTIVES

- Frequency can change vs. time
  - Basis of Frequency Modulation (FM)
  - Define "instantaneous frequency"
- Chirp Signals (LFM)
  - Quadratic phase

$$x(t) = A\cos(\alpha t^2 + 2\pi f_0 t + \varphi)$$

Time-Varying FREQUENCIES Diagram

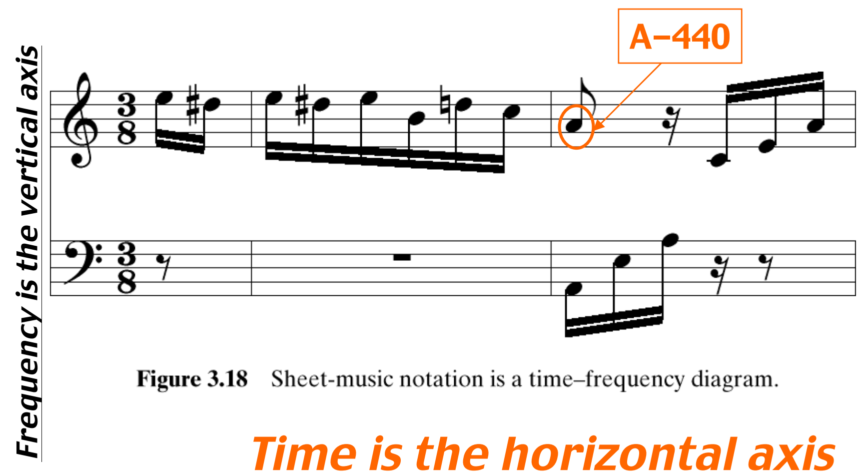


Figure 3.18 Sheet-music notation is a time–frequency diagram.

Time is the horizontal axis

Fourier Series Expansion

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$\boxed{X_k = A_k e^{j\varphi_k}}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{N} \tfrac{1}{2} X_{-k} e^{-j2\pi k f_0 t}$$

$N \to \infty$???

Fourier Series Integral

- Determine X_k from x(t)

$$X_k = \frac{2}{T_0} \int_0^{T_0} x(t) e^{-j2\pi kt/T_0} \, dt$$

$\boxed{f_0 = 1/T_0}$

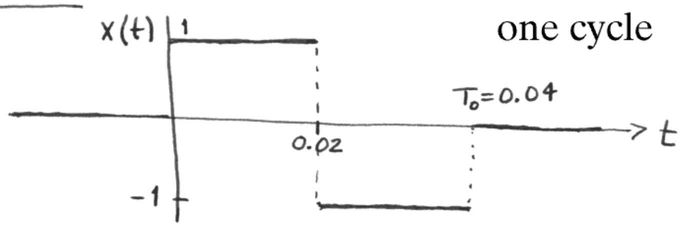
$$X_0 = \frac{1}{T_0} \int_0^{T_0} x(t) \, dt$$

SQUARE WAVE (50% duty cycle)

$$x(t) = \begin{cases} 1 & 0 \le t < \frac{1}{2}T_0 \\ -1 & \frac{1}{2}T_0 \le t < T_0 \end{cases} \qquad (3.4.4)$$

Draw a plot of the square wave defined in (3.4.4) for $T_0 = 0.04$ sec.

Ex 3.3



FS for 50% SQUARE WAVE

- Only the ODD HARMONIC are present
  - Depends on 50% duty cycle !
  - Phase is $\pi/2$ for k>0; $-\pi/2$ for k<0

$$X_k = \begin{cases} \dfrac{4}{j\pi k} & k = \pm 1, \pm 3, \pm 5, \ldots \\ 0 & k = 0, \pm 2, \pm 4, \pm 6, \ldots \end{cases} \qquad (3.4.5)$$

Fourier Series Spectrum

The magnitude of these coefficients is shown in Fig. 3.12. The phase angles are $-\pi/2$ for $k > 0$, and $\pi/2$ for $k < 0$. Note that if $f_0 = 1/T_0 = 25$ Hz, only the frequencies at ± 25, ± 75, ± 125, etc. are in the spectrum.

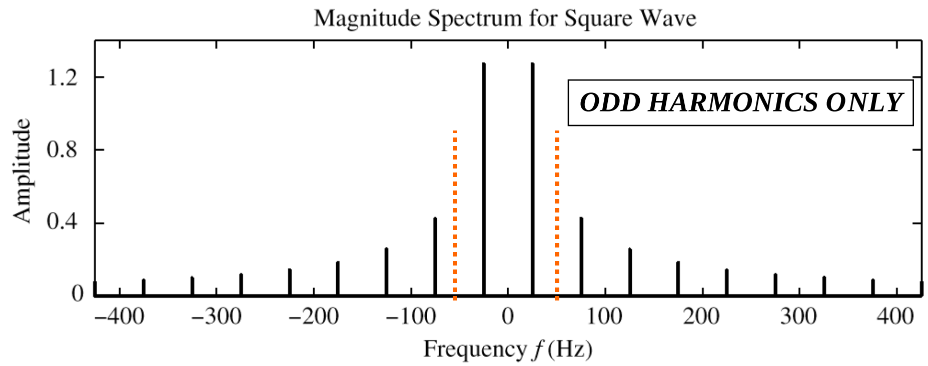


Figure 3.12 Spectrum of the square-wave signal whose Fourier series coefficients are given in (3.4.5) with $f_0 = 1/T_0 = 25$ Hz.

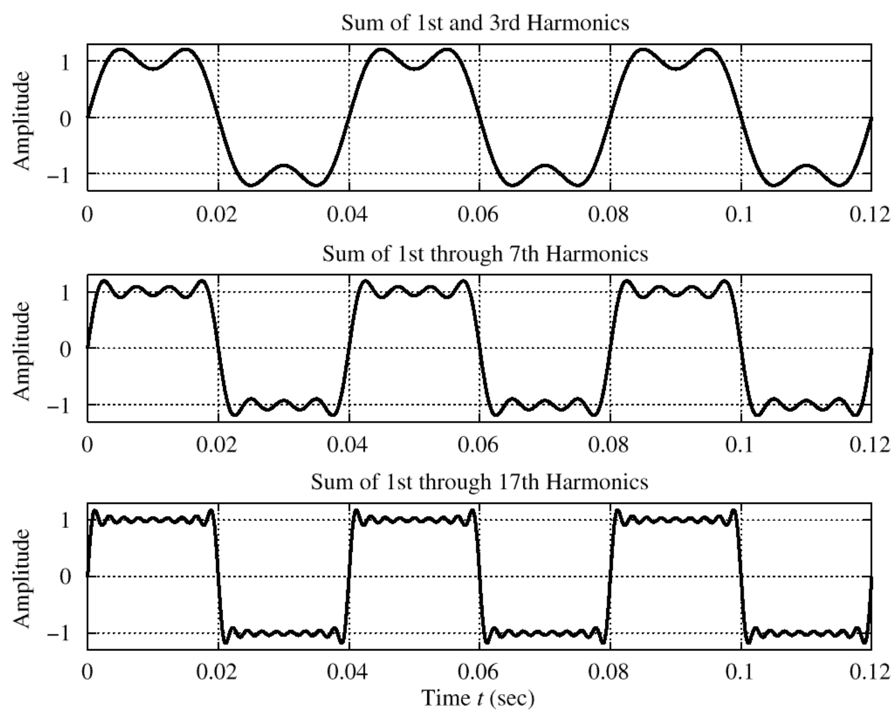
Fourier Series SUM

- **Add a FINITE number of terms**
  - **How close to the original x(t) ?**

$$X_k = A_k e^{j\varphi_k}$$

$$x_N(t) = X_0 + \sum_{k=1}^{N} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{N} \tfrac{1}{2} X_{-k} e^{-j2\pi k f_0 t}$$

$$x(t) = \lim_{N \to \infty} x_N(t) \quad ???$$



Convergence of Fourier Series

$$x_N(t) = X_0 + \sum_{k=1}^{N} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{N} \tfrac{1}{2} X_{-k} e^{-j2\pi k f_0 t}$$

$$x(t) = \lim_{N \to \infty} x_N(t) \quad ???$$

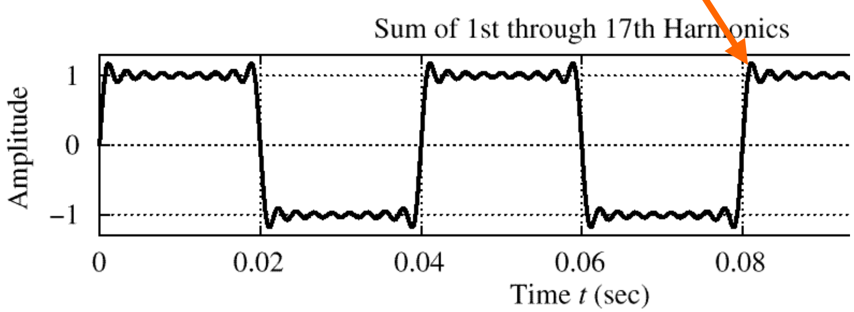
$$\int_0^{T_0} \left| x(t) - x_N(t) \right|^2 dt \to 0$$

ERROR ENERGY

$$x(t) = X_0 + \sum_{k=1}^{\infty} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{\infty} \tfrac{1}{2} X_{-k} e^{-j2\pi k f_0 t}$$

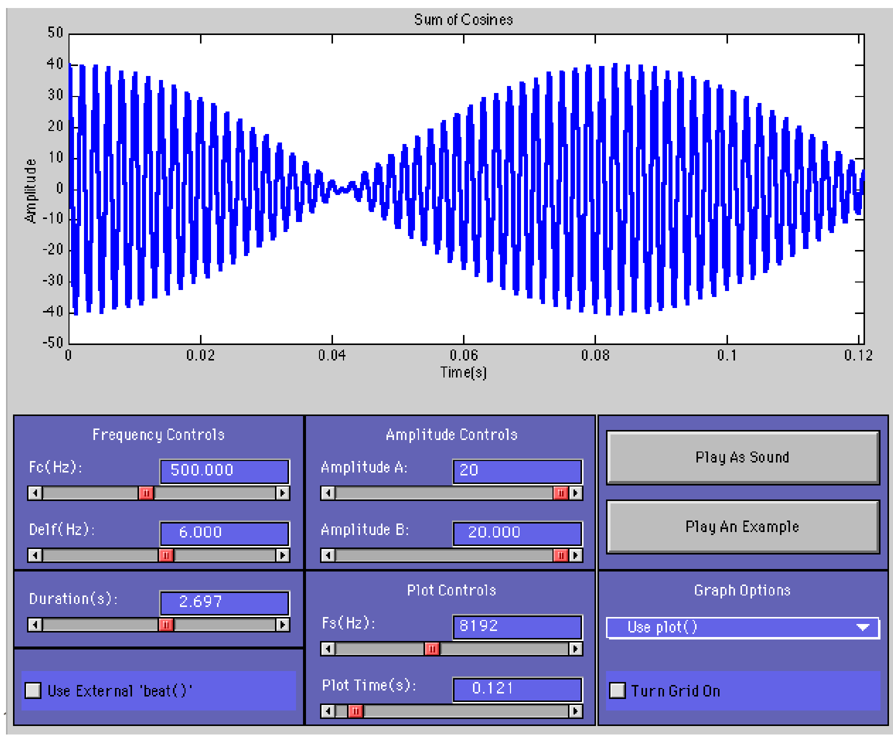
Gibbs' Phenomenon

- Convergence at DISCONTINUITY of x(t)
  - There is ALWAYS an overshoot
  - 9% for the Square Wave case

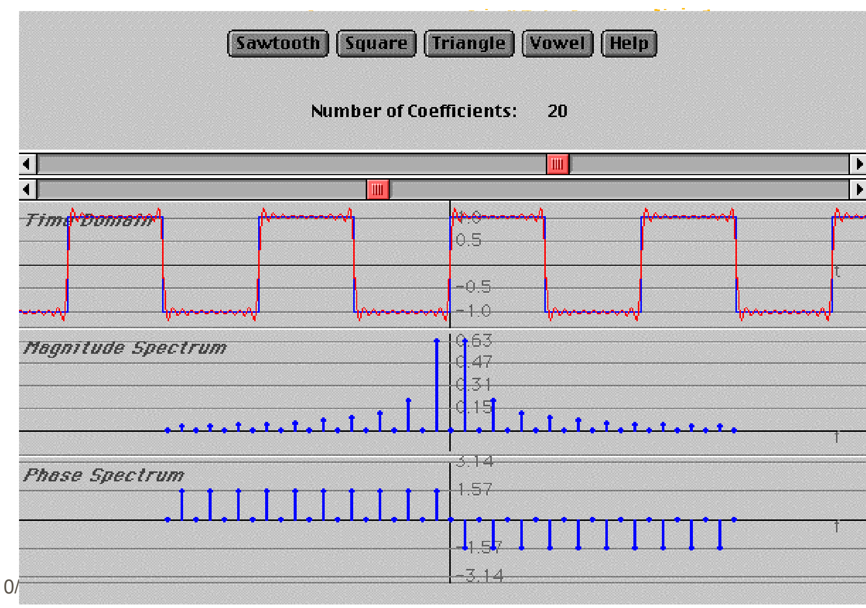


A Couple of DEMOS

- Beat Control GUI
  - DSPFirst Toolbox: MATLAB
    - DSPFIRST/beatcon.m
- Fourier Series Java Applet
  - Interactive
  - http://users.ece.gatech.edu/~slabaugh/java/fourier/fourier.html



Fourier Series Java Applet



Time-Varying FREQUENCIES Diagram

Frequency is the vertical axis

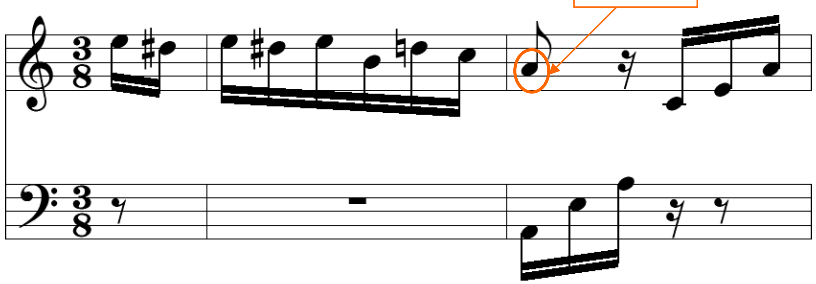
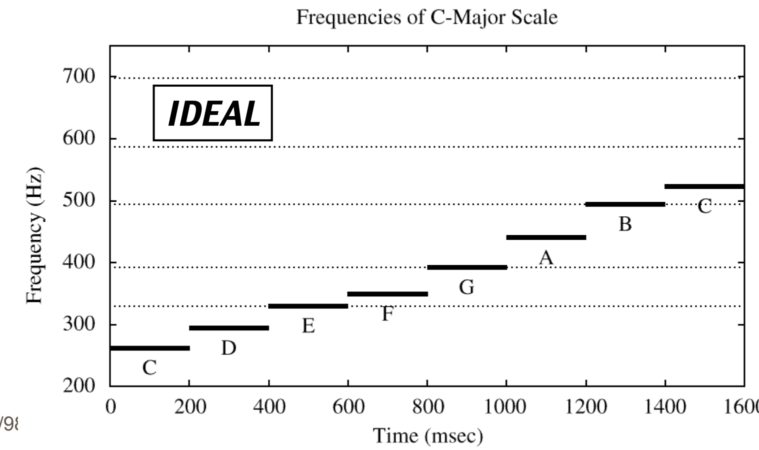


Figure 3.18 Sheet-music notation is a time–frequency diagram.

Time is the horizontal axis

STEPPED FREQUENCIES

- C–major SCALE: successive sinusoids
  - Frequency is constant for each note

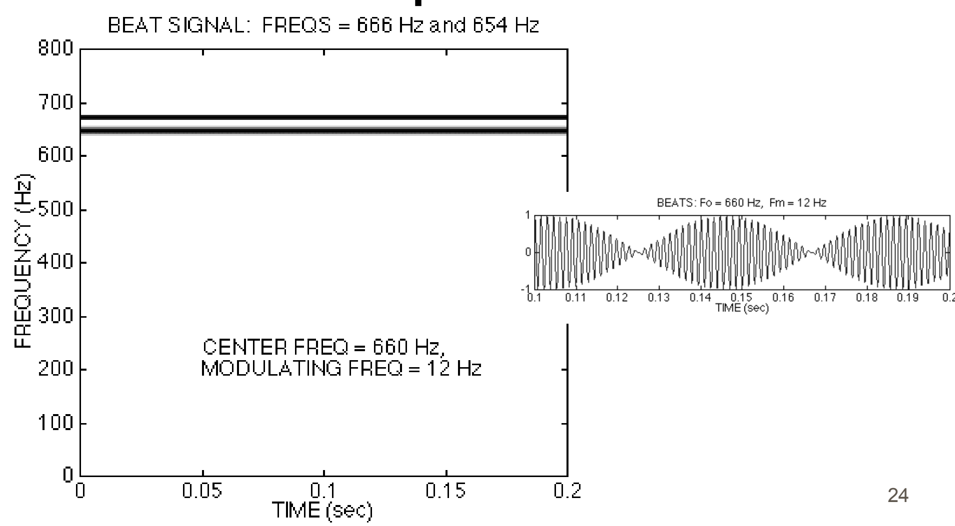


R-rated: ADULTS ONLY

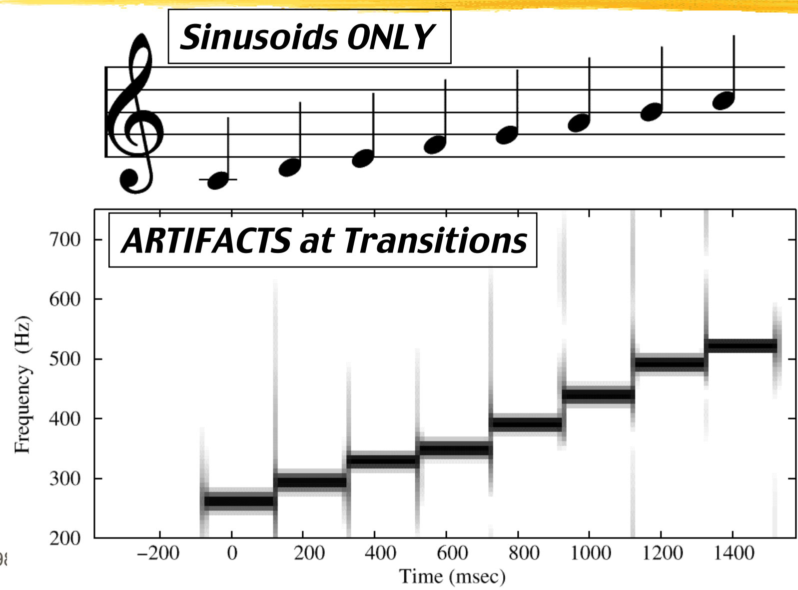
- SPECTROGRAM Tool
  - MATLAB function is specgram.m
  - DSP First has spectgr.m
- ANALYSIS program
  - Takes x(t) as input
  - Produces spectrum values X_k
  - OVER a SHORT TIME: the ANALYSIS FRAME
    - Uses the FFT (Fast Fourier Transform)

SPECTROGRAM EXAMPLE

- Two Constant Frequencies: Beats



SPECTROGRAM of C-Scale



New Signal: Linear FM

- Called Chirp Signals (LFM)
  - Quadratic phase

$$x(t) = A\cos(\alpha t^2 + 2\pi f_0 t + \varphi)$$

- Freq will change LINEARLY vs. time
  - Example of Frequency Modulation (FM)
  - Define "instantaneous frequency"

INSTANTANEOUS FREQ

- Definition

$$x(t) = A\cos(\psi(t))$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t)$$

- For Sinusoid:

$$\psi(t) = 2\pi f_0 t + \varphi$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = 2\pi f_0$$

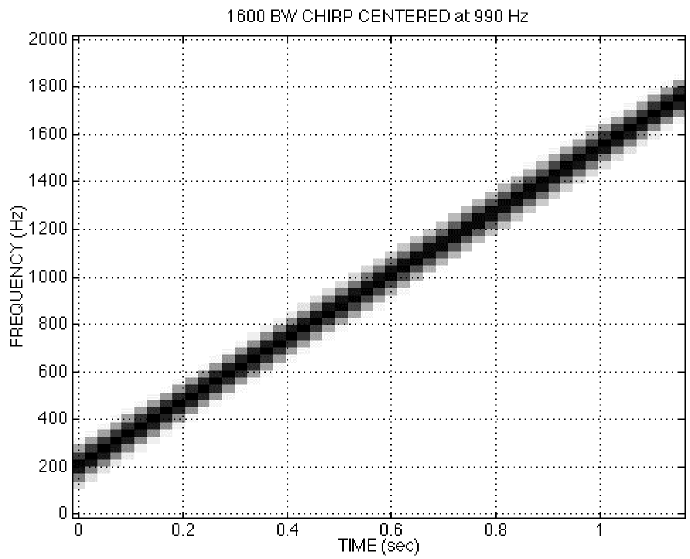
INSTANTANEOUS FREQ of the Chirp

- Chirp Signals (LFM)
  - Quadratic phase
- Freq will change LINEARLY vs. time

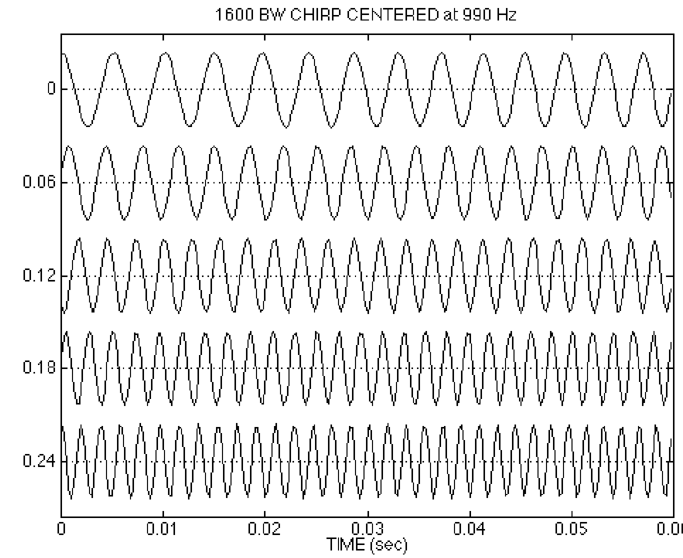
$$x(t) = A\cos(\alpha t^2 + \beta t + \varphi)$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = 2\alpha t + \beta$$

CHIRP SPECTROGRAM



CHIRP WAVEFORM



OTHER CHIRPS

- ψ(t) can be anything:

$$x(t) = A\cos(\alpha\cos(\beta t) + \varphi)$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = -\alpha\beta\sin(\beta t)$$

- ψ(t) could be speech or music:
  - FM radio

SINE-WAVE FREQUENCY MODULATION (FM)