# EE-2200 Winter-99

**Lecture 6**

**Time-Varying Frequency**

**29-Jan-99**

# Information

- Prob Set #3 due TODAY in Lecture
  - Solutions will be posted later on
- Quiz #1 on 1–Feb (Monday)
  - In ECE Auditorium
  - Super Bowl make-up: Sat at 4 PM
    - Also in VanLeer (room 240)
- Lab #4 will be posted over weekend
  - Music Synthesis Lab (learn notation)

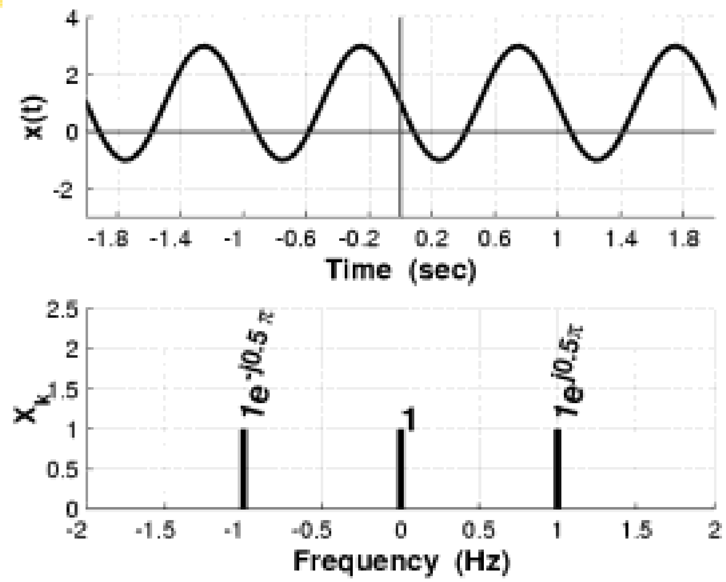


# READING ASSIGNMENTS

- This Lecture:
  - Chapter 3, pp. 68–77
- Other Reading:
  - Notes on Fourier Series
    - (3 pages posted to WebCT)
  - Next Lecture: start Chapter 4



# X(t) <---> $X_k$ ?



# LECTURE OBJECTIVES

- Frequency can change **vs. time**
  - Introduce Spectrogram Visualization
- Sinusoidal Synthesis
  - Add lots of short–duration sinusoids
    - With DIFFERENT starting times
    - And DIFFERENT frequencies
  - Speech Modeling Example



# Time-Varying FREQUENCIES Diagram

*Frequency is the vertical axis*


**Figure 3.18** Sheet-music notation is a time–frequency diagram.

*Time is the horizontal axis*



# Fourier Series Expansion

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$X_k = A_k e^{j\varphi_k}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{N} \tfrac{1}{2} X_{-k} e^{-j2\pi k f_0 t}$$

$N \rightarrow \infty$ ???



# Fourier Series Integral

- Determine $X_k$ from x(t)

$$X_k = \frac{2}{T_0} \int_0^{T_0} x(t) e^{-j2\pi kt/T_0} \, dt$$

$f_0 = 1/T_0$

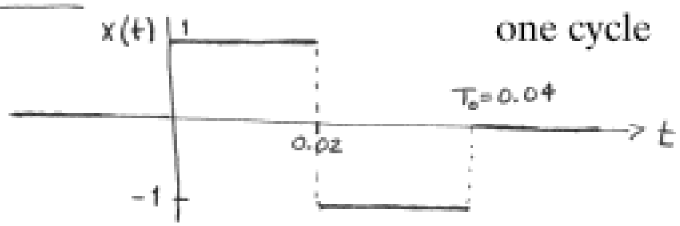
$$X_0 = \frac{1}{T_0} \int_0^{T_0} x(t) \, dt$$

# SQUARE WAVE (50% duty cycle)

$$x(t) = \begin{cases} 1 & 0 \le t < \frac{1}{2}T_0 \\ -1 & \frac{1}{2}T_0 \le t < T_0 \end{cases} \qquad (3.4.4)$$

Draw a plot of the square wave defined in (3.4.4) for $T_0 = 0.04$ sec.

Ex 3.3






# FS for 50% SQUARE WAVE

- Only the ODD HARMONIC are present
  - Depends on 50% duty cycle !
  - Phase is $\pi/2$ for k>0; $-\pi/2$ for k<0

$$X_k = \begin{cases} \dfrac{4}{j\pi k} & k = \pm1, \pm3, \pm5, \ldots \\ 0 & k = 0, \pm2, \pm4, \pm6, \ldots \end{cases} \qquad (3.4.5)$$



# Fourier Series Spectrum

The magnitude of these coefficients is shown in Fig. 3.12. The phase angles are $-\pi/2$ for $k > 0$, and $\pi/2$ for $k < 0$. Note that if $f_0 = 1/T_0 = 25$ Hz, only the frequencies at $\pm25$, $\pm75$, $\pm125$, etc. are in the spectrum.

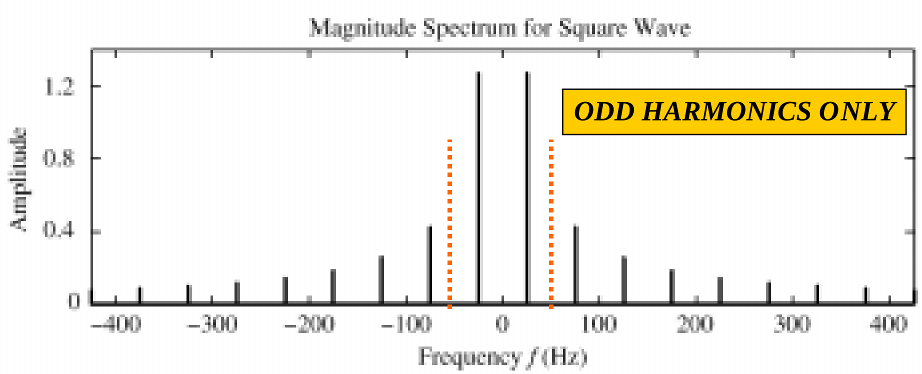



**Figure 3.12** Spectrum of the square-wave signal whose Fourier series coefficients are given in (3.4.5) with $f_0 = 1/T_0 = 25$ Hz.

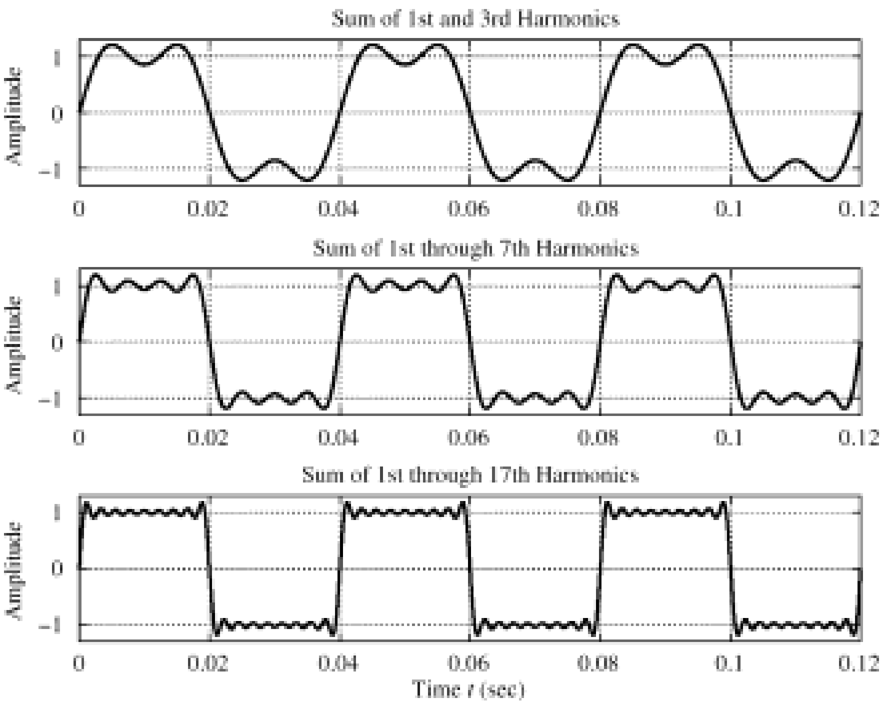
# Fourier Series SUM

- Add a FINITE number of terms
  - How close to the original x(t) ?

$$X_k = A_k e^{j\varphi_k}$$

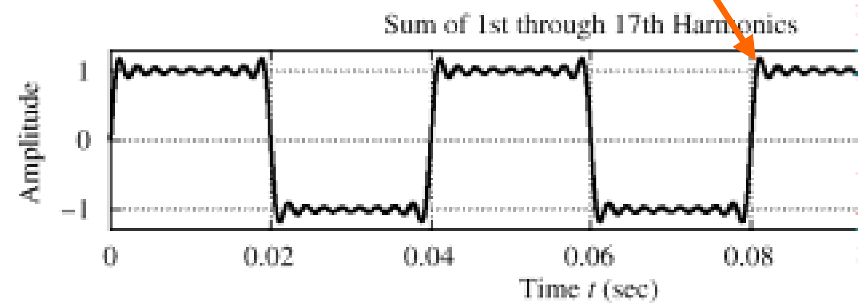
$$x_N(t) = X_0 + \sum_{k=1}^{N} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{N} \tfrac{1}{2} X_{-k} e^{-j2\pi k f_0 t}$$

$$x(t) = \lim_{N\to\infty} x_N(t) \quad ???$$




# Gibbs' Phenomenon

- Convergence at DISCONTINUITY of x(t)
  - There is ALWAYS an overshoot
  - 9% for the Square Wave case

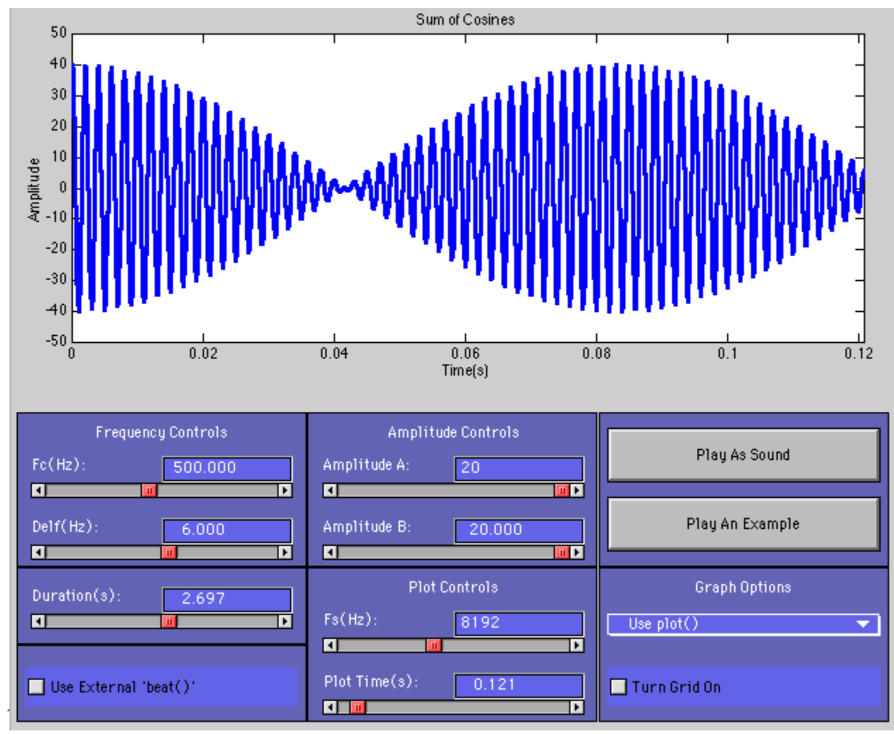




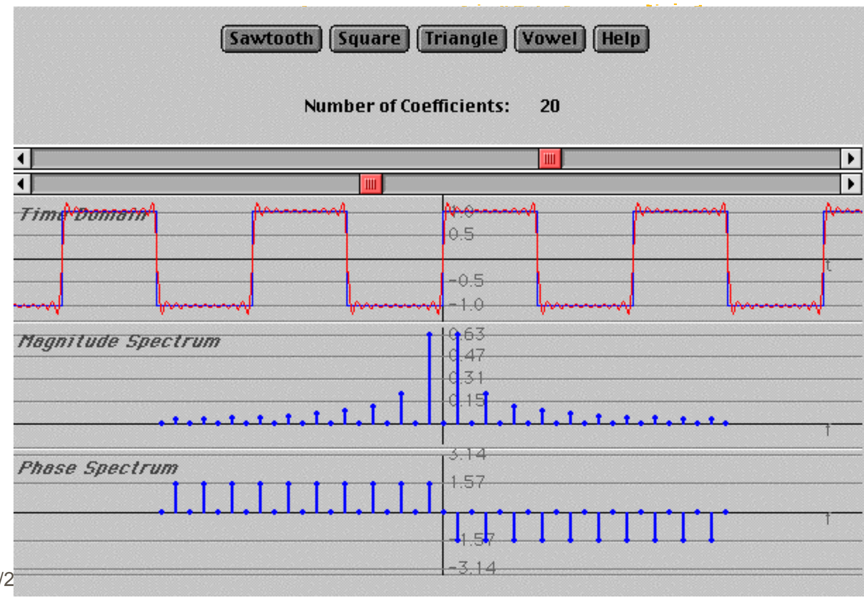

# A Couple of DEMOS

- Beat Control GUI
  - DSPFirst Toolbox: MATLAB
    - DSPFIRST/beatcon.m
- Fourier Series Java Applet
  - Interactive
  - http://users.ece.gatech.edu/~slabaugh/java/fourier/fourier.html





# Fourier Series Java Applet






# Time-Varying FREQUENCIES Diagram


Frequency is the vertical axis

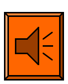
**Figure 3.18** Sheet-music notation is a time–frequency diagram.

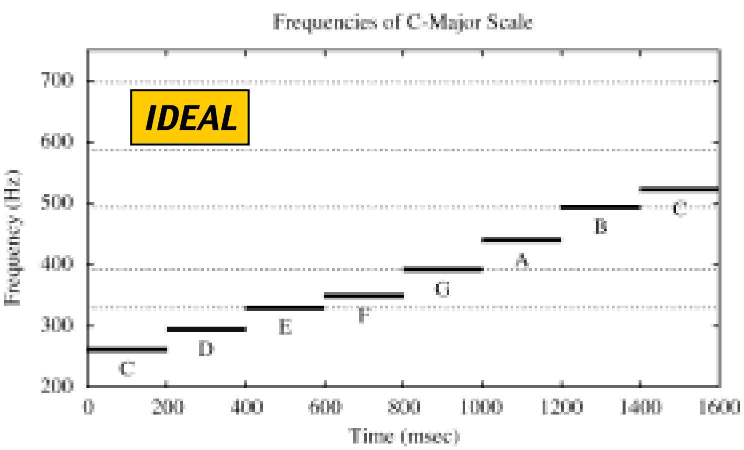
*Time is the horizontal axis*



# STEPPED FREQUENCIES

- C–major SCALE: successive sinusoids
  - Frequency is constant for each note





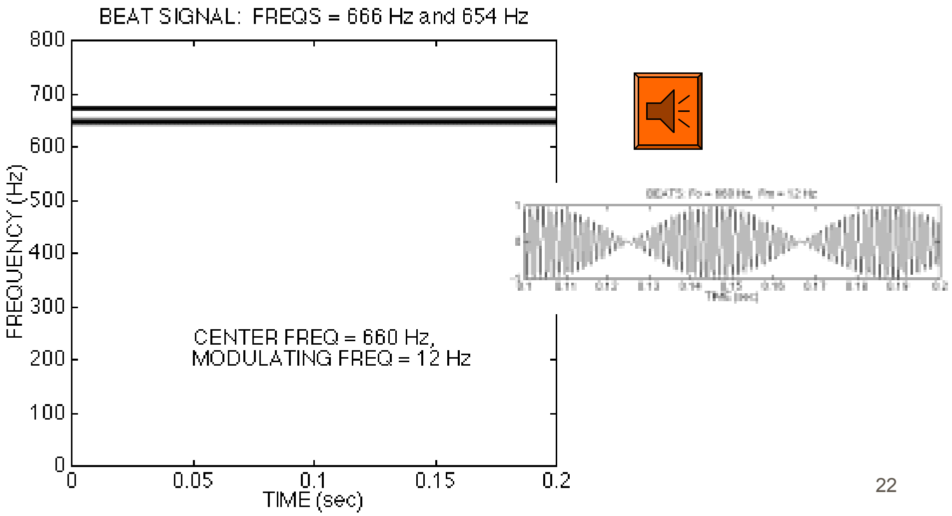



# R-rated: ADULTS ONLY

- SPECTROGRAM Tool
  - MATLAB function is specgram.m
  - DSP First has spectgr.m (NO PLOTTING)
- ANALYSIS program
  - Takes x(t) as input
  - Produces spectrum values $X_k$
  - OVER a SHORT TIME: the ANALYSIS FRAME
    - Uses the FFT (Fast Fourier Transform)

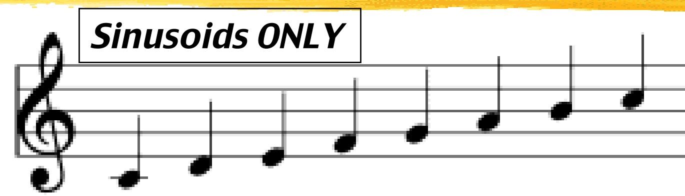
# SPECTROGRAM EXAMPLE

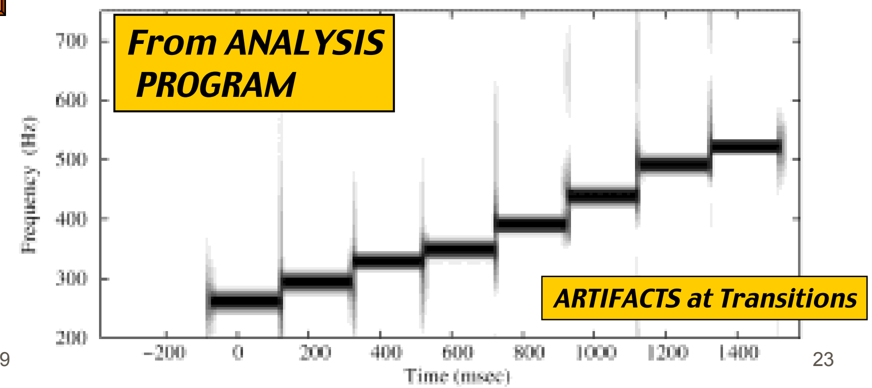
- Two Constant Frequencies: Beats





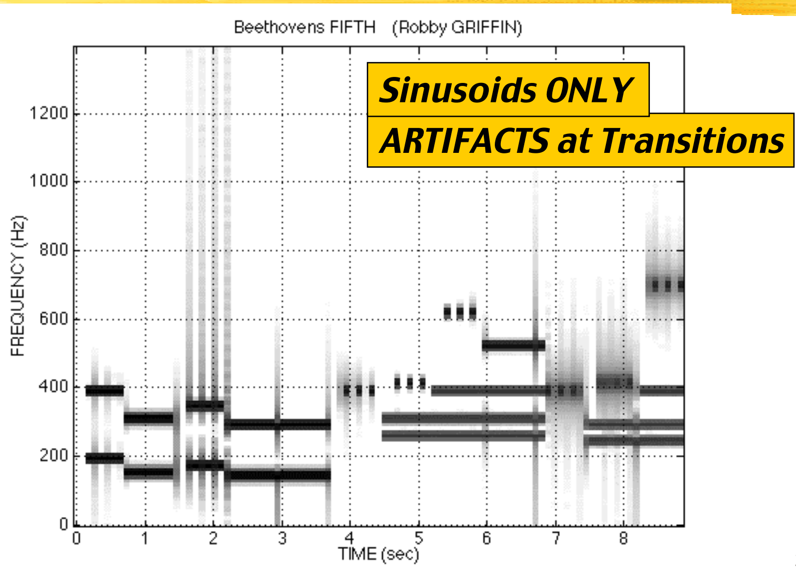

# SPECTROGRAM of C-Scale






# Spectrogram of LAB SONG






# Sinusoidal Synthesis

- Use Short-Duration Sinusoids:
  - Amp, Phase, Frequency & Duration

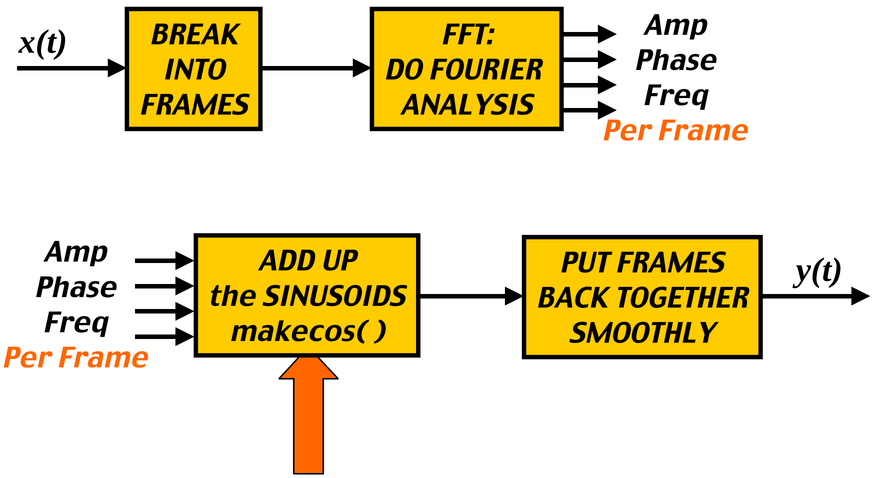
$$x(t) = A_k \cos(2\pi f_k t + \varphi_k) \quad \text{for} \quad t_k \le t \le t_{k+1}$$

- Freq will change every FRAME

$$t_k \le t \le t_{k+1}$$

- Then ADD several sinusoids together

# ANALYSIS --> SYNTHESIS





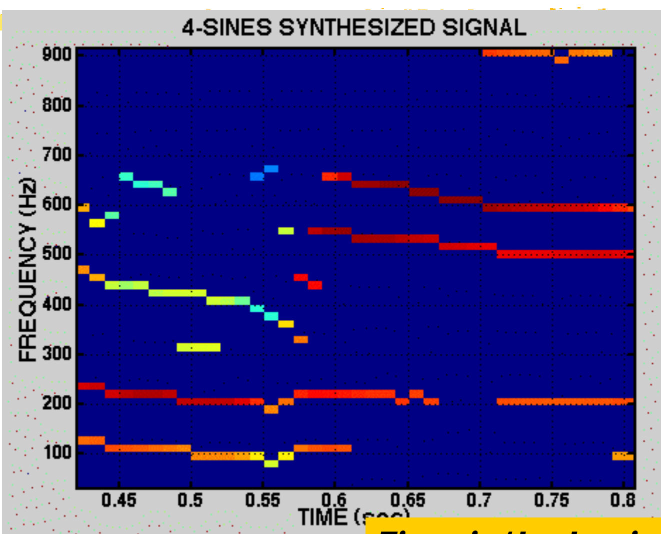

# Sine Synthesis: SPEECH

- FRAME Length = 10 millisec
- Examples:
  - Original
  - 9 sinusoids per frame
  - 4 sinusoids
  - 2 sinusoids
- Need to SMOOTH Boundaries



# Time-Varying FREQUENCIES Diagram

*Frequency is the vertical axis*

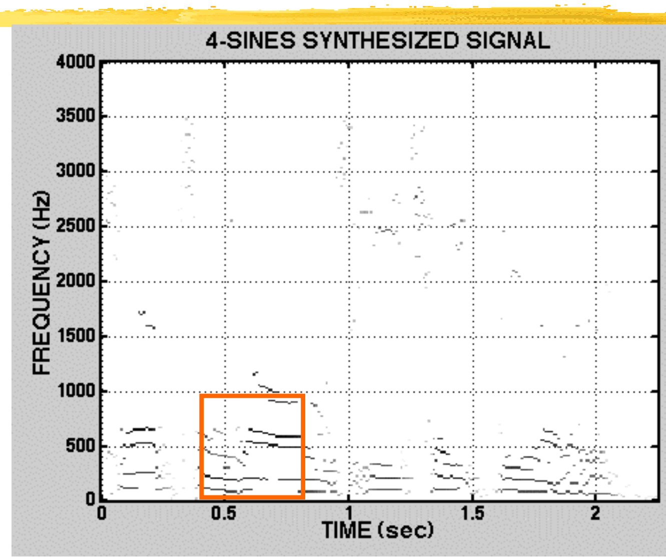



*Time is the horizontal axis*



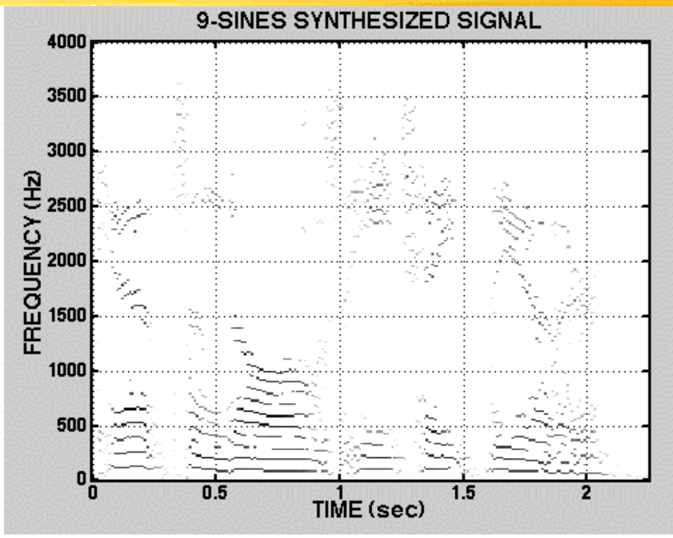
# 4-SINES Diagram

*Frequency is the vertical axis*



# 9-SINES Diagram

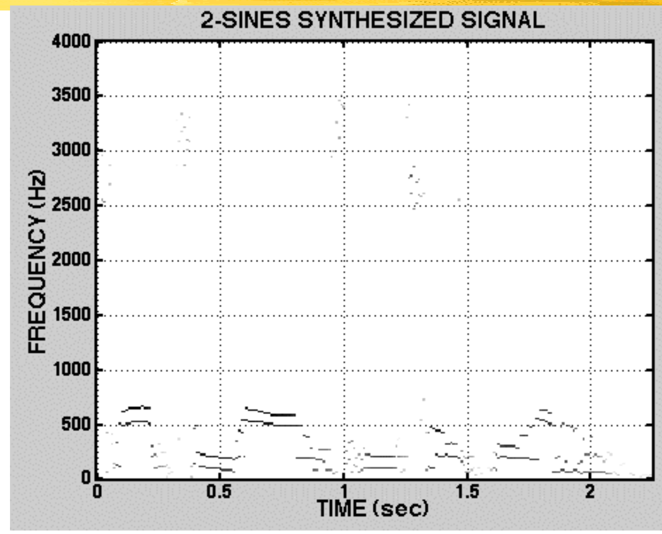
*Frequency is the vertical axis*





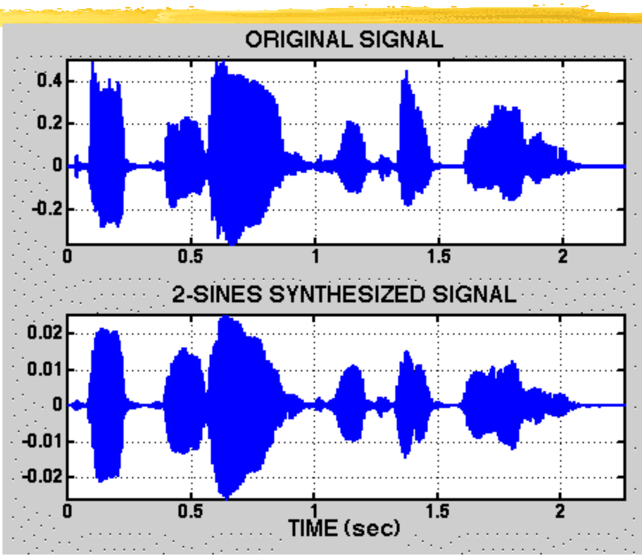

# 2-SINES Diagram

*Frequency is the vertical axis*






# TIME SIGNALS: COMPARE






# TIME SIGNALS: ZOOM