GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Solution to Homework Assignment No. 7

Date Assigned: February 20, 1998
Date Due: February 27, 1998

Problem 7.4*

(a) From the spectrogram and the spectral slices, it appears that the signal starts out as a single component at frequency 400 Hz. Then at time $t = 50$ msec the amplitude decreases but the frequency remains the same. Then at time $t = 150$ msec a second component is added at frequency 800 Hz. Finally the 400 Hz component goes away at time $t = 200$ msec. If we measure the duration of the disturbances beginning at 50, 150, and 200 msec we see that it is about 25 msec. For the sampling rate of 4000 samples/sec, this means that the length of the analysis window is $N = 4000 * .025 = 100$ samples.

(b) For a moving average filter of length $N = 100$, the gain at d.c. is 100. Therefore the spectrogram amplitude of 50 means a complex exponential amplitude of 0.5. Thus, from the spectral slices, we can infer that the signal is

$$x(t) = \begin{cases} \cos(800\pi t + \theta_1) & 0 \le t < 50 \\ 0.4\cos(800\pi t + \theta_1) & 50 \le t < 150 \\ 0.4\cos(800\pi t + \theta_1) + \cos(1600\pi t + \theta_2) & 150 \le t < 200 \\ \cos(1600\pi t + \theta_2) & 200 \le t < 400 \end{cases}$$

The spectrogram does not give any information on phase so we cannot determine the phase angles θ_1 and θ_2.

Problem 7.3*:

(a) The sequence determined by the given MATLAB statements is

$$x[n] = \begin{cases} \cos(400\pi n/2000) = \frac{1}{2}e^{j.2\pi n} + \frac{1}{2}e^{-j.2\pi n} & 0 \le n \le 399 \\ \cos(1200\pi n/2000) = \frac{1}{2}e^{j.6\pi n} + \frac{1}{2}e^{-j.6\pi n} & 400 \le n \le 799 \end{cases}$$

(b) The spectrogram is shown in Figure 1 and two spectral slices are shown in Figure 2. Note that the first part of the signal has frequency 200 Hz in the interval $0 \le n < 400$, which corresponds to the time interval $0 \le t < 200$ msec. The second part of the signal has frequency 600 Hz and it occupies the time interval $200 \le t < 400$ msec. Now, the spectrogram is computed using

$$X[k,n] = \sum_{m=n-99}^{n} x[m]e^{-j(2\pi/100)kn} \tag{1}$$

The 100-point moving average filter has gain of 100 at $\hat{\omega} = 0$ and the equivalent time duration of the analysis window is $100/2000 = 50$ msec. The continuous-time frequency 200 Hz will correspond to a normalized discrete-time frequency $\hat{\omega} = 2\pi 200/2000 = (2\pi/100)10$, which corresponds exactly to frequency index $k = 10$ in Equation (1). Therefore, we should have

$$X[k,n] = \begin{cases} 0 & k \ne 10 \\ 100(.5) & k = 10 \end{cases} \qquad \text{for } 0 \le n < 400$$

When $n = 400$, the signal changes abruptly to a frequency of 600 Hz. Likewise the 600 Hz frequency corresponds to $\hat{\omega} = 2\pi 600/2000 = (2\pi/100)30$ or exactly $k = 30$ in Equation (1). Therefore, for $400 \le n \le 498$ (about 50 msec.), the spectrum analysis window will include some samples of frequency 200 Hz and some samples of frequency 600 Hz. Thus, we will not see a clear indication of a single frequency. This interval corresponds to $200 \le t < 250$ msec., and we see in Figure 1 that the spectrogram has a "smeared" appearance in this time interval. For $499 \le n < 800$, the spectrum window does not include the jump in frequency, and we again obtain a single spectrum line corresponding to 600 Hz frequency. Therefore

$$X[k,n] = \begin{cases} 0 & k \ne 30 \\ 100(.5) & k = 30 \end{cases} \qquad \text{for } 499 \le n < 800$$

Look at Figures 1 and 2 to verify that all this is true.

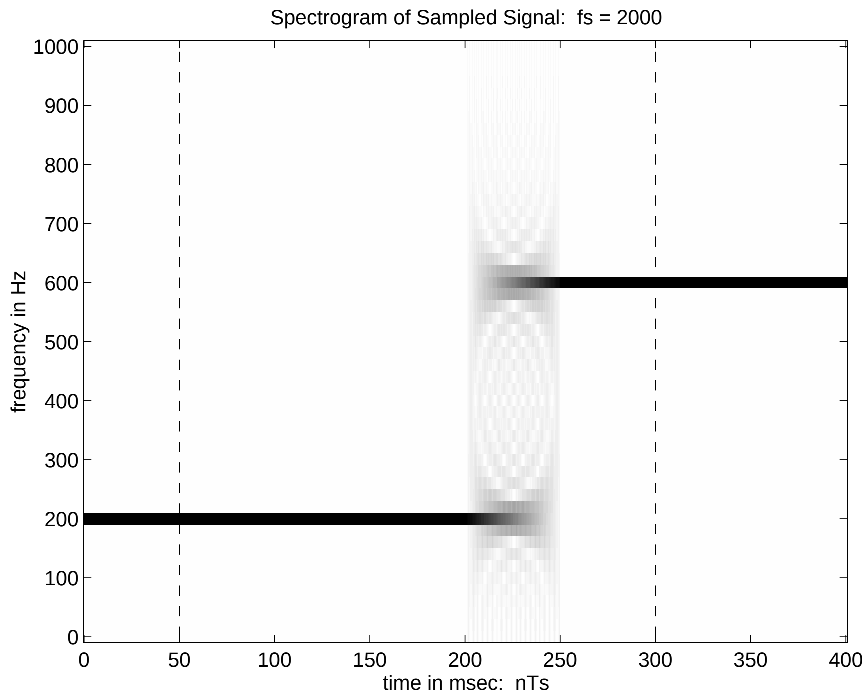

Figure 1: Spectrogram of 200/600 Hz signal.

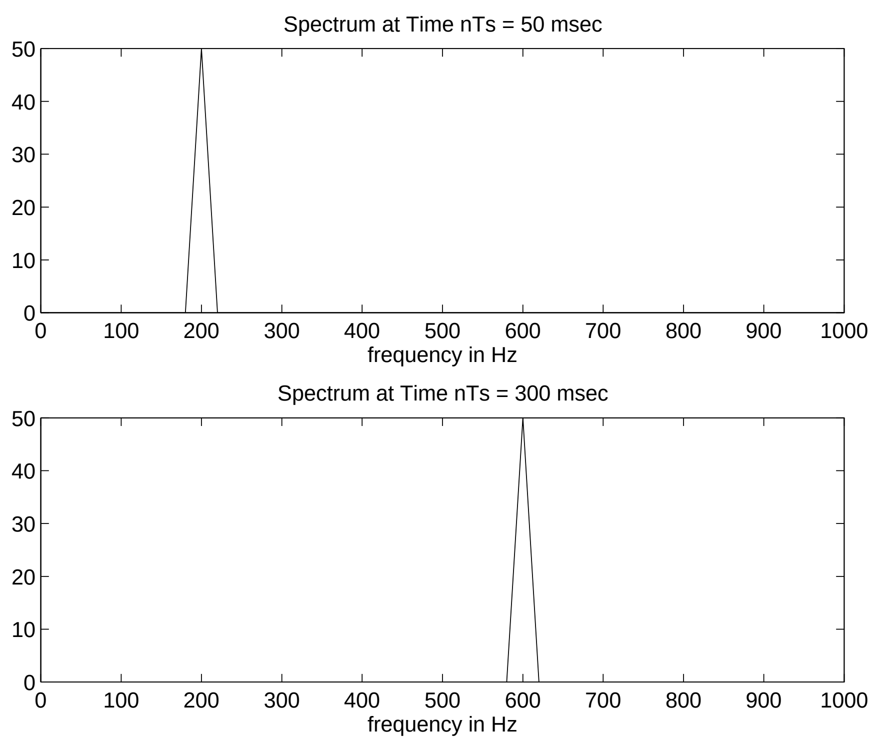

Figure 2: Spectral slices for 200/600 Hz signal.