

Lecture 15

H(z) & Frequency Response

8-March-99

Info: Web-CT, Lab, HW

- Calendar:
 - Final Exam is Period 11
 - Review Session planned
- Grade Weightings soon
- Prob Set #7 due on Friday
- Last Lab is on-Line & in-Lab

READING ASSIGNMENTS

- This Lecture:
 - Chapter 8, pp. 263–279
- Other Reading:
 - Recitation: Ch. 8, pp. 261–272
 - POLES & ZEROS
 - Next Lecture: Chapter 8, pp. 279–300

LECTURE OBJECTIVES

- SYSTEM FUNCTION: $H(z)$
- $H(z)$ has POLES and ZEROS
- FREQUENCY RESPONSE of IIR
 - Get $H(z)$ first

$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- THREE-DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

IIR FILTER REVIEW

ADD PREVIOUS OUTPUTS

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

FEEDBACK COEFFICIENT

SIGN CHANGE

MATLAB

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yy = filter([3,-2],[1,-0.8],xx)
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IMPULSE RESPONSE

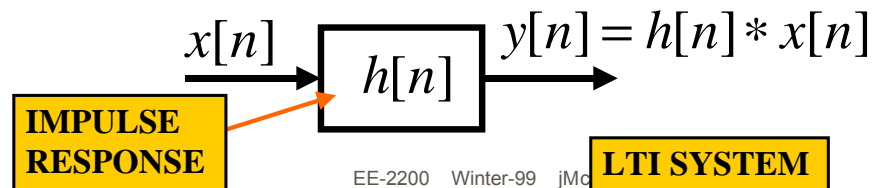
DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 3x[n]$$

Find $h[n]$

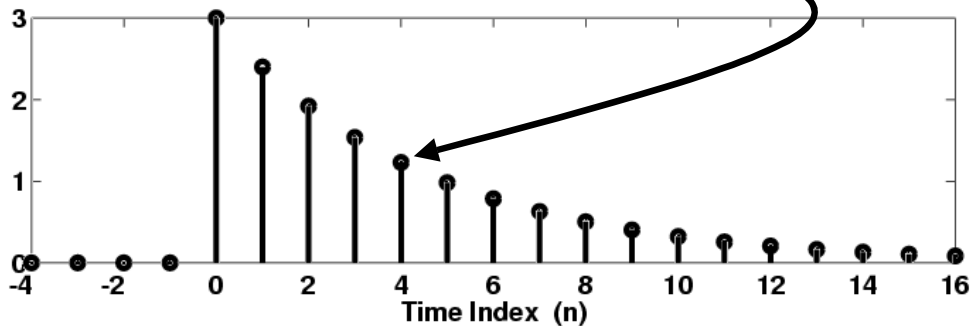
$$h[n] = 3(0.8)^n u[n]$$

CONVOLUTION in TIME-DOMAIN



PLOT IMPULSE RESPONSE

$$h[n] = b_0(a_1)^n u[n] = 3(0.8)^n u[n]$$



THREE DOMAINS

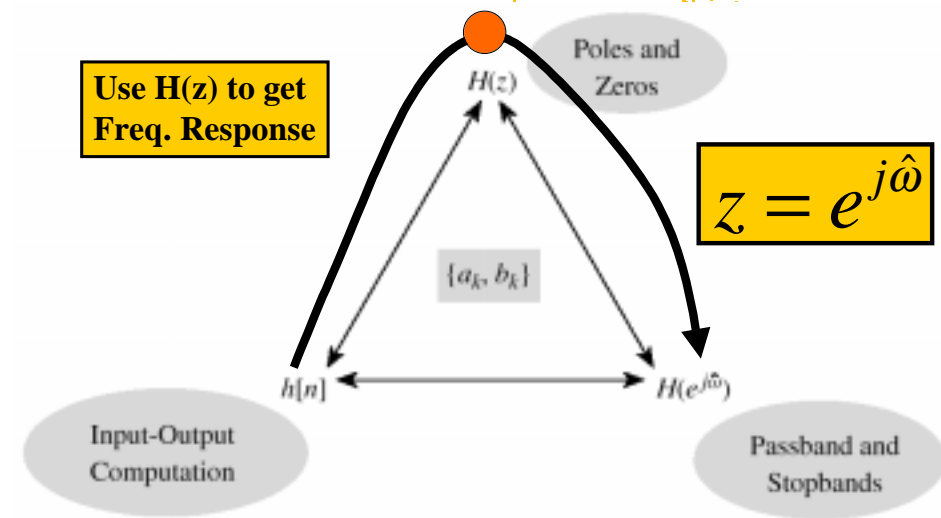


Figure 8.13 Relationship among the n -, z -, and $\hat{\omega}$ -domains. The filter coefficients $\{a_k, b_k\}$ play a central role.

Z-Transform DEFINITION

POLYNOMIAL Representation

$$H(z) = \sum_n h[n] z^{-n}$$

APPLIES to Any SIGNAL

IIR EXAMPLE?

$$h[n] = b_0 (a_1)^n u[n] = 3(0.8)^n u[n]$$

$$H(z) = \sum_n 3(0.8)^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} 3(0.8)^n z^{-n} = \frac{3}{1 - 0.8z^{-1}}$$

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Z-Transform of IIR Filter

DERIVE the SYSTEM FUNCTION $H(z)$

Use DELAY PROPERTY

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

$$Y(z) = a_1 z^{-1} Y(z) + b_0 X(z) + b_1 z^{-1} X(z)$$

EASIER with DELAY PROPERTY

Time delay of n_0 samples multiplies the z -transform by z^{-n_0}

$$x[n - n_0] \iff z^{-n_0} X(z)$$

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SYSTEM FUNCTION of IIR

NOTE the FILTER COEFFICIENTS

$$Y(z) - a_1 z^{-1} Y(z) = b_0 X(z) + b_1 z^{-1} X(z)$$

$$(1 - a_1 z^{-1}) Y(z) = (b_0 + b_1 z^{-1}) X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{B(z)}{A(z)}$$

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CONVOLUTION PROPERTY

MULTIPLICATION of z -TRANSFORMS

$$X(z) \xrightarrow{\quad} \boxed{H(z)} \xrightarrow{\quad} Y(z) = H(z) X(z)$$

CONVOLUTION in TIME-DOMAIN

$$x[n] \xrightarrow{\quad} \boxed{h[n]} \xrightarrow{\quad} y[n] = h[n] * x[n]$$

IMPULSE RESPONSE

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SYSTEM FUNCTION

DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

SYSTEM FUNCTION:

$$Y(z) = \left(\frac{3 - 2z^{-1}}{1 - 0.8z^{-1}} \right) X(z)$$

H(z) →

H(z) = z-Transform{ h[n] }

FIRST-ORDER CASE:

$$b_0 = 1, a_1 = a$$

$$h[n] = a^n u[n]$$

$$H(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \frac{1}{1 - az^{-1}}$$

$$|a| < z$$

$$a^n u[n] \iff \frac{1}{1 - az^{-1}}$$

POLES & ZEROS

ROOTS of NUMERATOR & DENOMINATOR

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0 z + b_1}{z - a_1}$$

$$b_0 z + b_1 = 0 \implies z = -\frac{b_1}{b_0} \quad \text{ZERO}$$

$$z - a_1 = 0 \implies z = a_1 \quad \text{POLE}$$

INTERPRET ROOTS

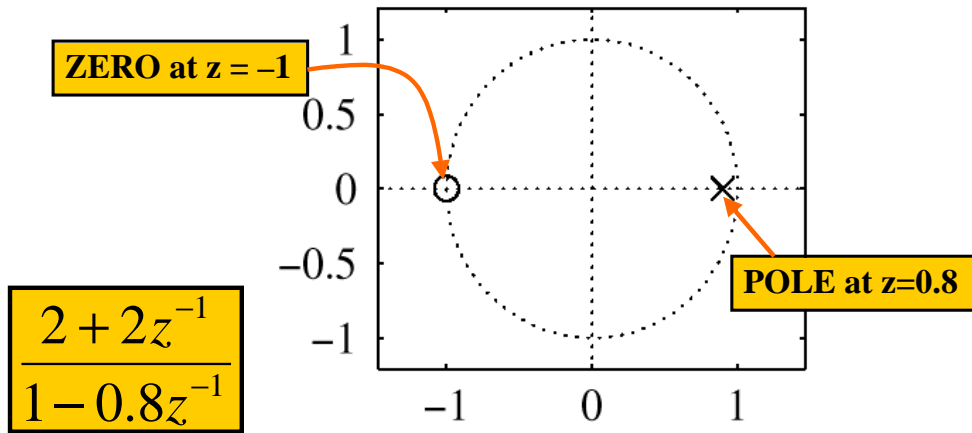
VALUE of H(z) at POLES is INFINITE

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0 z + b_1}{z - a_1}$$

$$H(z) \Big|_{z=-(b_1/b_0)} = 0 \quad \text{ZERO}$$

$$H(z) \Big|_{z=a_1} \rightarrow \infty \quad \text{POLE}$$

POLE-ZERO PLOT



FREQUENCY RESPONSE

- SYSTEM FUNCTION: $H(z)$
- $H(z)$ has DENOMINATOR
- FREQUENCY RESPONSE of IIR

■ We have $H(z)$

$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- THREE-DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

FREQUENCY RESPONSE

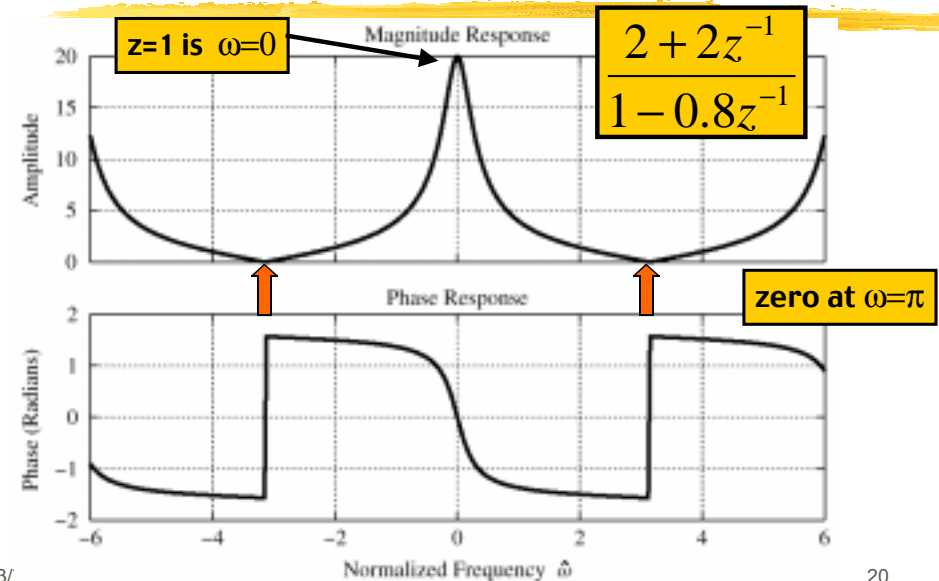
- EVALUATE on the UNIT CIRCLE

$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

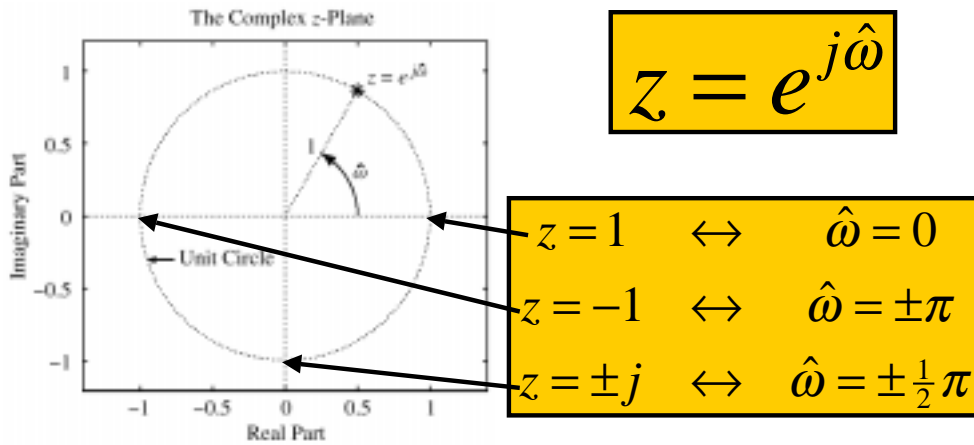
$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}} = \frac{b_0 + b_1 e^{-j\hat{\omega}}}{1 - a_1 e^{-j\hat{\omega}}}$$

PLOT FREQ. RESPONSE



UNIT CIRCLE

MAPPING BETWEEN z and $\hat{\omega}$



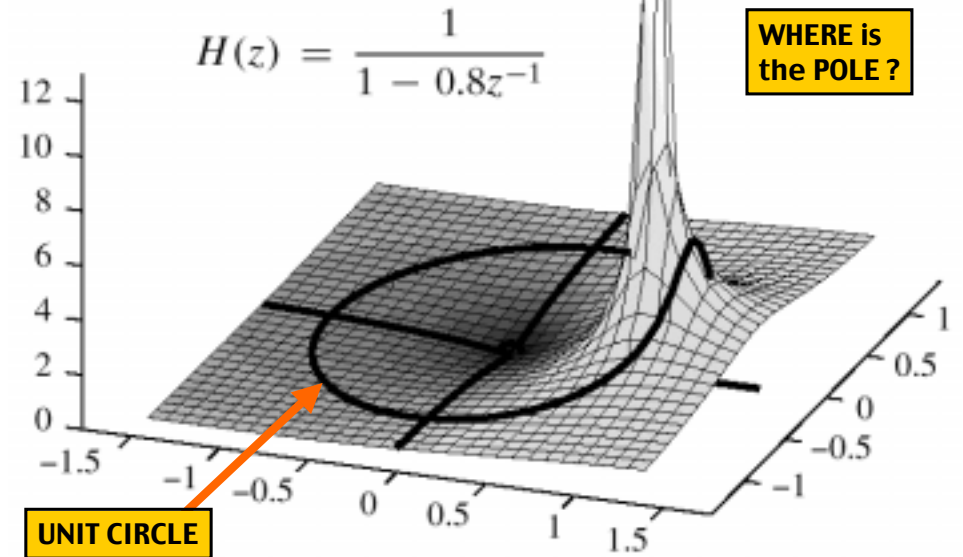
$$z = e^{j\hat{\omega}}$$

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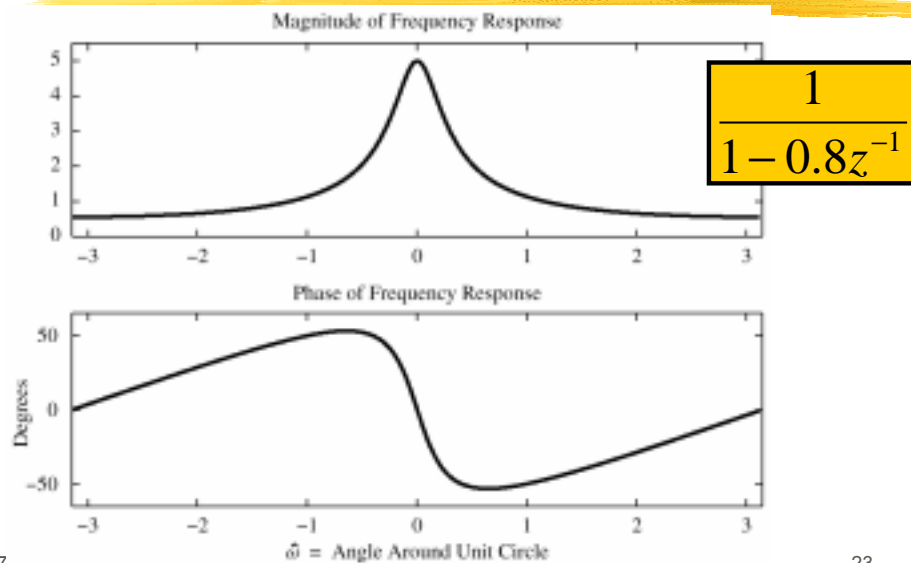
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3-D VIEWPOINT: EVALUTE $H(z)$ EVERYWHERE



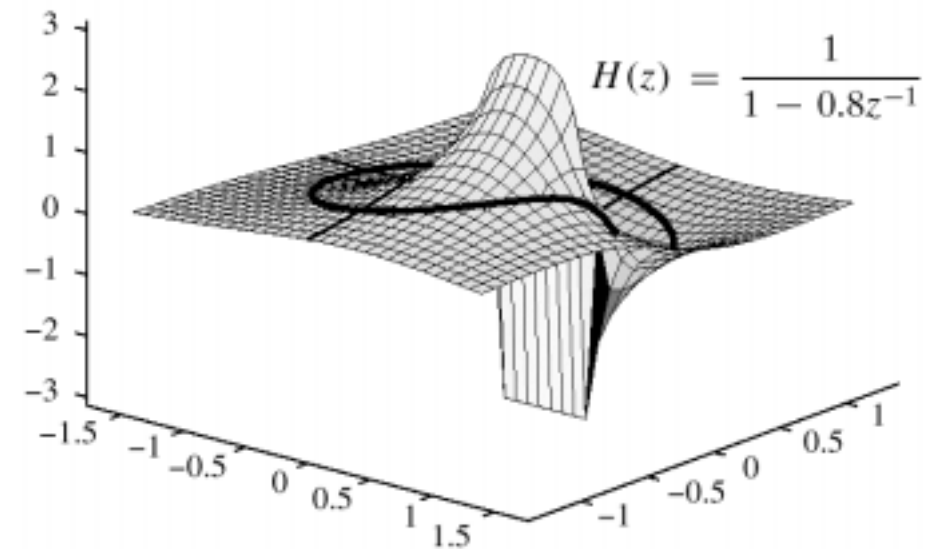
FREQ. RESPONSE from 3-D



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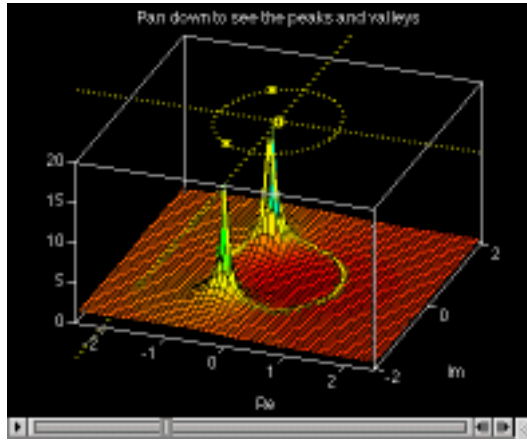
PHASE from 3-D PLOT



MOVIE for H(z) in 3-D

POLES to H(z) to Frequency Response

TWO POLES SHOWN



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THREE DOMAINS

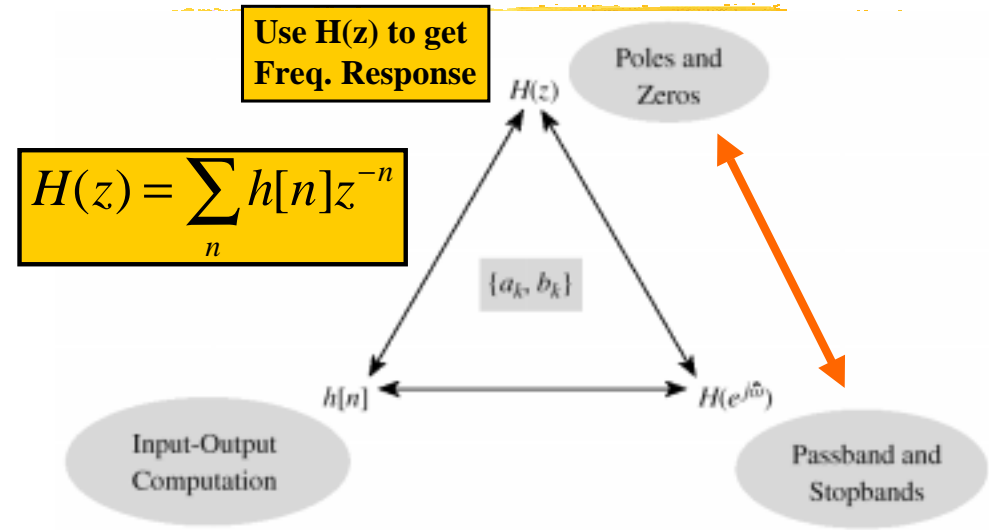
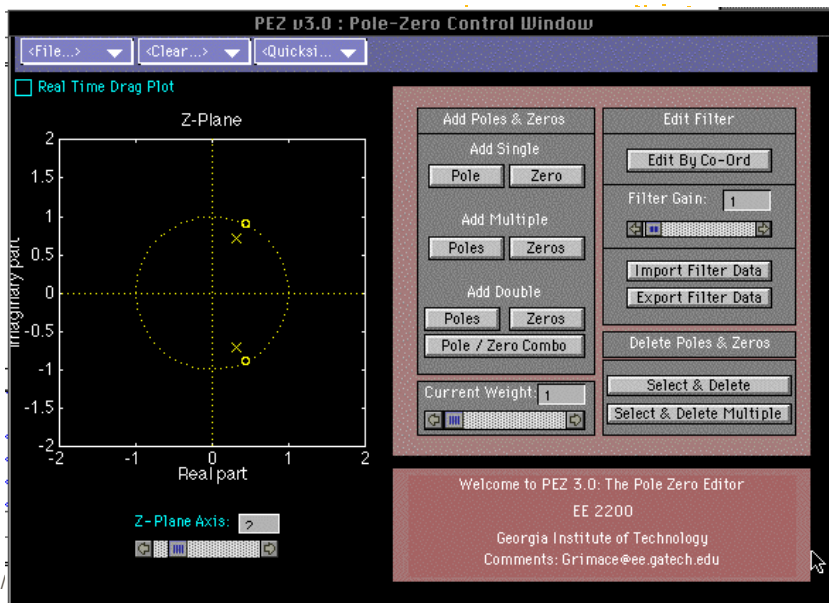


Figure 8.13 Relationship among the n -, z -, and $\hat{\omega}$ -domains. The filter coefficients $\{a_k, b_k\}$ play a central role.

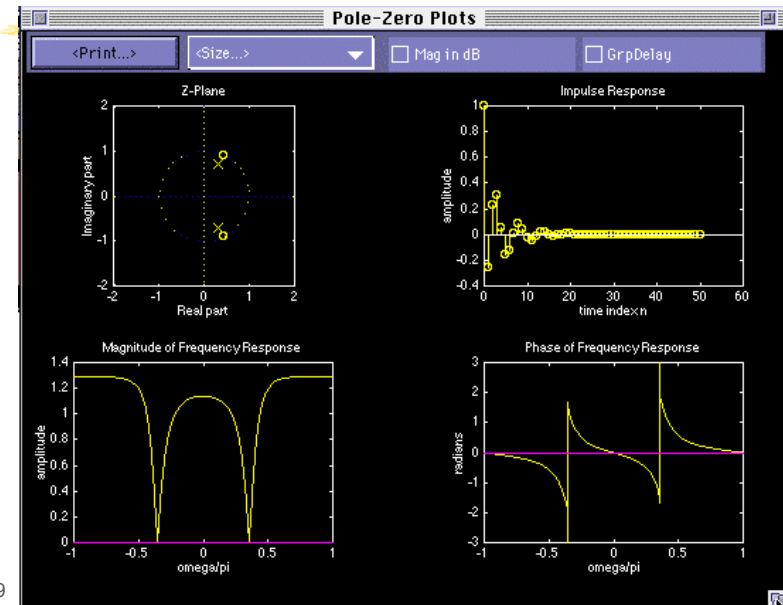
PEZ for Lab #9



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PEZ shows 3 DOMAINS



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