GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Fall 1998
Problem Set #3

Assigned: 13, 20 Oct 1998
Due Date: 26 Oct 1998 (MONDAY)

Quiz #1 will be held in lecture on Friday 16-Oct-98. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1 and #2.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$)

Reading: In *DSP First*, all of Chapter 4 on *Sampling.* Finish Chap. 3 on *Spectrum Representation*, pp. 68–77.

The web site for the course uses Web-CT: `http://webct.ece.gatech.edu`

$\Longrightarrow$ The six(6) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the "unstarred" problems.

PROBLEM 3.1*:

A periodic signal $x(t) = x(t + T_0)$ is described over one period $-T_0/2 \le t \le T_0/2$ by the equation

$$x(t) = \begin{cases} 1 & |t| < t_c \\ 0 & t_c < |t| \le T_0/2 \end{cases}$$

where $t_c < T_0/2$.

(a) Sketch the periodic function $x(t)$ for $-T_0 < t < 2T_0$ for the case $t_c = T_0/8$.

(b) Determine the D.C. coefficient X_0. (This answer will depend on t_c and T_0.)

(c) Use the Fourier *analysis* integral[1] (for $k \neq 0$)

$$X_k = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-jk\omega_0 t}dt$$

to determine a general formula for the Fourier coefficients X_k in the representation

$$x(t) = X_0 + \Re e\left\{\sum_{k=1}^{\infty} X_k e^{jk\omega_0 t}\right\}$$

Your final result should depend on t_c and T_0. Note: the frequency ω_0 is given in radians/sec.

(d) Sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(100)$ rad/sec and $t_c = T_0/4$ for frequencies between $-5\omega_0$ and $+5\omega_0$. Label each component with its complex amplitude (magnitude and phase).

(e) Compare your answer in part (d) to the formula for the Fourier coefficients of a 50% duty cycle square wave given in class (and in equation (3.4.5) in the book). Compare both the magnitudes and phases of X_k, as well as the trend versus k. State the similarities and also the differences.

(f) Sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(100)$ rad/sec and $t_c = T_0/8$ for frequencies between $-5\omega_0$ and $+5\omega_0$. Label each component with its complex amplitude (magnitude and phase).

---

[1] The integral can be done over any period of the signal; in this case, the most convenient choice is from $-T_0/2$ to $T_0/2$.

PROBLEM 3.2*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp.

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α, β and T_2. For this problem, assume that the starting time of the "chirp" is $t = 0$.

(b) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(-33t^2 + 98t - 0.2)}\right\}$$

derive a formula for the *instantaneous* frequency versus time.

(c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.

(d) *(Optional part)* What would happen if the instantaneous frequency were to become negative? Since instantaneous frequency often corresponds to what we hear, would we hear negative frequency?

PROBLEM 3.3*:

Consider the cosine wave

$$x(t) = 10\cos(880\pi t + \phi)$$

Suppose that we obtain a sequence of numbers by sampling the waveform at equally spaced time instants nT_s. In this case the resulting sequence would have values

$$x[n] = x(nT_s) = 10\cos(880\pi n T_s + \phi) \qquad -\infty < n < \infty$$

Suppose that $T_s = 0.0001$.

(a) How many samples will be taken in one period of the cosine wave?

(b) Now consider another waveform $y(t)$ such that

$$y(t) = 10\cos(\omega_0 t + \phi)$$

Find a frequency $\omega_0 > 880\pi$ such that $y(nT_s) = x(nT_s)$ for all integers n.
Hint: Use the fact that $\cos(\theta + 2\pi n) = \cos(\theta)$ *if n is an integer.*

(c) For the frequency found in part (b), what is the total number of samples taken in one period of $x(t)$?

PROBLEM 3.4:

Let $x(t) = 7\sin(11\pi t)$. In each of the following the discrete-time signal $x[n]$ is obtained by sampling $x(t)$ at a rate f_s; and the resultant $x[n]$ can be written:

$$x[n] = A\cos(\omega_0 n + \phi)$$

So for each part below, determine the values of A, ϕ and ω_0. In addition, state whether or not the signal has been oversampled or undersampled.

(a) Let the sampling frequency be $f_s = 10$ samples/sec.

(b) Let the sampling frequency be $f_s = 5$ samples/sec.

PROBLEM 3.5:

An amplitude modulated (AM) cosine wave is represented by the formula

$$x(t) = [10 + \cos(2\pi(2000)t)]\cos(2\pi(10^4)t)$$

Sketch the two-sided spectrum of this signal to determine the maximum frequency and also the minimum sampling rate f_s such that $x(t)$ can be reconstructed from its samples.

PROBLEM 3.6*:

Suppose that a discrete-time signal $x[n]$ is given by the formula

$$x[n] = 3\cos(0.2\pi n - \pi/2)$$

and that it was obtained by sampling a continuous-time signal at a sampling rate of $f_s = 8000$ samples/second.

(a) Determine two *different* continuous-time signals $x_1(t)$ and $x_2(t)$ whose samples are equal to $x[n]$; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/8000$ sec. Both of these signals should have a frequency less than 8000 Hz. Give a formula for each sinusoidal signal.

(b) If $x[n]$ is given by the equation above, what signal will be reconstructed by an ideal D-to-C converter operating at a sampling rate of 11,025 samples/second?

PROBLEM 3.7*:

In the rotating disk and strobe demo described in Chapter 4 of *DSP First*, we observed that different flashing rates of the strobe light would make the spot on the disk stand still.

(a) Assume that the disk is rotating in the clockwise direction at a constant speed of 900 rpm (revolutions per minute). Express the movement of the spot on the disk as a rotating complex phasor.

(b) If the strobe light can be flashed at a rate of n flashes *per second* where n is an integer greater than zero, determine all possible flashing rates such that the disk can be made to stand still.
NOTE: the only possible flashing rates are 1 per second, 2 per second, 3 per second, etc.

(c) If the flashing rate is 13 times per second, explain how the spot will move and write a complex phasor that gives the position of the spot at each flash.

(d) Draw a spectrum plot of the discrete-time signal in part (c) to explain your answer.

PROBLEM 3.8*:
A digital chirp signal is synthesized according to the following formula:

$$x[n] = \Re e\{e^{j\theta[n]}\} = \cos(\pi(0.7\times 10^{-3})n^2) \qquad \text{for } n = 0, 1, 2, \ldots, 200$$

(a) Make a plot of the rotating phasor $e^{j\theta[n]}$ for $n = 10$, 50 and 100.

(b) If this signal is played out through a D-A converter whose sampling rate is 8 kHz, make a plot of the instantaneous analog frequency (in hertz) versus time for the analog signal.

(c) If the *constant frequency* digital signal $v[n] = \cos(0.7\pi n)$ is played out through a D-A converter whose sampling rate is 8 kHz, what (analog) frequency will be heard?

PROBLEM 3.9:
A non-ideal D-to-C converter takes a sequence $y[n]$ as input and produces a continuous-time output $y(t)$ according to the relation

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

where $T_s = 0.001 = 10^{-3}$ second. The input sequence is given by the formula

$$y[n] = \begin{cases} \frac{1}{5}(n+1) & 0 \le n \le 4 \\ (0.5)^{(n-4)} & 5 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $y[n]$ versus n.

(b) For the pulse shape

$$p(t) = \begin{cases} 1 & -0.0005 \le t \le 0.0005 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

(c) For the pulse shape

$$p(t) = \begin{cases} 1 - 1000|t| & -0.001 \le t \le 0.001 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.