GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Fall 1998
Problem Set #4

Assigned: 27 Oct 1998
Due Date: 2 Nov 1998 (MONDAY)

Quiz #2 will be held in lecture on Friday 13-Nov-98. It will cover material from Chapters 3, 4, 5, and 6, as represented in Problem Sets #3, #4 and #5.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$)

Reading: In *DSP First*, Chapter 5 on *FIR Filters*.

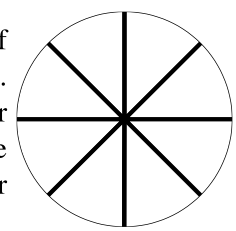
The web site for the course uses Web-CT: `http://webct.ece.gatech.edu`

$\Longrightarrow$ The six(6) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the "unstarred" problems.

PROBLEM 4.1*:

When watching old TV movies, all of us have seen the phenomenon where a wagon wheel appears to move backwards. The same illusion can also be seen in automobile commercials, when the car's hubcaps have a spoked pattern. Both of these are due to the 30 frames/sec sampling used in transmitting TV images.



In the figure to the right, an eight-spoked wheel is shown. Assume that the diameter of this wheel is two feet, which is almost exactly the tire diameter of a typical automobile. In addition, assume that the wheel is rotating CCW, so that if attached to a car, the car would be traveling to the left *at a constant speed*. However, when seen on TV the spoke pattern of the car wheel appears to stand still. How fast is the car traveling (in miles per hour)? Derive a general equation that will make it easy to give all possible answers.

PROBLEM 4.2:

A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{5} x[n-k]$$

The input to this system is *unit step* signal, denoted by $u[n]$:

$$x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

Compute $y[n]$, over the range $-5 \leq n \leq \infty$. Make a plot of $y[n]$ vs. n.

PROBLEM 4.3:

A linear time-invariant system is described by the difference equation: $y[n] = \sum_{k=0}^{5} x[n-k]$

The input to this system is a complex exponential signal:

$$x[n] = je^{j0.4\pi n} \qquad -\infty < n < \infty$$

Compute $y[n]$, over the range $-\infty \le n \le \infty$. Simplify as much as possible.

PROBLEM 4.4*:

A linear time-invariant system is described by the difference equation

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) When the input to this system is

$$x[n] = \begin{cases} 0 & n < 0 \\ n+1 & n = 0, 1, 2 \\ 5-n & n = 3, 4 \\ 1 & n \ge 5 \end{cases}$$

Compute the values of $y[n]$, over the range $0 \le n \le 10$.

(b) For the previous part, plot both $x[n]$ and $y[n]$.

(c) Determine the response of this system to a unit impulse input; i.e., find the output $y[n] = h[n]$ when the input is $x[n] = \delta[n]$. Plot $h[n]$ as a function of n.

PROBLEM 4.5*:

A linear time-invariant system is described by the difference equation

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2].$$

(a) Draw the implementation of this system as a block diagram in *direct form*.

(b) Give the implementation as a block diagram in *transposed direct form*.

PROBLEM 4.6*:

Consider a system defined by

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

(a) Suppose that the input $x[n]$ is non-zero only for $0 \le n \le N-1$; i.e., it has a support of N samples. Show that $y[n]$ is non-zero at most over a finite interval of the form $0 \le n \le P-1$. Determine P and the support of $y[n]$ in terms of M and N.

(b) Suppose that the input $x[n]$ is non-zero only for $N_1 \le n \le N_2$. What is the support of $x[n]$? Show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \le n \le N_4$. Determine N_3 and N_4 and the support of $y[n]$ in terms of N_1, N_2, and M.

Hint: Draw a sketch similar to Fig. 5.5 to illustrate the zero regions of the output signal.

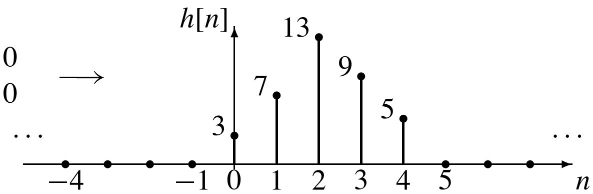
PROBLEM 4.7*:

Answer the following questions about the time-domain response of FIR digital filters:

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

(a) When tested with an input signal that is an impulse, $x[n] = \delta[n]$, the observed output from the filter is the signal $h[n]$ shown below:

$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases} \quad \longrightarrow$$



Determine the filter coefficients $\{b_k\}$ of the difference equation for the FIR filter.

(b) If the filter coefficients are $\{b_k\} = \{13,\ -13,\ 13\}$ and the input signal is

$$x[n] = \begin{cases} 0 & \text{for } n \text{ even} \\ 1 & \text{for } n \text{ odd} \end{cases}$$

determine the output signal $y[n]$ for all n. Give your answer as either a plot or a formula.

PROBLEM 4.8*:

For a particular LTI system, when the input is the *unit step* signal:

$$x_1[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

the corresponding output is

$$y_1[n] = \delta[n] + 2\delta[n-1] - \delta[n-2] = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ 2 & n = 1 \\ -1 & n = 2 \\ 0 & n \geq 3 \end{cases}$$

Determine the output when the input to the LTI system is $x_2[n] = 3u[n] - 2u[n-4]$. Give your answer as a formula expressing $y_2[n]$ in terms of known sequences, or give a list of values for $-\infty < n < \infty$.

PROBLEM 4.9:

Factor the following polynomial and plot its the root locations in the complex plane.

$$P(z) = 1 + \tfrac{1}{2}z^{-1} + \tfrac{1}{2}z^{-2} + z^{-3}$$

In MATLAB see the functions called `roots` and `zplane` (or `zzplane.m` from the EE-2200 Web-CT page). Note: $P(z)$ has a finite number of roots and is equal to zero at the root locations, so we often refer to the plot as a plot of the zeros of $P(z)$.