GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2823A
Problem Set No. 5

Date Assigned: February 5, 1999
Date Due: February 12, 1999

Reading Assignment: In Kamen and Heck, read pp. 582-610 and 514-545.

Homework Assignment: Turn in for grading only the starred problems: 5.2*, 5.4*, and 5.5*.

Practice Problems:
Look at Problems on Problem Sets 4 and 5 of EE3230, Winter, Spring, Fall of 1998.

Problem 5.1
Consider the impulse response $h(t)$ whose Fourier transform is

$$H(j\omega) = [1 + \cos(\omega T/4)]H_r(j\omega) \quad \text{where} \quad H_r(j\omega) = \begin{cases} T/4 & |\omega| < 4\pi/T \\ 0 & |\omega| > 4\pi/T \end{cases}$$

(a) Sketch $H(j\omega)$ as a function of ω.

(b) Determine an equation for $h(t)$ in terms of $h_r(t)$ and sketch it for $-3T/2 \le t \le 3T/2$.

(c) Determine $h(0)$ and $h(nT/2)$ for n an integer.

Problem 5.2*: (Correction to original handout.)

The discrete-time Fourier transform (DTFT) of a sampled signal can be computed directly from the samples $x[n] = x_c(nT)$ by the formula

$$X(e^{j\omega T}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega Tn}. \tag{0.1}$$

It also can be related to the continuous-time Fourier transform $X_c(j\omega)$ by the formula

$$X(e^{j\omega T}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\omega - 2\pi k/T)). \tag{0.2}$$

(a) Suppose the continuous-time signal is $x_c(t) = e^{-200t}u(t)$ and $T = .001$ sec so that $x[n] = e^{-0.2n}u[n]$. Use (**??**) with the formula for the sum of terms of a geometric series,

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1,$$

to determine the DTFT of the sequence $x[n]$.

(i) Sketch (or plot it with Matlab) $|X_c(j\omega)/T|$ as a function of ω for $-2\pi/T < \omega < 2\pi/T$.

(ii) Sketch (or plot it with Matlab) $|X(e^{j\omega T})|$ as a function of ω for $-2\pi/T < \omega < 2\pi/T$.

(iii) Compare $|X_c(j\omega)/T|$ to $|X(e^{j\omega T})|$ over the range of frequencies plotted. The following Matlab program would be useful if you fill in the missing statements.

```
alpha=200;T=.001;
omega=(-1:.005:1)*2*pi/T;
Xc=
X=
plot(omega,abs(Xc)/T,'r',omega,abs(X),'--g')
```

Fill in the missing statements and run the program. Hand in a plot with your solution.

(b) Suppose the continuous-time signal is $x_c(t) = \dfrac{\sin(\omega_m t)}{\pi t}$ where $\omega_m = \pi/(2T)$ so that $x[n] = \dfrac{1000\sin(0.5\pi n)}{\pi n}$ when $T = 0.001$. Find $X_c(j\omega)$ for this case and then use (**??**) to determine the DTFT of the sequence $x[n]$.

(i) Sketch $|X_c(j\omega)/T|$ as a function of ω for $-2\pi/T < \omega < 2\pi/T$.

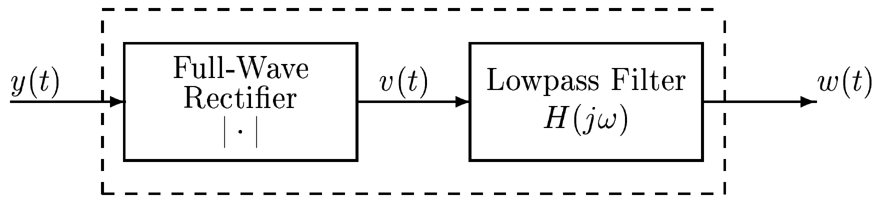
(ii) Sketch $|X(e^{j\omega T})|$ as a function of ω for $-2\pi/T < \omega < 2\pi/T$.

(iii) Compare $|X_c(j\omega)/T|$ to $|X(e^{j\omega T})|$ over the range of frequencies plotted.

(c) In which case [(a) or (b)] does aliasing cause $X(e^{j\omega T})$ to differ from $X_c(j\omega)/T$ over the range of frequencies $-\pi/T < \omega < \pi/T$?

Problem 5.3

An *envelope detector* is a device for demodulating an AM signal. The following is an idealized representation of an envelope detector.



The full-wave rectifier and the lowpass filter are described by the equations

$$v(t) = |y(t)| \qquad \text{and} \qquad H(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

Assume that $y(t) = e(t)\cos(\omega_c t)$, where $e(t) = [A + x(t)] > 0$ is the positive envelope of the carrier $\cos(\omega_c t)$. Furthermore, assume that $X(j\omega) = 0$ for $|\omega| > \omega_b$, where $\omega_c \gg \omega_b$.

(a) Is the full-wave rectifier a linear system? Is it time-invariant?

(b) Show that $v(t) = |e(t)\cos(\omega_c t)| = e(t)|\cos(\omega_c t)|$.

(c) Sketch the function $|\cos(\omega_c t)|$, and show that it is periodic. What is its fundamental frequency ω_0?

(d) Since $|\cos(\omega_c t)|$ is a periodic function, we can write

$$v(t) = e(t) \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 t}$$

Use this result to determine an expression for $V(j\omega)$ and use the above information about $e(t)$ and $X(j\omega)$ to determine an expression for $w(t)$ in terms of $e(t)$ and $x(t)$. Determine a numerical value for the gain of the filter so that $w(t) = e(t) = [A + x(t)]$.

Problem 5.4*:

(a) A signal $y(t)$ is generated by convolving a bandlimited signal $x_1(t)$ with another bandlimited signal $x_2(t)$; i.e.,

$$y(t) = x_1(t) * x_2(t)$$

where

$$\begin{aligned} X_1(j\omega) &= 0 \qquad \text{for } |\omega| \geq 200\pi \\ X_2(j\omega) &= 0 \qquad \text{for } |\omega| \geq 600\pi \end{aligned}$$

Impulse train sampling is performed on $y(t)$ to obtain

$$y_p(t) = y(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} y(nT)\delta(t - nT)$$

Specify the range of values for the sampling period T that will ensure that $y(t)$ is recoverable from $y_p(t)$.

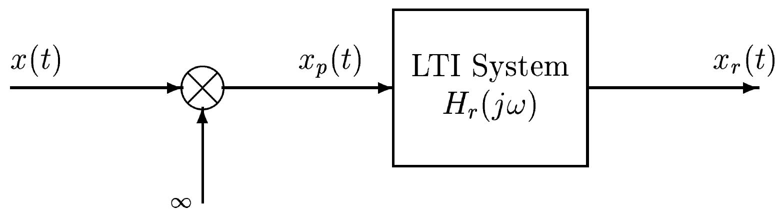
(b) Repeat part (a) for the band-limited signal obtained by *multiplying* $x_1(t)$ and $x_2(t)$; i.e.,

$$y(t) = x_1(t) \cdot x_2(t)$$

where $x_1(t)$ and $x_2(t)$ are as defined in part (a). *Hint: Sketch a "typical" $X_1(j\omega)$ and $X_2(j\omega)$ and determine the bandwidth of $Y(j\omega)$ by convolution.*

(c) Repeat part (a) for the band-limited signal $y(t) = x_1(t)\cos(1000\pi t)$ where $x_1(t)$ has a bandlimited Fourier transform as defined in part (a).

Problem 5.5*



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

The input signal for the above sampling/reconstruction system is

$$x(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3) \qquad -\infty < t < \infty$$

and the frequency response of the lowpass reconstruction filter is

$$H_r(j\omega) = \begin{cases} T & |\omega| < \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$$

where T is the sampling period.

(a) Determine the Fourier transform $X(j\omega)$ and plot the Fourier transform $X_p(j\omega)$ for $-2\pi/T < \omega < 2\pi/T$ for the case where $2\pi/T = 1000\pi$. Carefully label your sketch to receive full credit. What is the output $x_r(t)$?

(b) Now assume that $\omega_s = 2\pi/T = 500\pi$. Determine an equation for the output $x_r(t)$.

(c) Is it possible to choose the sampling rate so that

$$x_r(t) = A + 2\cos(100\pi t - \pi/4)$$

where A is a constant? If so, what is the value of T and what is the numerical value of A?