GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Project No. 1

Date Assigned: May 8, 1998
Date Due: May 25, 1998

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Overview
In order to get a better understanding of signals and systems, it is helpful to work a few problems where you get to create your own systems and test them with different kinds of inputs.
In this project, you will be using MATLAB a new way. You will be using the Symbolic Math Toolbox. This toolbox is found on the HP PC's and UNIX workstations in CoC rooms 304, 309 and 311. (Please make sure that you have access to these machines. The Symbolic Math Toolbox is NOT found in the student edition of MATLAB. The functions in the Symbolic Math Toolbox can be used to represent, manipulate and analyze continuous-time signals and systems symbolically rather than numerically. The warm up section will prepare you not only for this project but for the future ones as well

Warm up:
Sinusoids and complex exponentials are very useful for analyzing signals and systems since they form the building blocks for a large class of signals.

(a) Consider the continuous time sinusoid

$$x(t) = \sin(2\pi t/T) \tag{1}$$

This creates a symbolic variable x that is represented by the above equation. The variables of x are the single character string 't' and 'T'. You can use the function `ezplot` to plot a symbolic expression. However, the expression can only have one variable. Suppose you desire to use T=5, you can use the substitution function subs as follows:
```
>> x5 = subs(x,5,'T');
```
To plot two periods of the function x5, you can use:
```
>> ezplot(x5,0,10);
```

(b) Now, create a symbolic expression for the following signal

$$x(t) = \cos(2\pi t/T)\sin(2\pi t/T)$$

The two sinusoids should be created separately, and then combined using `symmul`. For T = 4,8, and 16 use `ezplot` to plot the signal on the interval $0 \le t \le 32$. What is the fundamental period of x(t) in terms of T? You can also do many symbolic functions such as integration and differentiation. In MATLAB create an expression that performs the integral of $x(t)$ with respect to the variable t

$$y = \int x(t)\ dt$$

Now evaluate the following integral:

$$v = \int_{-6}^{7.5} \cos(2\pi t/3)\sin(2\pi t/9)\ dt$$

Hint: Do help on the function `int`.
What is v equal to? Explain your results (Include your MATLAB code in your report).

(c) Store in x a symbolic expression for

$$x(t) = e^{j2\pi t/16} + e^{j2\pi t/8}$$

The function `ezplot` cannot be used to plot $x(t)$ since it is complex.

i) Write a function `xr=sreal(x)` which returns a symbolic expression representing the real part of $x(t)$.

ii) Write a function the returns the imaginary part of x(t), i.e. `xi = simag (x)`.

iii) Create two functions `xm=sabs(x)` and `xa=sangle(x)` which create the symbolic expressions representing the magnitude and phase respectively.

iv) Use ezplot to plot the magnitude and phase of x(t) as well as its real and imaginary parts on the interval $0 \le t \le 32$. Why is the phase plot discontinuous?

(d) Step functions are a little trickier to create. Look at the following function:

$$f(t) = t(u(t) - u(t-2))$$

where $u(t)$ is the unit step function

$$u(t) = \begin{cases} 1, & t \ge 0; \\ 0, & t < 0. \end{cases}$$

The Symbolic Math Toolbox calls the unit step function Heaviside. Download the function code from the course web page at:
`http://www.ece.gatech.edu/users/rschafer/ee3230/mfiles`
Use the Symbolic toolbox function compose to create and plot the following functions:

g1(t) = f(-t)
g2(t) = f(t+1)
g3(t) = f(t-3)
g4(t) = f(-t+1)
g5(t) = f(-2t+1)

Try to visualize the result before you see the plot.

Synthesis of Continuous Time Signals

For these problems you will construct symbolic expressions for periodic signals with a small number of nonzero Fourier coefficients.

(a) $x_1(t) : T = 1; a_1 = a_{-1} = 5, a_3 = a_{-3} = 2;$

(b) $x_2(t) : T = 2; a_1 = a^*_{-1} = j, a_2 = a^*_{-2} = j/2, a_3 = a^*_{-3} = j/4, a_4 = a^*_{-4} = j/8;$

(c) $x_2(t) : T = 4; a_1 = a^*_{-1} = j, a_2 = a^*_{-2} = j/2, a_3 = a^*_{-3} = j/4, a_4 = a^*_{-4} = j/8;$

Plot each signal using `ezplot`. How could you have predicted from the Fourier coefficients that each of that each of the signals would be real?

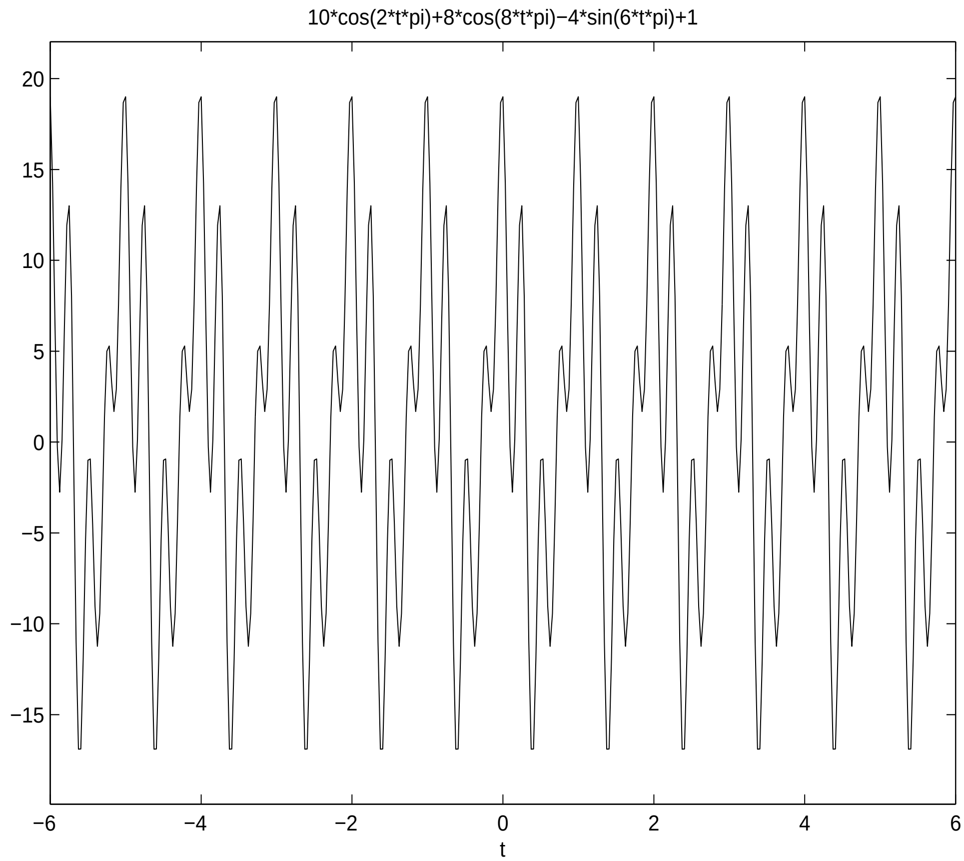
Now you should be able to write an M-file which synthesizes a signal $x(t)$ from its Continuous Time Fourier Series Coefficients when the coefficients are zero outside the interval $-N \leq k \leq N$. The first line of your function should look like

`function x = ctsynth(a,T,N)`

x should be in its simplest form. MATLAB has the ability to simplify expressions. If a function can be simplified, MATLAB has a function simplify() that does that for you. The commands below show how you should use your function.

`>> T = 1; >> N = 4; >> a = sym('[4 -2*j 0 5 1 5 0 2*j 4]'); >> x = ctsynth(a,T,N);`

Then use `ezplot` to verify your signal. Your plot should look like figure (1).



Triangular Wave

Consider a periodic striangle wave with period T = 2. Over the interval $-1 \leq t \leq 1$ the triangle wave is described by $x(t) = t$.

(a) Using `int`, create a symbolic expression, a, which contains the Fourier series coefficients of the triangular wave for each value of k. The symbolic expression will be a function of k, e.g. a_5 is given by `numeric(subs(a,5,'k'))`. Simplify the expression as much as possible. Use the stem function to plot the Fourier series coefficients for k between -10 and 10. Note: MATLAB may not be able to evaluate a for k = 0 since MATLAB cannot evaluate expressions like $sin(x)/x$ at $x = 0$. In this case, calculate a_0 separately.

(b) Create a symbolic expression for $x_N(t)$ for N = 1,3,5, and 9. Use ezplot to plot $x_N(t)$ on the interval $-1 \leq t \leq 1$. Plot all four signals on the same figure. What is the value of $x_N(t)$ at $t = \pm 0.5$? Does the value change as N increases?

(c) *Differentiation in Time.* A signal $x(t)$ can be differentiated in the time domain if its Fourier transform $X(j\omega)$ is multiplied by the factor $j\omega$. Refer to section 4.3.4 in your text. Create a symbolic expression for $x(t)$ using N=12 and T=4. Plot 3 periods of x(t). Now create a new vector of coefficients b_k by multiplying the coefficients a_k by $j\omega$. Using the new coefficients b_k synthesize a new signal $xd(t)$. On the same figure, plot 3 periods of xd. What can you tell about the relationship between $x(t)$ and $xd(t)$? What would $xd(t)$ look like as N increases?

(d) *Integration in Time.* How would you create a signal $xi(t)$ that is the integral of the original signal $x(t)$? Be careful when you try to evaluate the Fourier coefficient at

$\omega = 0$. You might want to do that by hand and then place it in the proper position in your vector of Fourier coefficeints. Show your plot on a different figure using 3 preiods.

(e) *Time Shift*. Going back to $x(t)$ create, $xs(t) = x(t - t0)$, a shifted synthesized verion of $x(t)$. Use $t_0 = 3$. Note how a phase shift in the frequency domain translate into a time shift. Show your plot on a different figure using 3 periods.

Note: You need to create $xd(t)$,$xi(t)$ and $xs(t)$ by varying the Fourier coefficients a_k and *NOT* by changing the original signal $x(t)$.

Report
Your report should be *TYPED*. It should be concise and to the point. Include *ALL* MATLAB code that you write. Include *ALL* plots. You can mark with a pencil on the plots to indicate characteristics (e.g. period) of a function. In your write up, you should include all MATLAB results that you get. You can do this by copying and pasting from the MATLAB to your word processor. Answer all questions asked and provide the appropriate explanations when necessary.
If you have any questions, please send email or stop by during office hours.
Good Luck.