

EE-2200

Fall-98

Lecture 7

Sampling & Aliasing

19-Oct-98

Homework Info

- Prob Set #3 due **NEXT MONDAY**
 - In Lecture, before NOON
- Solutions will be posted on Tues
 - HW #2 solutions are on the web site

Web-CT Info

- Check the Bulletin Board for msgs
 - New version of **mgnotes.m**
 - Spectrogram image display info
 - New M-file: **plotspec.m**
- Problem Set #3 postponed
- Calendar has entries:
 - **Quiz #2 on 13-Nov (Friday)**

Lab Info

- Lab #3 Report
 - New version of mgnotes.m
 - Info on Displaying Spectrogram image
 - FORMAL Lab Report
- Lab #4 was posted on Sunday
 - AM and FM signals

READING ASSIGNMENTS

■ This Lecture:

- Chapter 4, pp. 83–94

■ Other Reading:

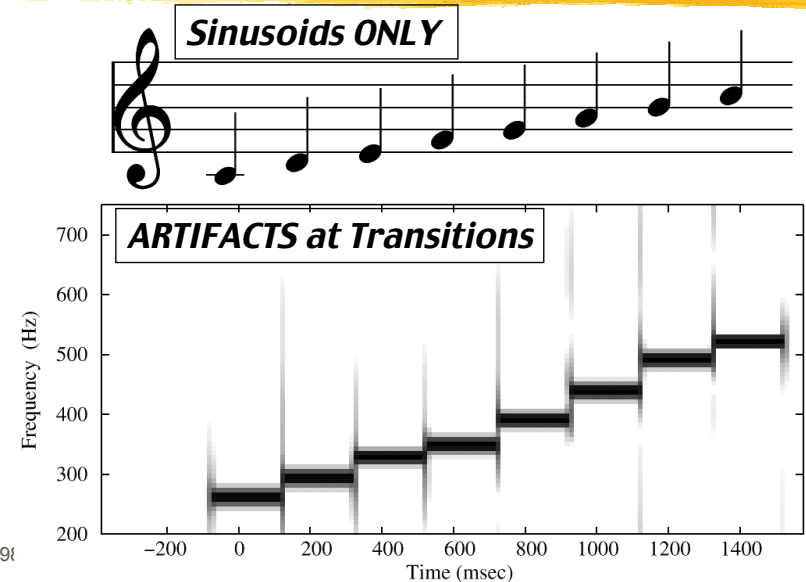
- Recitation: Chapter 4, pp. 90–100
 - Strobe Demo
- Next Lecture: Chap. 4, pp. 100–111

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SPECTROGRAM of C-Scale



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R-rated: ADULTS ONLY

■ SPECTROGRAM Tool

- MATLAB function is **specgram.m**
- DSP First has **spectgr.m**

■ ANALYSIS program

- Takes $x(t)$ as input
- Produces spectrum values X_k
- OVER a **SHORT TIME**: the **ANALYSIS FRAME**
 - Uses the FFT (Fast Fourier Transform)

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LECTURE OBJECTIVES

■ SAMPLING can cause ALIASING

- **Sampling Theorem**
- Sampling Rate $> 2(\text{Highest Frequency})$

■ Spectrum for digital signals, $x[n]$

- Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

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SYSTEMS Process Signals

PROCESSING GOALS:

- Change $x(t)$ into $y(t)$
 - For example, more BASS
- Improve $x(t)$, e.g., image deblurring
- Extract Information from $x(t)$



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System IMPLEMENTATION

ANALOG/ELECTRONIC:

- Circuits: resistors, capacitors, op-amps



DIGITAL/MICROPROCESSOR

- Convert $x(t)$ to **numbers** stored in memory



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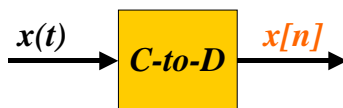
SAMPLING $x(t)$

SAMPLING PROCESS

- Convert $x(t)$ to **numbers** $x[n]$
- "n" is an integer; $x[n]$ is a sequence
- "n" is the storage address in memory

UNIFORM SAMPLING at $t = nT_s$

- IDEAL: $x[n] = x(nT_s)$



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SAMPLING RATE

SAMPLING RATE (f_s)

- $1/T_s =$ NUMBER of SAMPLES PER SECOND
- 125 microsec \rightarrow 8000 samples/sec
 - UNITS ARE HERTZ: 8000 Hz

UNIFORM SAMPLING at $t = nT_s = n/f_s$

- IDEAL: $x[n] = x(nT_s) = x(n/f_s)$



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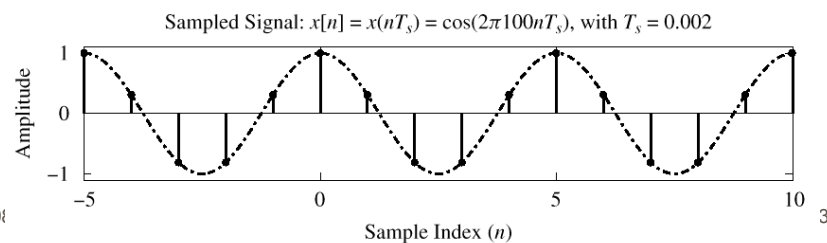
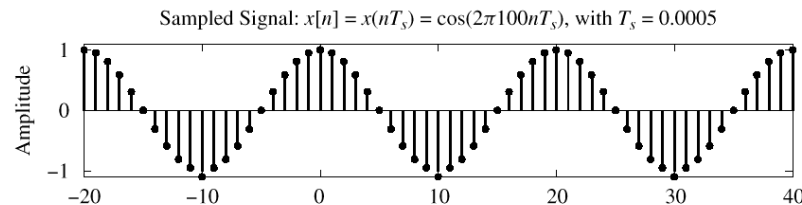
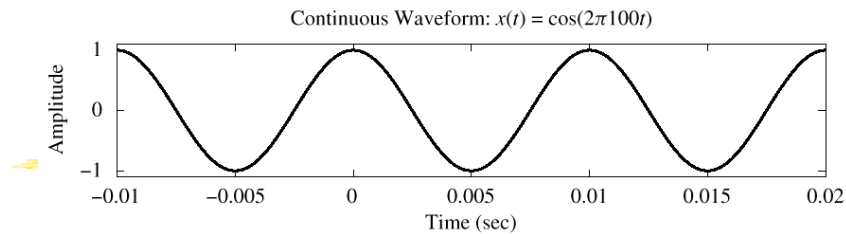
SAMPLING A SINUSOID

HOW OFTEN ?

- DEPENDS on FREQUENCY of SINUSOID
- ANSWERED by SHANNON/NYQUIST
- DEPENDS on "RECONSTRUCTION"

Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than f_{\max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{\max}$.



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STORING DIGITAL SOUND

- $x[n]$ is a SAMPLED SINUSOID
 - A list of numbers stored in memory
- CD rate is 44,100 samples per second
- 16-bit samples
- Stereo uses 2 channels
- Number of bytes for 1 minute is
 - $2 \times (16/8) \times 60 \times 44100 = 10.584$ Mbytes

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DISCRETE-TIME SINUSOID

- Change $x(t)$ into $x[n]$

$$x(t) = A \cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A \cos(\omega n T_s + \varphi)$$

$$x[n] = A \cos((\omega T_s)n + \varphi)$$

$$x[n] = A \cos(\hat{\omega} n + \varphi)$$

$$\hat{\omega} = \omega T_s$$

DIGITAL FREQUENCY

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DIGITAL FREQUENCY

- **DIGITAL FREQUENCY is NORMALIZED**
- **UNITS are radians, not rad/sec**
- **GOES from 0 to 2π , as f goes from 0 to the sampling frequency**

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

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ALIASING

- **ADDING f_s or $2f_s$ TO THE FREQ of $x(t)$**
- **SAMPLED SIGNAL: $x[n] = x(n/f_s)$**
 - **$x[n]$ STAYS THE SAME**
- **CAN'T TELL f_0 FROM $(f_0 + f_s)$ or $(f_0 + 2f_s)$**
- **CALLED **ALIASING****

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ALIASING DERIVATION

$$x(t) = A \cos(2\pi f_0 t + \phi) \quad (4.1.4)$$

If we sample $x(t)$ with a sampling period of T_s , we get the sequence $x[n]$ with values

$$x[n] = x(nT_s) = A \cos(2\pi f_0 nT_s + \phi) \quad (4.1.5)$$

Now consider another sinusoid with the same amplitude and phase, but with frequency $f_0 + \ell f_s$, where ℓ is an integer and $f_s = 1/T_s$.

$$y(t) = A \cos(2\pi(f_0 + \ell f_s)t + \phi)$$

If this second waveform, $y(t)$, is sampled with period T_s , we get

$$\begin{aligned} y[n] &= y(nT_s) = A \cos(2\pi(f_0 + \ell f_s)nT_s + \phi) \\ &= A \cos(2\pi f_0 nT_s + 2\pi \ell f_s nT_s + \phi) \\ &= A \cos(2\pi f_0 nT_s + 2\pi \ell + \phi) \\ &= A \cos(2\pi f_0 nT_s + \phi) \\ &= x[n] \end{aligned}$$

NORMALIZED FREQUENCY

- **DIGITAL FREQUENCY**

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s$$

Normalized Cyclic Frequency

$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

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SPECTRUM for $x[n]$

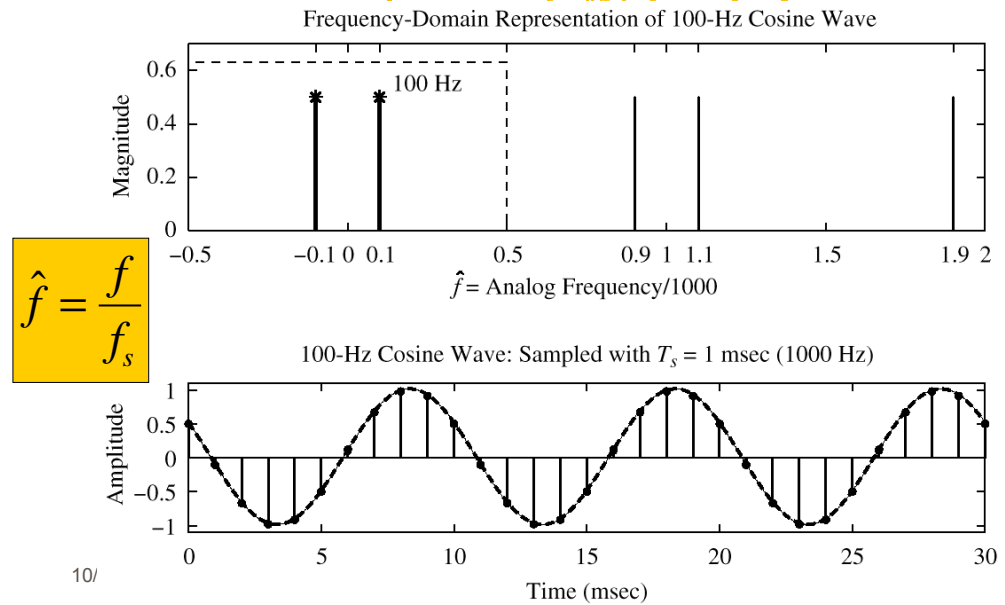
- PLOT versus NORMALIZED FREQUENCY
- INCLUDE ALL SPECTRUM LINES
 - ALIASES
 - FOLDED ALIASES
 - ALIASES of NEGATIVE FREQS

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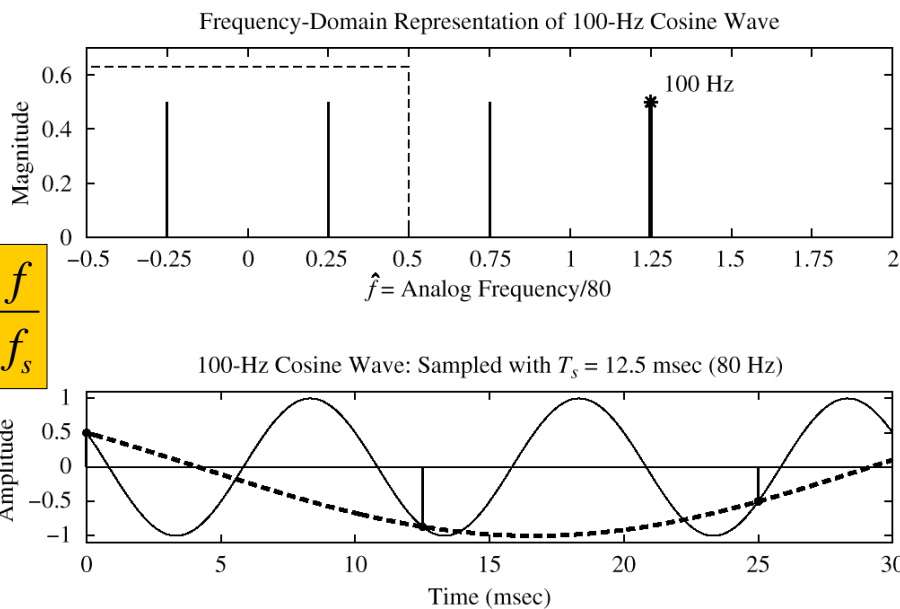
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SPECTRUM (DIGITAL)



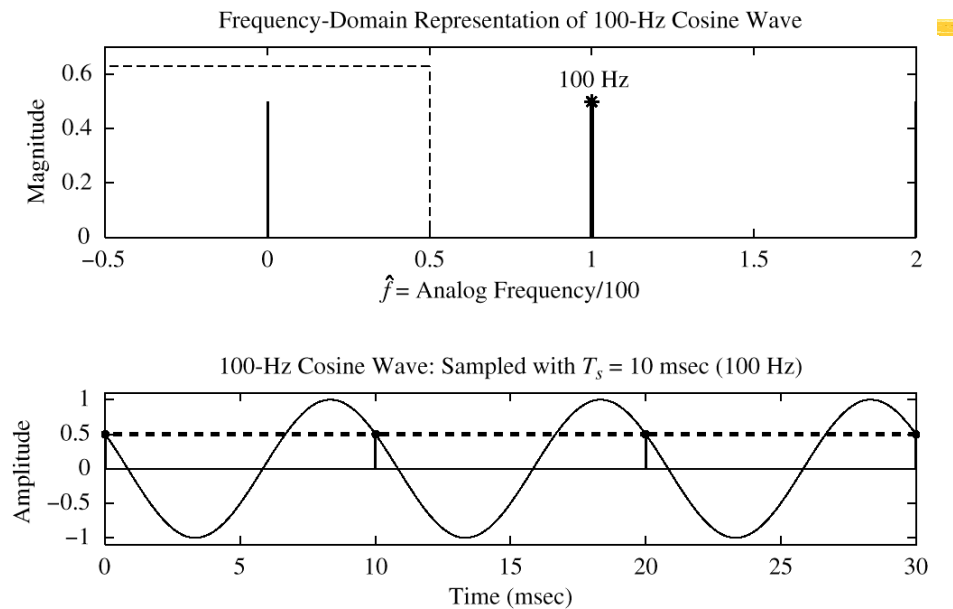
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SPECTRUM of $x[n]$ ALIASING CASE



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SPECTRUM of $x[n]$ ALIASING to ZERO FREQ



DEMOS from CHAPTER 4

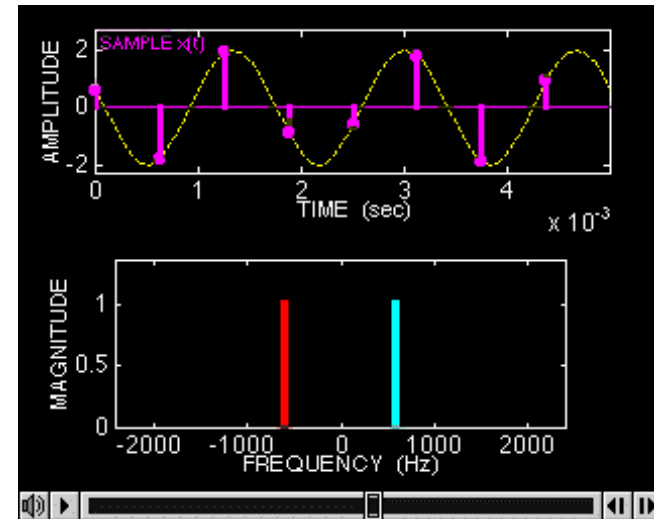
- CD-ROM DEMOS
- SAMPLING DEMO
 - Different Sampling Rates
 - Aliasing of a Sinusoid
- STROBE DEMO
 - Synthetic vs. Real
 - Television **SAMPLES** at 30 fps
- Sampling & Reconstruction

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SAMPLING DEMO

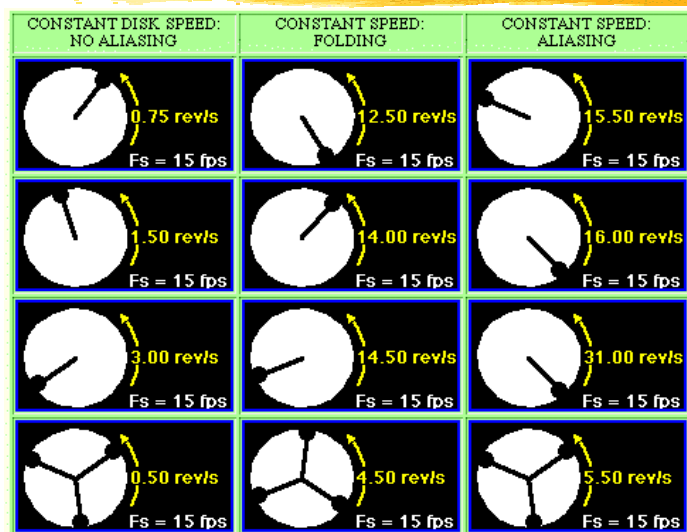


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STROBE DEMO (Synthetic)



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FOLDING DERIVATION

A second source of aliased signals actually comes from the negative frequency component of the cosine wave. These frequencies are $-f_0 + \ell f_s$, where ℓ is a positive or negative integer. Consider a third signal

$$w(t) = A \cos(2\pi(-f_0 + \ell f_s)t - \phi)$$

whose initial phase is the negative of that in (4.1.4). If we sample $w(t)$ with a sampling period of T_s , we now get

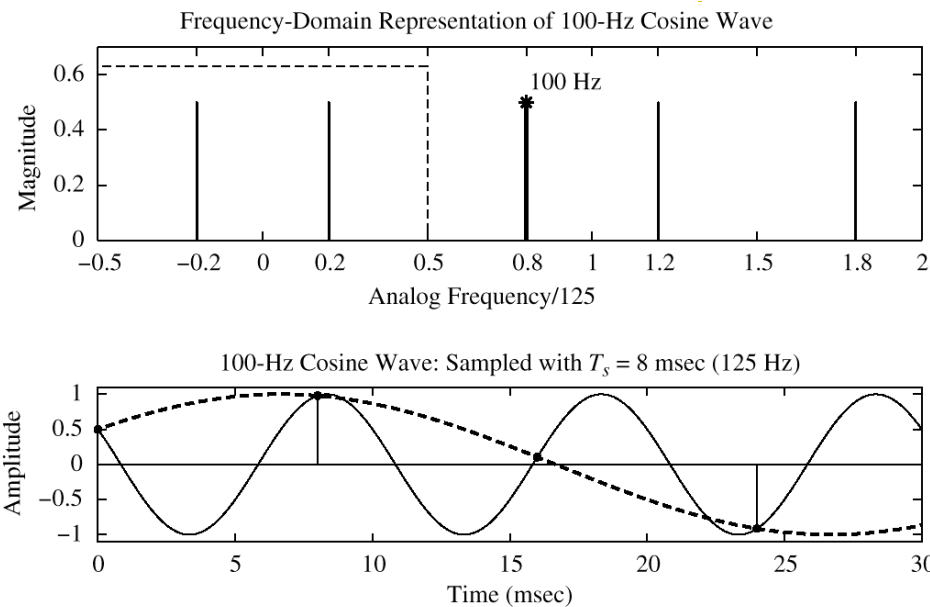
$$\begin{aligned}
 w[n] &= w(nT_s) = A \cos(2\pi(-f_0 + \ell f_s)nT_s - \phi) \\
 &= A \cos(-2\pi f_0 nT_s + 2\pi \ell f_s nT_s - \phi) \\
 &= A \cos(-2\pi f_0 nT_s + 2\pi \ell - \phi) \\
 &= A \cos(2\pi f_0 nT_s + \phi) \\
 &= x[n]
 \end{aligned}$$

The fourth line in this equation is true because the cosine function is an even function; i.e., $\cos(-\theta) = \cos \theta$.

FOLDING

- ANOTHER $x[n]$ THAT IS IDENTICAL
- CAN'T TELL f_0 FROM $(f_s - f_0)$
 - Or, $(2f_s - f_0)$ or, $(3f_s - f_0)$
- EXAMPLE:
 - $y(t)$ has 1000 Hz component
 - SAMPLING FREQ = 1500 Hz
 - WHAT is the "FOLDED" ALIAS ?

SPECTRUM of $x[n]$ FOLDING CASE



FOLDING DIAGRAM

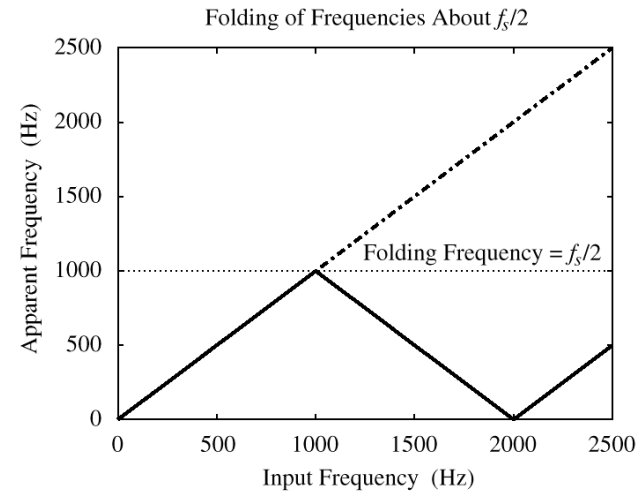


Figure 4.4 Folding of a sinusoid sampled at $f_s = 2000$ samples/sec. The apparent frequency is the lowest frequency of a sinusoid that has exactly the same samples as the input sinusoid.