GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2823
Problem Set No. 3

Date Assigned: January 22, 1999
Date Due: January 29, 1999

Reading Assignment: In Kamen and Heck, read pp. 141-184, 191-194, and 206-235.

Homework Assignment: Turn in for grading only the starred problems: 3.1*, 3.3*, 3.4*, 3.6*, and 3.7*.

Problem 3.1*

Consider the signal $x(t)$, whose Fourier transform is

$$X(j\omega) = \begin{cases} 10 & -2\pi < \omega < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

This is the signal $x(t)$ that is referred to in all the problems below.

(a) $x(t)$ is the input to a linear time-invariant system whose impulse response is

$$h(t) = \frac{\sin[\pi(t-2)]}{\pi(t-2)}$$

Use Fourier transforms to determine an equation for the output $y(t) = x(t) * h(t)$ of the LTI system.

(b) Another signal has Fourier transform $Y(j\omega) = X(j(\omega - 10\pi)) + X(j(\omega + 10\pi))$. Plot the Fourier transform $Y(j\omega)$ of this signal. Use a theorem of Fourier transforms to determine the signal $y(t)$.

(c) Still another signal is $v(t) = (x(t))^2$. Determine and plot the Fourier transform $V(j\omega)$ of this signal.

(d) Give an equation for the Fourier transform of $w(t) = x(t-5)$.

(e) Plot the magnitude and phase of $R(j\omega)$, the Fourier transform of the signal $r(t) = x^{(1)}(t)$.

Problem 3.2:

Consider the periodic signal $x(t)$, which is defined over one period by

$$x(t) = \begin{cases} 1 & -2 < t < 0 \\ 0 & 0 < t < 2 \end{cases}$$

The period of the signal is $T = 4$.

(a) The signal $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k. Sketch the spectrum of the input signal as a function of ω.

(b) The frequency response of a LTI highpass filter is

$$H(j\omega) = \begin{cases} 0 & |\omega| < 5\pi/4 \\ e^{-j\omega} & 5\pi/4 < |\omega| \end{cases}$$

Plot the magnitude of the frequency response, $|H(j\omega)|$ on the same graph as your spectrum plot. What is the effect of the factor $e^{-j\omega}$ on the output waveform?

(c) Determine the output of the system for the given input $x(t)$. Give the simplest possible equation for your answer.

Problem 3.3*:

Consider an LTI system for which the input $x(t)$ and output $y(t)$ satisfy the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + x(t)$$

(a) Determine the frequency response, $H(j\omega)$, of this system.

(b) If the input signal is

$$x(t) = \delta(t-1) - e^{-(t-1)}u(t-1),$$

determine the Fourier transform $Y(j\omega)$ of the output signal $y(t)$.

(c) Determine the output $y(t)$.

Problem 3.4*:

(a) Show that in general if a signal $x(t)$ is real, then its Fourier transform is "conjugate symmetric", i.e., $X(-j\omega) = X^*(j\omega)$, where * denotes complex conjugation.

(b) Furthermore, show that conjugate symmetry implies that

$$\begin{aligned}
\mathcal{R}e\{X(-j\omega)\} &= \mathcal{R}e\{X(j\omega)\} && \text{even symmetry} \\
\mathcal{I}m\{X(-j\omega)\} &= -\mathcal{I}m\{X(j\omega)\} && \text{odd symmetry} \\
|X(-j\omega)| &= |X(-j\omega)| && \text{even symmetry} \\
\angle X(-j\omega) &= -\angle X(-j\omega) && \text{odd symmetry}
\end{aligned}$$

(c) Find the Fourier transform of the signal $x(t) = e^{-\alpha t}u(t)$ where α is real and $0 < \alpha$. Sketch the real and imaginary parts of $X(j\omega)$ as a function of ω and verify that the above symmetry conditions hold for this real signal.

Problem 3.5:

Consider the following periodic signal, which is the input to a LTI system:

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n4)$$

(a) The input $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k.

(b) The impulse response of the LTI system is

$$h(t) = \alpha e^{-\alpha(t-1)}u(t-1)$$

Use convolution to obtain an equation for the output $y(t)$ when the input is the signal in part (a). *Hint: Use superposition and time invariance to find the output due each impulse.* Make a sketch of the output signal as a function of time for the case $\alpha = 2$.

(c) Determine the frequency response of the LTI system. Sketch $|H(j\omega)|$ as a function of ω. How does the shape of the frequency response depend on α?

(d) Use the frequency response and the Fourier series result of part (a) to determine a Fourier series expression for the output of the system for the given input $x(t)$. How would you choose α if you wanted the output to be essentially equal to a constant?

Problem 3.6*

The frequency response of a LTI system is

$$H(j\omega) = \frac{10 - j\omega}{10 + j\omega}$$

(a) The input $x(t)$ and the output $y(t)$ satisfy a differential equation. What is that equation?

(b) From the table of Fourier transform pairs and properties, find the impulse response $h(t)$ of the system.

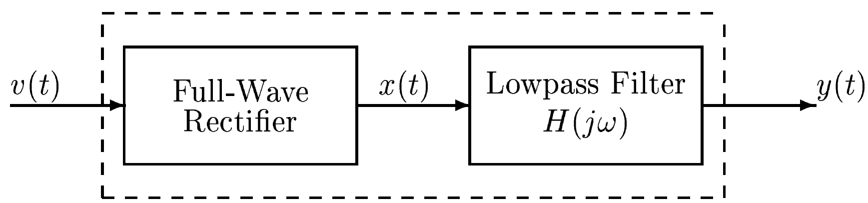
(c) Determine a general expression for the magnitude-squared of the frequency response $|H(j\omega)|^2 = H(j\omega)H^*(j\omega)$ and reduce it to its simplest form.

(d) Determine a general expression for the phase of the frequency response $\arg[H(j\omega)]$.

(e) If the input to the LTI system is $x(t) = 4 + \cos(10t)$ for $-\infty < t < \infty$, what is the corresponding output $y(t)$?

Problem 3.7*

A DC power supply can be modeled as the following system:



The full-wave rectifier and the lowpass filter are described by the equations

$$x(t) = |v(t)| \qquad \text{and} \qquad H(j\omega) = \begin{cases} G & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

The input signal is $v(t) = 155\cos(120\pi t)$.

(a) Carefully sketch the signal $x(t)$ at the output of the full-wave rectifier. Label the time axis carefully. What frequencies will be present in the Fourier transform of $x(t)$?

(b) The objective of the lowpass filter is to remove all frequency components except the DC component. How should ω_c be chosen so that only the DC component remains?

(c) For the choice of ω_c in part (b) how should the gain G of the lowpass filter be chosen so that $y(t) = 10$ for all t?