GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Winter 1999
Problem Set #3

Assigned: 22 Jan 1998
Due Date: 29 Jan 1998 (FRIDAY)

Quiz #1 will be held in lecture on Monday 1-Feb-99. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$)

Reading: In *DSP First*, all of Chapter 3 on *Spectrum Representation*, especially pp. 48–73.

The web site: `http://webct.ece.gatech.edu/SCRIPT/WIN99EE2200/scripts/serve_home`
You should change your password; look under `COURSE TOOLS`. Please check the "Bulletin Board" often.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

The Homework must be turned in at the Friday lecture. After 2:00 PM on Friday, the homework is considered late and will be given a zero.

PROBLEM 3.1*:

A signal composed of sinusoids is given by the equation

$$x(t) = 2\cos(6\pi t) + 3\cos(10\pi t - \pi/4)$$

(a) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.

(b) Is $x(t)$ periodic? If so, what is the smallest period?

(c) Now consider a new signal $w(t) = x(t) - \cos(6t)$. Draw a carefully labelled sketch of the spectrum for $w(t)$. Explain why $w(t)$ is *not* periodic.

PROBLEM 3.2*:

Consider a signal $x(t)$ such that

$$x(t) = 2\cos(\omega_1 t)\cos(\omega_2 t) = \cos[(\omega_2 + \omega_1)t] + \cos[(\omega_2 - \omega_1)t]$$

where $0 < \omega_1 < \omega_2$.

(a) Suppose that $\omega_1 = 25\pi$ and $\omega_2 = 60\pi$. Determine the (minimum) period of $x(t)$.

(b) Draw the spectrum for $x(t)$, using the parameters from part (a).

PROBLEM 3.3*:

A periodic signal $x(t) = x(t + T_0)$ is described over one period $-T_0/2 \le t \le T_0/2$ by the equation

$$x(t) = \begin{cases} 1 & |t| < t_c \\ 0 & t_c < |t| \le T_0/2 \end{cases}$$

where $t_c < T_0/2$. In this problem assume that $T_0 = 4$ and $t_c = 1$.

(a) Sketch the periodic function $x(t)$ for t in the range $-T_0 < t < 2T_0$.

(b) Determine the D.C. coefficient X_0 using the parameters $T_0 = 4$ and $t_c = 1$.

(c) Determine the *fundamental frequency* ω_0 in the Fourier Series representation.

(d) Use the Fourier *analysis* integral[1] (for $k \neq 0$)

$$X_k = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt$$

to determine a general formula for the Fourier coefficients X_k in the representation

$$x(t) = X_0 + \Re e \left\{ \sum_{k=1}^{\infty} X_k e^{jk\omega_0 t} \right\}$$

Your final result could depend on t_c and T_0, but use $t_c = 1$ and $T_0 = 4$.

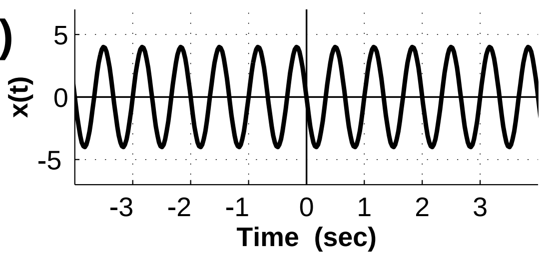
(e) Sketch the spectrum of $x(t)$ for the case $t_c = 1$ and $T_0 = 4$. Include the first 3 non-zero frequency components in both positive and negative frequency. Label each component with its complex amplitude (magnitude and phase).

[1]The integral can be done over any period of the signal; in this case, the most convenient choice is from $-T_0/2$ to $T_0/2$.

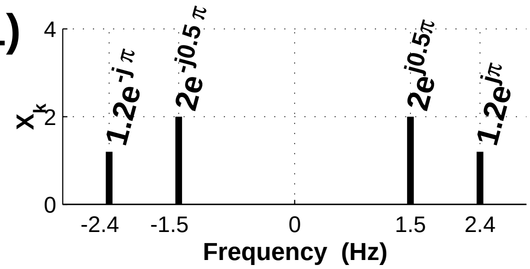
PROBLEM 3.4*:

Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Explain your answers by deriving the formula for a time signal from each of the spectrum plots.

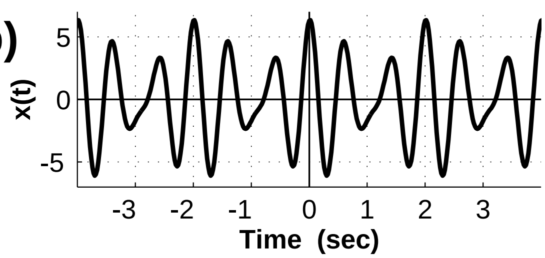
(a)



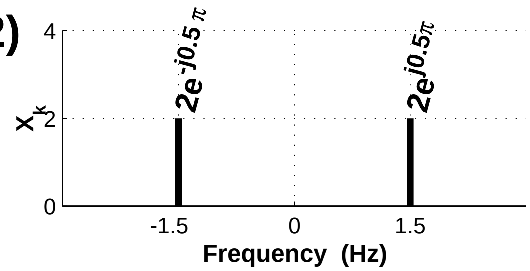
(1)



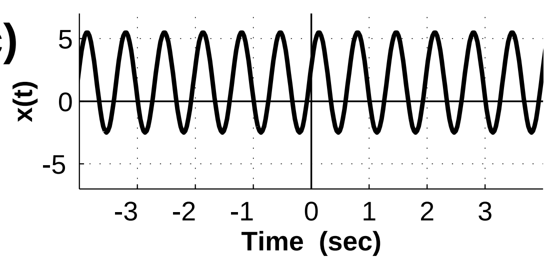
(b)



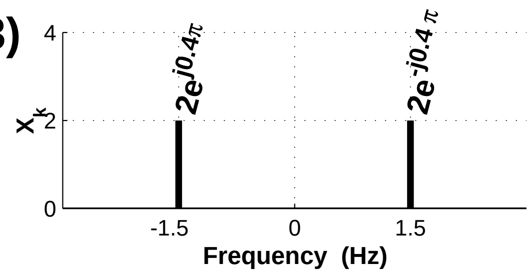
(2)



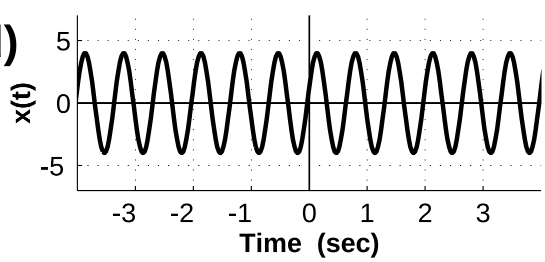
(c)



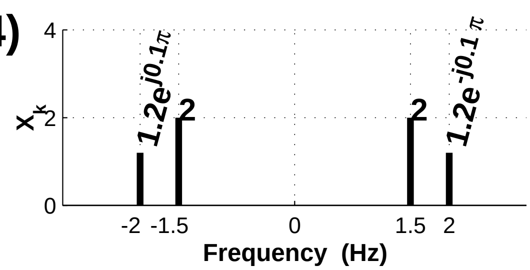
(3)



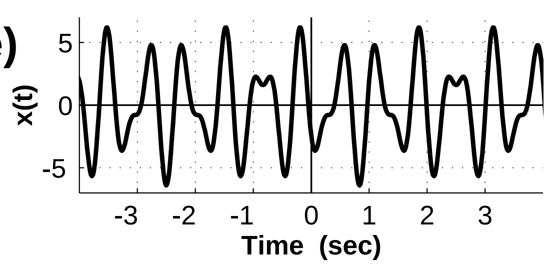
(d)



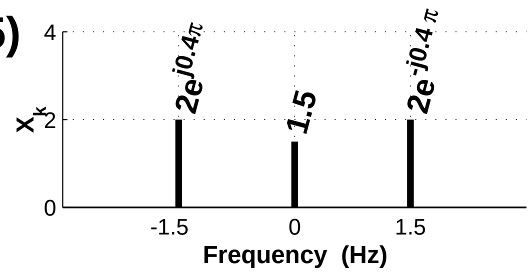
(4)



(e)



(5)



PROBLEM 3.5*:

DSP First, Chapter 3, Problem 8, page 80.

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with middle-C and ending with high-C are:

note name	C	$C^{\#}$	D	$E^{\flat}$	E	F	$F^{\#}$	G	$G^{\#}$	A	$B^{\flat}$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C (note #49) is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

(d) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The C Minor chord is composed of the tones of $C\ \ E^{\flat}\ \ G$ sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the C Minor chord assuming that each note is realized by a pure sinusoidal tone and that each note is equally loud. (You do not have to specify the complex amplitudes precisely.)