GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

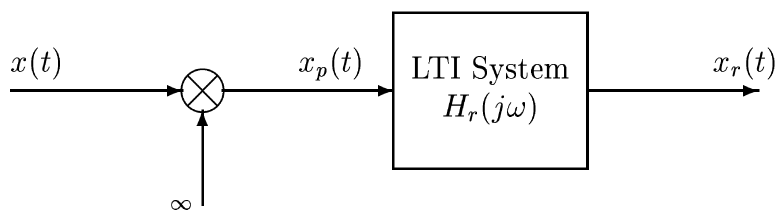
EE3230
Addendum to Problem Set No. 5

Date Assigned: October 30, 1998
Date Due: November 2, 1998

Reading Assignment: In Oppenheim and Willsky, read pp. 582-610 and 514-545.

Homework Assignment: Turn in for grading only the starred problems: 5.2*, 5.4*, 5.5* and 5.6*.

Problem 5.6* *Replace previous problem with this one.*



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

The input signal for the above sampling/reconstruction system is

$$x(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3) \qquad -\infty < t < \infty$$

and the frequency response of the lowpass reconstruction filter is

$$H_r(j\omega) = \begin{cases} T & |\omega| < \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$$

where T is the sampling period.

(a) Determine the Fourier transform $X(j\omega)$ and plot the Fourier transform $X_p(j\omega)$ for $-2\pi/T < \omega < 2\pi/T$ for the case where $2\pi/T = 1000\pi$. Carefully label your sketch to receive full credit. What is the output $x_r(t)$?

(b) Now assume that $\omega_s = 2\pi/T = 500\pi$. Determine an equation for the output $x_r(t)$.

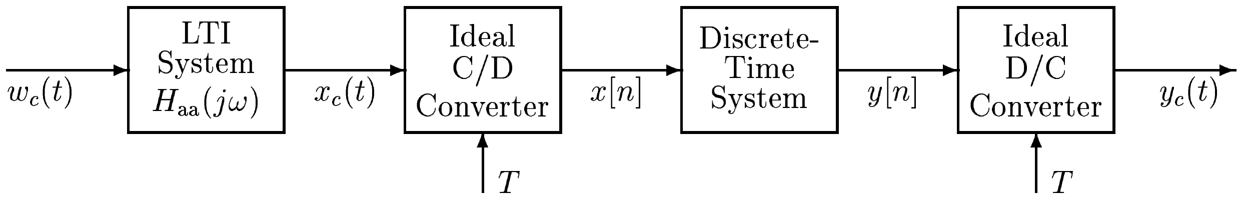
(c) Is it possible to choose the sampling rate so that

$$x_r(t) = A + 2\cos(100\pi t - \pi/4)$$

where A is a constant? If so, what is the numerical value of A?

Problem 5.7 *Work this problem to review for the test on Thursday November 5.*

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t - nT)}{\frac{\pi}{T}(t - nT)}$$

(a) The first LTI system has frequency response such that $H_{aa}(j\omega) = 0$ for $|\omega| \geq 1000\pi$. This system is a lowpass filter, but not necessarily an *ideal* lowpass filter; i.e., its passband may not be perfectly flat. A filter used like this in a sampling system is often called an *anti-aliasing filter*. Why?

(b) How should T be chosen if we want $y_c(t) = x_c(t)$ when the discrete-time system is defined by the difference equation $y[n] = x[n]$?

(c) If the filter $H_{aa}(j\omega)$ satisfies the condition given in part (a) and the sampling rate is chosen to avoid aliasing as in part (b), and the discrete-time system is defined by $y[n] = x[n]$, give an expression for $y_c(t)$ in terms of $w_c(t)$ and the impulse response $h_{aa}(t)$ of the anti-aliasing filter.

(d) If the filter $H_{aa}(j\omega)$ satisfies the condition given in part (a) and the sampling rate is chosen to avoid aliasing as in part (b), and the discrete-time system now is defined by $y[n] = x[n] + x[n-1]$, what is the effective frequency response $H_{\text{eff}}(j\omega)$ of the overall system from $w_c(t)$ to $y_c(t)$; i.e., what is $H_{\text{eff}}(j\omega)$ in the equation

$$Y_c(j\omega) = H_{\text{eff}}(j\omega) W_c(j\omega)$$

Be as specific as you can.