GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Winter 1999
Problem Set #1

Assigned: 8 Jan 1998

Due Date: 15 Jan 1998 (FRIDAY)

Reading: In *DSP First*, Appendix A on *Complex Numbers*, pp. 378–398; and Ch. 2 on *Sinusoids*, pp. 9–43.

The web site for the course uses Web-CT:

`http://webct.ece.gatech.edu`

Your initial password is digits 4 through 8 of your SSN.

ALL of the **STARRED** problems will have to be turned in for grading.

Some of the problems have solutions that can be found on the CD-ROM. Next week a solution will be posted to the web. After 2:05 on Friday, the homework is considered late and will be given a zero.

Several different mathematical notations can be used to represent complex numbers. In *rectangular form* we will use all of the following notations:

$$\begin{aligned} z &= (x, y) \\ &= x + jy \qquad\qquad \text{where } j = \sqrt{-1} \\ &= \Re e\{z\} + j\Im m\{z\} \end{aligned}$$

The pair (x, y) can be drawn as a vector, such that x is the horizontal coordinate and y the vertical coordinate.

In *polar form* we will use these notations:

$$\begin{aligned} z &= |z|e^{j\arg z} \\ &= re^{j\theta} \\ &= r\angle\theta \end{aligned}$$

where $|z| = r = \sqrt{x^2 + y^2}$ and $\arg z = \theta = \arctan(y/x)$. Again, in a vector drawing, r is the length and θ the direction of the vector.

Euler's Formula:

$$re^{j\theta} = r\cos\theta + j\,r\sin\theta$$

can be used to convert between Cartesian and polar forms.

In these review problems you will manipulate some complex numbers. A calculator will be useful for this purpose, especially if it is one with complex arithmetic capability. It is convenient to learn how to use this feature. However, it is also worthwhile to be able to do the calculations by hand; i.e., it is important to *understand* what your calculator is doing!

PROBLEM 1.1:

Convert the following to polar form:

(a) $z = 0 - j3$

(b) $z = 3 - j4$

(c) $z = (1, -1)$

(d) $z = (0, 4)$

Give numerical values for the magnitude and angle (phase).

PROBLEM 1.2:

Convert the following to rectangular form:

(a) $z = \sqrt{2}\,e^{-j(3\pi/4)}$ (c) $z = 1.6\,\angle\,(\pi/3)$

(b) $z = 7e^{j(3\pi/2)}$ (d) $z = 3\,\angle\,(11\pi)$

Give numerical values for the real and imaginary parts.

PROBLEM 1.3*:

Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = -3 - j\sqrt{3}$ and $z_2 = 3e^{j\pi/6}$.

(a) Conjugate: z_1^* (d) z_2^2 (g) $z_1 + z_2^*$

(b) jz_2 (e) $z_1^{-1} = 1/z_1$ (h) z_1/z_2

(c) z_2/z_1 (f) $z_1 z_1^*$ (i) $z_1 z_2$

Note: z^* means the "conjugate" of z.

PROBLEM 1.4*:

The phase of a sinusoid can be related to time shift:

$$x(t) = A\cos(2\pi f_\circ t + \phi) = A\cos(2\pi f_\circ (t - t_1))$$

In the following parts, assume that the frequency of the sinusoidal wave is $f = 100$ Hz. Determine whether each of the following is TRUE or FALSE, and explain.

(a) "When $t_1 = -1/500$ sec, a correct value for the phase is $\phi = 2\pi/5$."

(b) "When $t_1 = 1/500$ sec, a correct value for the phase is $\phi = -\pi/5$."

(c) "When $t_1 = 0.002$ sec, a correct value for the phase is $\phi = 1.6\pi$."

PROBLEM 1.5*:

The figure below is a plot of two sinusoidal signals. From the plot, determine values for the amplitude (A), phase (ϕ), and frequency $(\omega_\circ)$ needed in the formula:

$$x_i(t) = A\cos(\omega_\circ t + \phi)$$

for both $x_1(t)$ and $x_2(t)$. Give the answer as numerical values *including the units* where applicable. Since you must make approximate measurements on the figure, your final answers will be estimates.