GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Problem Set No. 5

Date Assigned: October 23, 1998
Date Due: October 30, 1998

Reading Assignment: In Oppenheim and Willsky, read pp. 582-610 and 514-545.

Homework Assignment: Turn in for grading only the starred problems: 5.2*, 5.4*, 5.5* and 5.6*.

Practice Problems:

(a) For practice try Problem 8.11 in Oppenheim and Willsky. This problem has answers in the back of the book.

(b) Work Problem 8.23 in Oppenheim and Willsky. (Solution: Problem 6.1, Winter 98)

(c) Work Problem 7.41 in Oppenheim and Willsky. (Solution: Problem 6.2, Winter 98)

(d) Work Problem 8.29 in Oppenheim and Willsky. (Solution: Problem 6.3, Winter 98)

Problem 5.1

Consider the impulse response $h(t)$ whose Fourier transform is

$$H(j\omega) = [1 + \cos(\omega T/4)]H_r(j\omega) \quad \text{where} \quad H_r(j\omega) = \begin{cases} T/4 & |\omega| < 4\pi/T \\ 0 & |\omega| > 4\pi/T \end{cases}$$

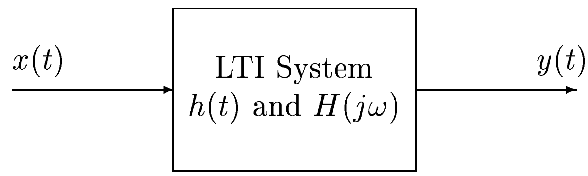
(a) Sketch $H(j\omega)$ as a function of ω.

(b) Determine an equation for $h(t)$ in terms of $h_r(t)$ and sketch it for $-3T/2 \leq t \leq 3T/2$.

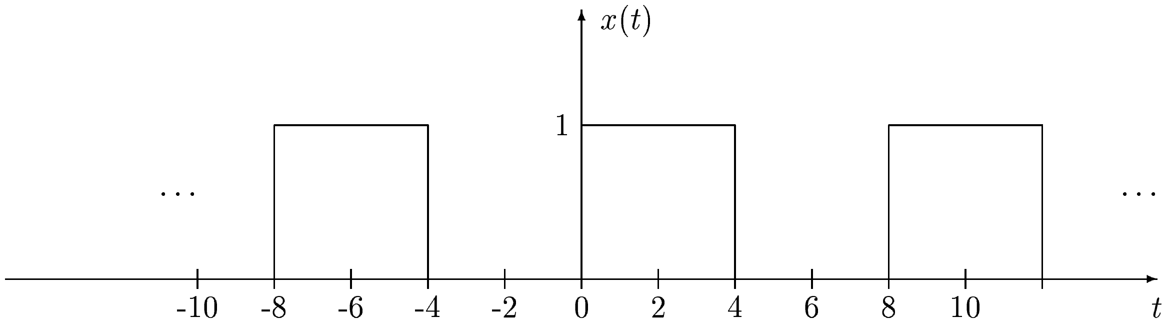
(c) Determine $h(0)$ and $h(nT/2)$ for n an integer.

Problem 5.2*:

Consider the LTI system below:



The input to this system is the periodic pulse wave $x(t)$ plotted below:



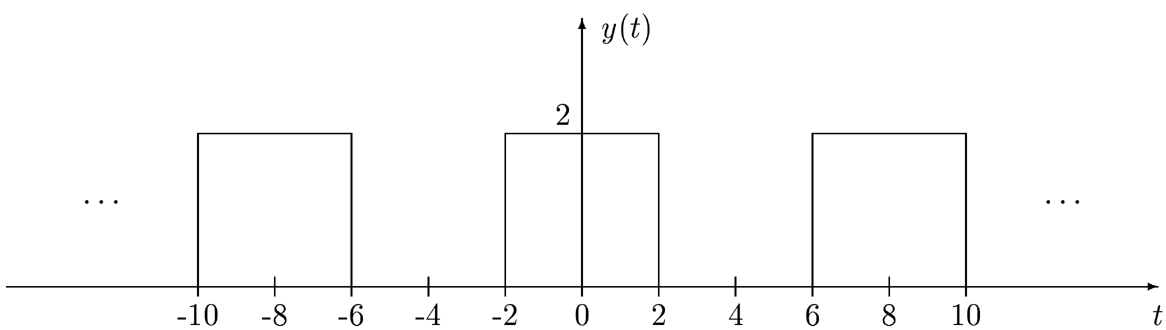
(a) Determine the Fourier transform of the input signal $x(t)$ and plot it for the frequency range $-5\omega_0 \leq \omega \leq 5\omega_0$.

(b) If the frequency response of the system is

$$H(j\omega) = 1 + \cos(4\omega) \qquad -\infty < \omega < \infty$$

what is the output of the system when the input is $x(t)$? Give an equation or plot for $y(t)$. *Use the spectrum plot of part (a) and a plot of the frequency response to help you solve this problem.*

(c) Determine the frequency response of the system, $H(j\omega)$, if the output is as shown below when the input is $x(t)$.



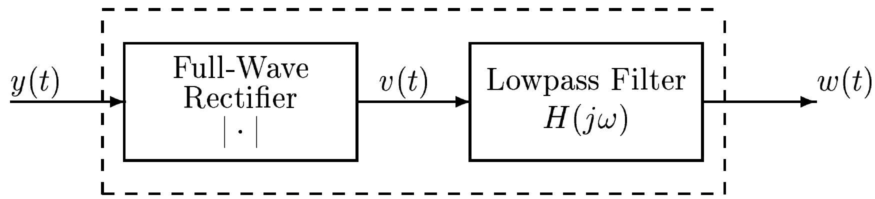
(d) Plot the output waveform $y(t)$ if the input is $x(t)$ and the frequency response is

$$H(j\omega) = (j\omega)e^{-j\omega 2}.$$

Also, determine the Fourier transform of $y(t)$ for this case and plot it for frequencies $-5\omega_0 \leq \omega \leq 5\omega_0$.

Problem 5.3

An *envelope detector* is a device for demodulating an AM signal. The following is an idealized representation of an envelope detector.



The full-wave rectifier and the lowpass filter are described by the equations

$$v(t) = |y(t)| \qquad \text{and} \qquad H(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

Assume that $y(t) = e(t)\cos(\omega_c t)$, where $e(t) = [A + x(t)] > 0$ is the positive envelope of the carrier $\cos(\omega_c t)$. Furthermore, assume that $X(j\omega) = 0$ for $|\omega| > \omega_b$, where $\omega_c \gg \omega_b$.

(a) Is the full-wave rectifier a linear system? Is it time-invariant?

(b) Show that $v(t) = |e(t)\cos(\omega_c t)| = e(t)|\cos(\omega_c t)|$.

(c) Sketch the function $|\cos(\omega_c t)|$, and show that it is periodic. What is its fundamental frequency ω_0?

(d) Since $|\cos(\omega_c t)|$ is a periodic function, we can write

$$v(t) = e(t) \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Use this result to determine an expression for $V(j\omega)$ and use the above information about $e(t)$ and $X(j\omega)$ to determine an expression for $w(t)$ in terms of $e(t)$ and $x(t)$. Determine a numerical value for the gain of the filter so that $w(t) = x(t)$.

Problem 5.4*:

(a) A signal $y(t)$ is generated by convolving a bandlimited signal $x_1(t)$ with another bandlimited signal $x_2(t)$; i.e.,

$$y(t) = x_1(t) * x_2(t)$$

where

$$\begin{aligned} X_1(j\omega) &= 0 \qquad \text{for } |\omega| \geq 200\pi \\ X_2(j\omega) &= 0 \qquad \text{for } |\omega| \geq 600\pi \end{aligned}$$

Impulse train sampling is performed on $y(t)$ to obtain

$$y_p(t) = y(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} y(nT)\delta(t - nT)$$

Specify the range of values for the sampling period T that will ensure that $y(t)$ is recoverable from $y_p(t)$.

(b) Repeat part (a) for the band-limited signal obtained by *multiplying* $x_1(t)$ and $x_2(t)$; i.e.,

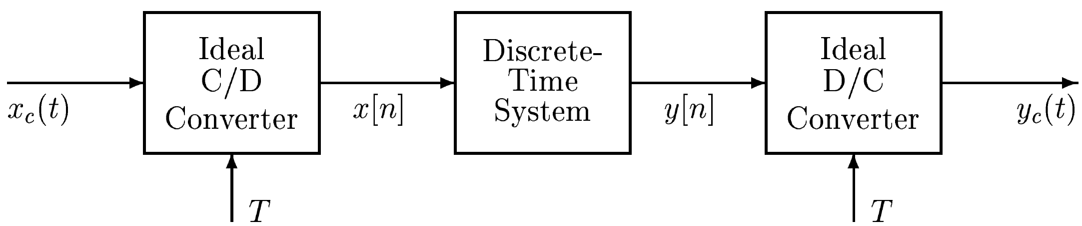
$$w(t) = x_1(t) \cdot x_2(t)$$

where $x_1(t)$ and $x_2(t)$ are as defined in part (a). *Hint: Sketch a "typical"* $X_1(j\omega)$ *and* $X_2(j\omega)$ *and determine the bandwidth of* $W(j\omega)$ *by convolution.*

(c) Repeat part (a) for the band-limited signal $y(t) = x_1(t)\cos(1000\pi t)$ where $x_1(t)$ has a bandlimited Fourier transform as defined in part (a).

Problem 5.5*

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t - nT)}{\frac{\pi}{T}(t - nT)}$$

Assume that $X_c(\omega) = 0$ for $|\omega| \geq 1000\pi$.

(a) Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T$ such that $y_c(t) = x_c(t)$?

(b) Determine the relationship between $y[n]$ and $x[n]$ so that if the sampling rate satisfies the condition of (a), then $y_c(t) = x_c(t - 10T)$.

(c) If the input is $x_c(t) = \cos(1200\pi t + pi/3)$, the sampling frequency is $2\pi/T = 2000\pi$, and $y[n] = x[n]$, what is $y_c(t)$?

(d) The input/output relation for the discrete-time system is

$$y[n] = \frac{1}{3}\left(x[n] - x[n-1] + x[n-2]\right)$$

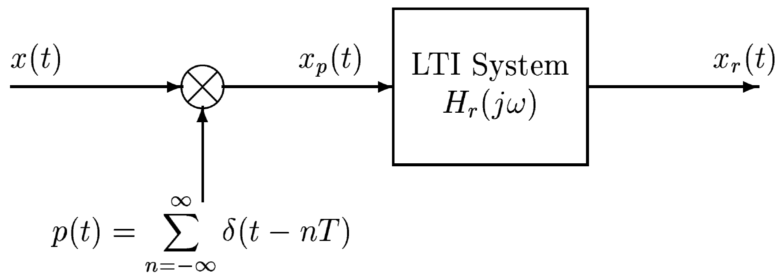
For the value of T chosen in part (a), the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{eff}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{eff}(j\omega)$. Plot the magnitude and phase of $H_{eff}(j\omega)$. Use MATLAB to do this or sketch it by hand.

(e) Suppose that the frequency response of the discrete-time system is defined over one period $(-\pi \leq \hat{\omega} \leq \pi)$ by

$$H(e^{j\hat{\omega}}) = \begin{cases} 0 & |\hat{\omega}| < \pi/2 \\ e^{-j\hat{\omega}10} & \pi/2 < |\hat{\omega}| \leq 3\pi/4 \\ 0 & 3\pi/4 < |\hat{\omega}| \leq \pi \end{cases}$$

where $\hat{\omega} = \omega T$. Plot the magnitude and phase of $H_{eff}(j\omega)$.

Problem 5.6*



The input signal for the above sampling/reconstruction system is

$$x(t) = 2\cos(400\pi t - pi/4) + \cos(700\pi t + \pi/3) \qquad -\infty < t < \infty$$

and the frequency response of the lowpass reconstruction filter is

$$H_r(j\omega) = \begin{cases} T & |\omega| < \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$$

where T is the sampling period.

(a) Sketch the Fourier transform $X_p(j\omega)$ for $-2\pi/T < \omega < 2\pi/T$ for the case where $2\pi/T = 1500\pi$. Carefully label your sketch to receive full credit. What is the output $x_r(t)$?

(b) Now assume that $\omega_s = 2\pi/T = 800\pi$. Determine an equation for the output $x_r(t)$.