GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Fall 1998
Lab #5: FM Synthesis for Musical Instruments

Date: week of 26 Oct 1998

Lab Quiz: we will have an on-line quiz given in lab this week (27–29 Oct)
It will cover MATLAB issues. There is a sample quiz on WebCT.

This is *the official* Lab#5 description; it is nearly *identical* to Lab-B in Appendix C.4 of the text.

The Warm-up section of each lab must be completed in Lab and the steps marked *Instructor Verification* must also be signed off **during the lab time**.

The lab report for this lab will be informal: discuss your results from section 4. Staple the **Instructor Verification** sheet to the end of your lab report.

The report will due during the week of **3-Nov** at the start of your lab.

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1 Introduction

The objective of this lab is to introduce more complicated signals that are related to the basic sinusoid. These are signals which implement frequency modulation (FM) and amplitude modulation (AM) are widely used in communication systems such as radio and television), but they also can be used to create interesting sounds that mimic musical instruments. There are a number of demonstrations on the CD-ROM that provide examples of these signals for many different conditions.

CD-ROM
FM Synthesis

2 Overview

Frequency modulation (FM) can be used to make interesting sounds that mimic musical instruments, such as bells, woodwinds, drums, etc. The goal in this lab is to implement one or two of these FM schemes and hear the results.

CD-ROM
DEMO: FM-Synthesis

We have already seen that FM defines the signal $x(t)$ to have a time-varying phase

$$x(t) = A\cos(\psi(t))$$

and that the instantaneous frequency changes according to the derivative of $\psi(t)$. If $\psi(t)$ is linear, $x(t)$ is a constant-frequency sinusoid; whereas, if $\psi(t)$ is quadratic, $x(t)$ is a chirp signal whose frequency changes linearly in time. FM music synthesis uses a more interesting $\psi(t)$, one that is sinusoidal. Since the derivative of a sinusoidal $\psi(t)$ is also sinusoidal, the instantaneous frequency of $x(t)$ will oscillate. This is useful for synthesizing instrument sounds because the proper choice of the modulating frequencies will produce a fundamental frequency and several overtones, as many instruments do.

The general equation for an FM sound synthesizer is:

$$x(t) = A(t)\cos\left(2\pi f_c t + I(t)\cos(2\pi f_m t + \phi_m) + \phi_c\right) \tag{1}$$

where $A(t)$ is the signal's amplitude. It is a function of time so that the instrument sound can be made to fade out slowly or cut off quickly. Such a function is called an *envelope*. The parameter f_c is called the *carrier* frequency. Note that when you take the derivative of $\psi(t)$ to find $f_i(t)$,

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi}\frac{d}{dt}\psi(t) \\ &= \frac{1}{2\pi}\frac{d}{dt}\left(2\pi f_c t + I(t)\cos(2\pi f_m t + \phi_m) + \phi_c\right) \\ &= f_c - I(t) f_m \sin(f_m t + \phi_m) + \frac{dI}{dt}\cos(2\pi f_m t + \phi_m) \end{aligned} \tag{2}$$

f_c will be a constant in that expression. It is the frequency that would be produced without any frequency modulation. The parameter f_m is called the *modulating* frequency. It expresses the rate of oscillation of $f_i(t)$. The parameters ϕ_m and ϕ_c are arbitrary phase constants, usually both set to $-\pi/2$ so that $x(0) = 0$.

The function $I(t)$ has a less obvious purpose than the other FM parameters in (1). It is technically called the *modulation index envelope*. To see what it does, examine the expression for the instantaneous frequency (2). The quantity $I(t)f_m$ multiplies a sinusoidal variation of the frequency. If $I(t)$ is constant or $\frac{dI}{dt}$ is relatively small, then $I(t)f_m$ gives the maximum amount by which the instantaneous frequency deviates from f_c. Beyond that, however, it is difficult to relate $I(t)$ to the sound made by $x(t)$ without some rather tedious mathematical analysis.

In our study of signals, we would like to characterize $x(t)$ as the sum of several constant-frequency sinusoids instead of a single signal whose frequency changes. In this regard, the following comments are relevant: when $I(t)$ is small (e.g., $I \approx 1$), low multiples of the carrier frequency (f_c) have high amplitudes. When $I(t)$ is large $(I > 4)$, both low and high multiples of the carrier frequency have high amplitudes. The net result is that $I(t)$ can be used to vary the harmonic content of the instrument sound (called overtones). When $I(t)$ is small, mainly low frequencies will be produced. When $I(t)$ is large, higher harmonic frequencies can also be produced. Since $I(t)$ is a function of time, the harmonic content will change with time. For more details see the paper by Chowning.[1]

3 Warm-up

3.1 Chirps and Aliasing

Use your MATLAB function `mychirp` (from the previous lab) to synthesize a "chirp" signal with the following parameters:

1. A total time duration of 2.5 secs. where the *desired* instantaneous frequency starts at 13,000 Hz and ends at 200 Hz.
2. Use a D/A conversion rate of $f_s = 8000$ Hz.

Listen to the signal. What comments can you make regarding the sound of the chirp (e.g., is it linear)? Does it chirp down, or chirp up, or both? Create a spectrogram of your chirp signal. Use the sampling theorem (from Chapter 4 in the text) to help explain what you hear and see.

In addition, make some theoretical calculations by hand: Determine the range of frequencies (in hertz) that will be synthesized by this MATLAB script. Make a sketch by hand of the instantaneous frequency versus time. Explain how *aliasing* affects the instantaneous frequency that is actually heard. Listen to the signal again to verify that it has the expected frequency content.

Instructor Verification (separate page)

[1] Ref: John M. Chowning, "The Synthesis of Complex Audio Spectra by means of Frequency Modulation," *Journal of the Audio Engineering Society*, vol. 21, no. 7, Sept. 1973, pp. 526–534.

3.2 Wideband FM

CD-ROM

Spectrograms & Sounds: Wide-band FM

Generate a set of wideband FM "chirps" with sinusoidal modulation according to the following formula:

$$x(t) = \cos(2\pi f_0 t + B\sin(2\pi f_m t))$$

over the time interval $0 \le t \le 1.35$ secs.

(a) First of all, do the case where $f_0 = 900$ Hz, $f_m = 3$ Hz and $B = 200$. Generate the signal, plot its spectrogram and listen to the signal to see if it corresponds to the spectrogram.

(b) Make a sketch (by hand) of the instantaneous frequency for the signal in part (a).
Instructor Verification (separate page)

(c) Now, change f_m to be $f_m = 30$ Hz and $B = 20$, but keep $f_0 = 900$ Hz. Generate the signal, plot its spectrogram and listen to the signal.

(d) One again, sketch (by hand) the instantaneous frequency for the signal in part (c) to see if it corresponds to the spectrogram.

(e) Now, change f_m to be $f_m = 300$ Hz and $B = 2$, but keep $f_0 = 900$ Hz. Generate the signal, plot its spectrogram and listen to the signal. This is a case of *wideband FM*, and the instantaneous frequency concept no longer explains the sound we hear, but you should write a reasonable explanation for the spectrum that you observe.
Instructor Verification (separate page)

When unsure about a command, use `help`.

4 Lab B: FM Synthesis of Instrument Sounds

4.1 Generating the Bell Envelopes

Now we take the general FM synthesis formula (1) and specialize for the case of a bell sound. The amplitude envelope $A(t)$ and the modulation index envelope $I(t)$ for the bell are both decaying exponentials. That is, they both have the following form:

$$y(t) = e^{-t/\tau} \qquad (3)$$

where τ is a parameter that controls the decay rate of the exponential. Notice that $y(0) = 1$ and $y(\tau) = 1/e$, so τ is the time it takes a signal of the form (3) to decay to $1/e = 36.8\%$ of its initial value. For this reason, the parameter τ is called the *time constant*.

Use (3) to write a MATLAB function that will generate a decaying exponential to be used later in synthesizing a bell sound. The file header should look like this:

```
function  yy = bellenv(tau, dur, fsamp);
%BELLENV produces envelope function for bell sounds
%
%        usage: yy = bellenv(tau, dur, fsamp);
%
%                where tau = time constant
```

```
%                     dur = duration of the envelope
%                   fsamp = sampling frequency
%                 returns:
%                      yy = decaying exponential envelope
%
%   note: produces exponential decay for positive tau
```

The function will be one or two lines of MATLAB code. The first line should define your time vector based on `fsamp` and `dur`, and the second generates the exponential (3).

The bell's amplitude envelope, $A(t)$, and modulation index envelope, $I(t)$ are identical, up to a scale factor.

$$A(t) = A_0 e^{-t/\tau} \qquad \text{and} \qquad I(t) = I_0 e^{-t/\tau}$$

Hence, one call to the `bellenv` function will generate the shape for both envelopes.

4.2 Parameters for the Bell

Now that we have the bell's amplitude and modulation index envelopes, we can create the actual sound signal for the bell by specifying all the parameters in the general FM synthesis formula (1). The frequencies f_c and f_m must be given numerical values. The ratio of carrier to modulating frequency is important in creating the sound of a specific instrument. For the bell, a good choice for this ratio is 1:2, e.g., $f_c = 110$ Hz and $f_m = 220$ Hz.

Now write a simple M-file `bell.m` that implements (1) to synthesize a bell sound. Your function should call `bellenv.m` to generate $A(t) = A_0 e^{-t/\tau}$ and $I(t) = I_0 e^{-t/\tau}$.

```
function  xx = bell(ff, Io, tau, dur, fsamp)
%BELL     produce a bell sound
%
%     usage:   xx = bell(ff, Io, tau, dur, fsamp)
%
%     where:  ff = frequency vector (containing fc and fm)
%             Io = scale factor for modulation index
%            tau = decay parameter for A(t) and I(t)
%            dur = duration (in sec.) of the output signal
%          fsamp = sampling rate
```

4.3 The Bell Sound

Test your `bell( )` function using the parameters of case #1 in the table. Play it with the `soundsc()` function at 11,025 Hz.[2] Does it sound like a bell? The value of $I_0 = 10$ for scaling the modulation index envelope is known to give a distinctive sound. Later on, you can experiment with other values to get a variety of bells.

[2] A higher sampling rate of 11,025 Hz is used because the signal contains many harmonics, some of which might alias if a lower f_s were used. You should experiment with lower values of f_s to see if you can hear a difference, e.g., $f_s = 8000$ Hz.

CASE	f_c (Hz)	f_m (Hz)	I_0	τ (sec)	T_{dur} (sec)	f_s (Hz)
1	110	220	10	2	6	11,025
2	220	440	5	2	6	11,025
3	110	220	10	12	3	11,025
4	110	220	10	0.3	3	11,025
5	250	350	5	2	5	11,025
6	250	350	3	1	5	11,025

The frequency spectrum of the bell sound is very complicated, but it does consist of spectral lines, which can be seen with a spectrogram. Among these frequencies, one spectral line will dominate what we hear. We would call this the *note frequency* of the bell. It is tempting to guess that the note frequency will be equal to f_c, but you will have to experiment to find the true answer. It might be f_m, or it might be something else—perhaps the fundamental frequency which is the greatest common divisor of f_c and f_m.

For each case in the table, do the following:

(a) Listen to the sound by playing it with the `soundsc()` function.

(b) Calculate the fundamental frequency of the "note" being played. Explain how you can verify by listening that you have the correct fundamental frequency.

(c) Describe how you can hear the frequency content changing according to $I(t)$. Plot $f_i(t)$ versus t for comparison.

(d) Display a spectrogram of the signal. Describe how the frequency content changes, and how that change is related to $I(t)$. Point out the "harmonic" structure of the spectrogram, and calculate the fundamental frequency, f_0.

(e) Plot the entire signal and compare it to the envelope $A(t)$ generated by `bellenv`.

(f) Plot about 100–200 samples from the middle of the signal and explain what you see, especially the frequency variation.

If you are making a lab report, do the plots for two cases—choose one of the first four and one of the last two. Write up an explanation only for the two that you choose.

4.4 Comments about the Bell

Cases #3 and #4 are extremes for choosing the decay rate τ. In case #3, the waveform does not decay very much over the course of three seconds and sounds a little like a sum of harmonically related sinusoids. With a "faster" decay rate, as in case #4, we get a percussion-like sound. Modifying the fundamental frequency f_0 (determined in part (d) above) should have a noticeable effect on the tone you hear. Try some different values for f_0 by changing f_c and f_m, but still in the ratio of 1:2. Describe what you hear.

Finally, experiment with different carrier to modulation frequency ratios. For example, in his paper, Chowning uses a fundamental frequency of $f_0 = 40$ Hz and a carrier to modulation frequency ratio of 5:7. Try this and a few other values. Which parameters sound "best" to you?

4.5 Optional: C-Major Scale

Finally, synthesize other note frequencies. For example, try to make the C-major scale (defined in Lab 3) consisting of seven consecutive notes.

Lab #5
EE-2200
Fall-1998
INSTRUCTOR VERIFICATION PAGE

Staple this page to the end of your Lab Report.

Name: ____ Date of Lab: __________

Part 3.1 Aliasing of the linear FM Chirp:

Verified:____________________ Date/Time:__________

Part 3.2(a),(b) Sinusoidal FM Signal Generation:

Verified:____________________ Date/Time:__________

Part 3.2(e) Explain why the Wideband FM Spectrogram has the characteristics that you observe. Does the spectrogram match your "hearing" experience when you listen to the sound in part (e)? Compare your observations to the spectrogram of other signals that you have seen in the lecture or in the lab. Write a short explanation in the space below:

Verified:____________________ Date/Time:__________