GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Homework Assignment No. 7

Date Assigned: February 20, 1998
Date Due: February 27, 1998

Reading Assignment: Read all of the notes "Spectrum Analysis of Discrete Signals."

Homework Assignment: Hand in Problems 7.1*, 7.3*, and 7.4*.

Problem 7.1*:

A discrete-time signal composed of sinusoids is given by the equation

$$x[n] = 10 + 10\cos(0.1\pi n) + 4\sin(0.5\pi n)$$

(a) Plot the spectrum of this signal for discrete-time frequencies $-\pi < \hat{\omega} < \pi$, indicating the complex size of each frequency component. You need not make separate plots for real/imaginary or magnitude/phase.

(b) Plot the spectrum of this signal for positive discrete-time frequencies $0 < \hat{\omega} < 2\pi$.

(c) Plot the spectrum of the signal $x_1[n] = e^{-j0.1\pi n}x[n]$ for positive frequencies $0 < \hat{\omega} < 2\pi$.

(d) Suppose that

$$y[n] = \sum_{m=0}^{M-1} x_1[n-m]$$

i.e., $y[n]$ is the output of a M-point moving averager with $x_1[n]$ as its input. For what value(s) of M will $y[n] = 5M$ for all n?

Problem 7.2:

Consider the periodic impulse train

$$x[n] = \sum_{r=-\infty}^{\infty} \delta[n - rN]$$

(a) Plot the sequence $x[n]$ for $N = 20$.

We know that any periodic discrete-time signal can be represented as a sum of complex exponentials in the form

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j(2\pi/N)kn}$$

where

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn}$$

(b) Determine $X[k]$ for $k = 0, 1, 2, \ldots, N-1$ for the periodic impulse signal.

(c) Now suppose that we let $N \to \infty$. What is the sequence $x[n]$ for $N = \infty$?

(d) What is the spacing of the frequencies as $N \to \infty$? State in words a description of the spectrum of an impulse.

Problem 7.3*:

A discrete-time signal is synthesized by the following MATLAB statements:

```
n1=0:399;
n2=400:799;
Ts=1/2000;
f0=200;
x=[cos(2*pi*f0*n1*Ts),cos(2*pi*3*f0*n2*Ts)];
```

(a) Write an equation for the sequence of samples represented by the MATLAB vector `x`. Plug in the numbers. Your equation should be in the form

$$x[n] = \begin{cases} ??? & ?? < n < ?? \\ ??? & ?? < n < ?? \end{cases}$$

(b) Sketch the spectrogram of the signal. Label the frequency and time axes appropriately for the sampling rate of the signal. *Assume that the spectrum analysis filter is a moving average filter of length* $M = 100$.

Problem 7.4*:

The spectrogram in Figure 1 and the spectral slices in Figure 2 are for an unknown signal $x(t)$ that was sampled with sampling rate $f_s = 4000$ samples/sec. The spectrogram was computed using the equation

$$X[k,n] = \sum_{m=0}^{N-1} x[n-m]e^{-j(2\pi/N)k(n-m)} = \sum_{m=n-N}^{n} x[m]e^{-j(2\pi/N)km}$$

(a) By considering the sampling rate and Figure 1, determine the length, N, of the spectrum analysis window.

(b) Explain the appearance of the spectrogram in Figure 1. Specifically, what accounts for the "smeared regions" around 50, 150, and 200 msec.? From your result in part (a) and the two figures, write an equation for the signal $x(t)$.

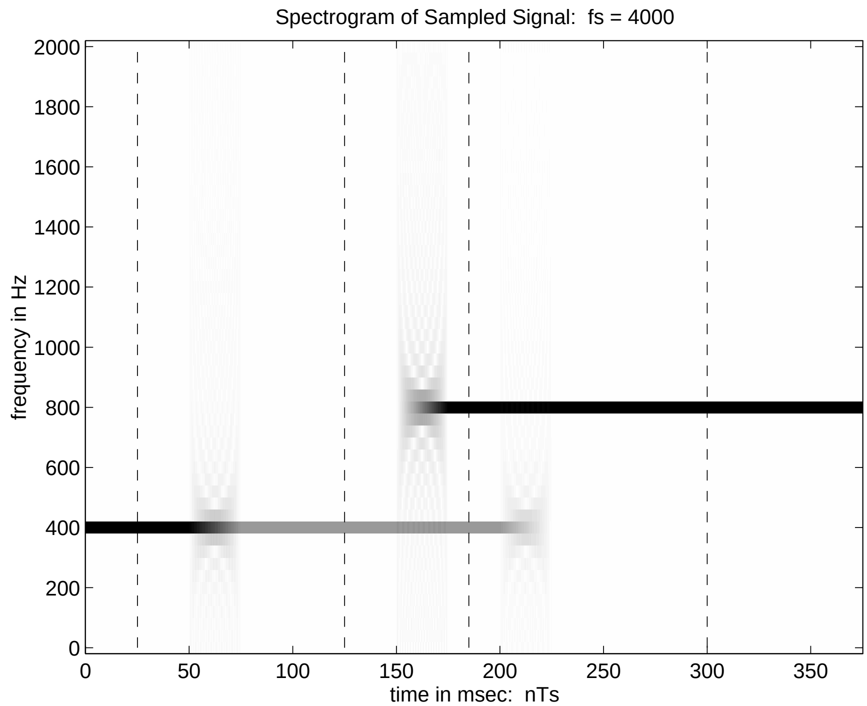

Figure 1: Spectrogram of unknown signal.

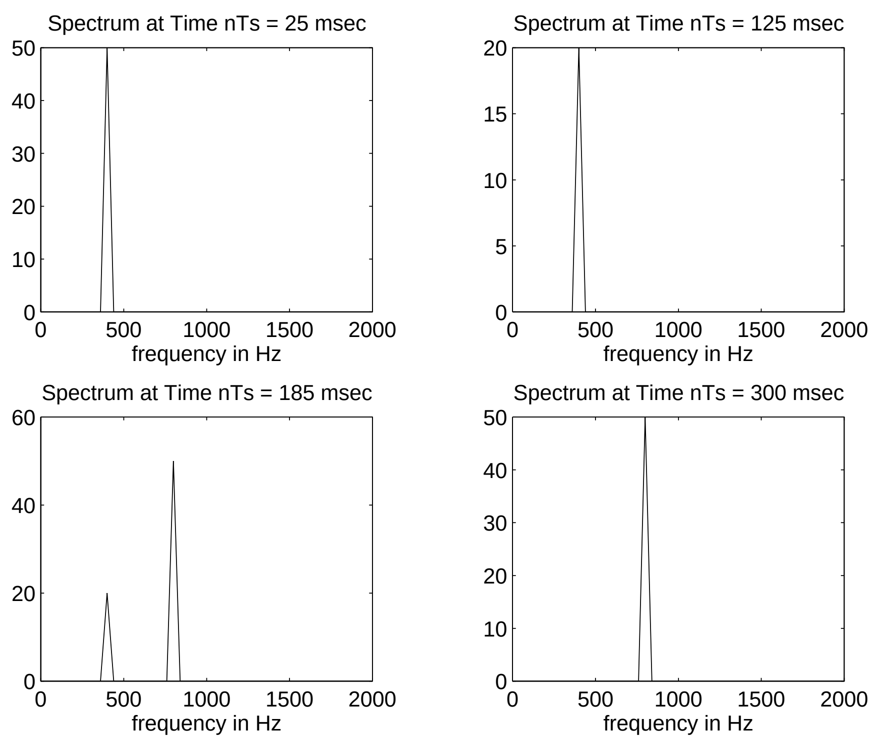

Figure 2: Spectral slices for unknown signal.