GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Fall 1998
Problem Set #6

Assigned: 16 Nov 1998
Due Date: 23 Nov 1998 (MONDAY)

Lab Quiz #2 will be held this week during the first part of your lab time.

$\Longrightarrow$ The five(5) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Many similar problems solutions can be found on the CD-ROM.

PROBLEM 6.1*:

Suppose that a LTI system has system function equal to

$$H(z) = 1 - 3z^{-2} - 7z^{-3} + 4z^{-5}$$

(a) Determine the difference equation that relates the output $y[n]$ of the system to the input $x[n]$.

(b) Determine and plot the output sequence $y[n]$ when the input is $x[n] = \delta[n]$.

PROBLEM 6.2:

A linear time-invariant filter is described by the difference equation

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

(a) Determine the system function $H(z)$ for this system.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.

(d) Sketch the frequency response (magnitude and phase) as a function of frequency for $-\pi \leq \hat{\omega} \leq \pi$.

(e) What is the output if the input is

$$x[n] = 4 + \cos[0.25\pi(n-1)] - 3\cos[(2\pi/3)n]$$

PROBLEM 6.3*:

Use the linearity and time delay properties to find the z-transforms of the following signals

$$\begin{aligned} x_1[n] &= \delta[n] \\ x_2[n] &= \delta[n-1] \\ x_3[n] &= \delta[n-7] \\ x_4[n] &= 2\delta[n] - 3\delta[n-1] + 4\delta[n-3] \end{aligned}$$

PROBLEM 6.4*:

The intention of the following MATLAB program is to filter a sinusoid via the `conv` function.

```
omega = pi/6;
nn = [ 0:300 ];
xn = cos(omega*nn - pi/4);
bb = [ 2  0  0  -2 ];
yn = conv( bb, xn );
```

(a) Determine $H(z)$ and also the zeros of the FIR filter.

(b) Determine a formula for $y[n]$, the signal contained in the vector `yn`. Give the individual values for $n = 0, 1, 2$, and then provide a general formula for $y[n]$ that is correct for $3 \leq n \leq 300$. This formula should give numerical values for the amplitude, phase and frequency of $y[n]$.

(c) Give a value of `omega` such that the output is guaranteed to be zero, for $n \geq 3$.

PROBLEM 6.5:

A linear time-invariant system has system function

$$\mathcal{H}(z) = (1 + z^{-2})(1 - 4z^{-2}) = 1 - 2z^{-2} - 4z^{-4}$$

The input to this system is

$$x[n] = 20 - 20\delta[n] + 20\cos(0.5\pi n + \pi/4)$$

Determine the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all n. (*Note: This is an easy problem if you approach it correctly!*)

PROBLEM 6.6*:

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\mathcal{S}_1: \quad y_1[n] = 3x_1[n] - 3x_1[n-1]$$

$$\mathcal{S}_2: \quad y_2[n] = 2x_2[n] + 2x_2[n-2]$$

$$\mathcal{S}_3: \quad y_3[n] = x_3[n-1] + x[n-2]$$

NOTE: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

Determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

(a) Determine the z-transform system function $H_i(z)$ for each system.

(b) Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.

PROBLEM 6.7*:

The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 2\cos(6000\pi t - \pi/4) + 11\cos(12000\pi t - \pi/3)$$

The system function for the LTI system is

$$H(z) = \frac{1}{4}(1 - z^{-4})$$

If $f_s = 8000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.