GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Winter 1999
Problem Set #4

Assigned: 5 Feb 99
Due Date: 12 Feb 99 (FRIDAY)

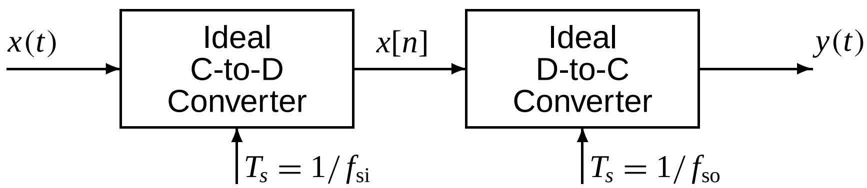
Reading: In *DSP First*, all of Chapter 4 on *Sampling*.

A lab quiz is planned for the labs on 16 & 18 Feb

⟹ The five(5) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the "unstarred" problems.

PROBLEM 4.1*:

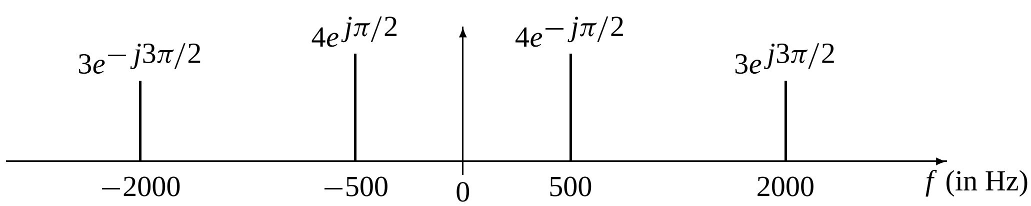


(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 10\cos(0.20\pi n - \pi/3)$$

If the sampling rate of the C-to-D converter is $f_{\text{si}} = 2500$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 2500 Hz; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_{\text{si}}) = x_2(nT_{\text{si}})$ if $T_{\text{si}} = 1/2500$.

(b) If the input $x(t)$ is given by the two-sided spectrum representation shown below,



Determine the spectrum for $x[n]$ when $f_{\text{si}} = 2500$ samples/sec. Make a plot for your answer, but label the frequency, amplitude and phase of each spectral component.

(c) Using the discrete-time spectrum from part (b), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_{\text{so}} = 8000$ Hz. In other words, the sampling rates of the two converters are different.

PROBLEM 4.2*:

Suppose that a MATLAB function has been written to calculate a sum of discrete-time sinusoids, e.g., something similar to the `makecos( )` that was written for the lab. Here is the actual function:

```
function  xn = makedcos(omegahat,ZZ,Length)
%MAKEDCOS  make a discrete-time sinusoid for x[n]
%
xn = real( exp( j*(0:Length-1)'*omegahat(:)' ) * ZZ(:) );
```

If the following MATLAB command is used to make an output sound:

```
soundsc( makedcos(pi*linspace(0,0.8,3),[-1,j,1-j],4000), 8000 )
```

(a) Draw a plot of the discrete-time spectrum (vs. $\hat{\omega}$) of the discrete-time signal defined by this MATLAB operation.

(b) Draw a plot of the continuous-time spectrum (vs. f in Hz) of the analog output signal defined by the `soundsc()` function.

PROBLEM 4.3*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly*.

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

There are examples on the CD-ROM in the Chapter 3 demos.

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α, β and T_2. For this problem, assume that the starting time of the "chirp" is $t = 0$.

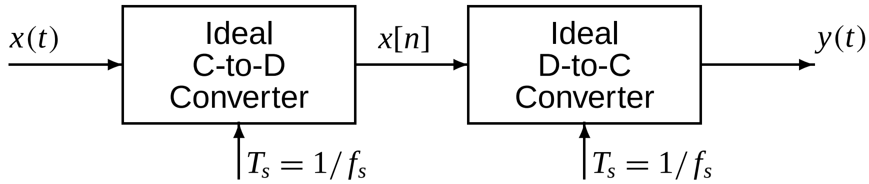
(b) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(-33t^2 + 98t - 0.2)}\right\}$$

derive a formula for the *instantaneous* frequency versus time.

(c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.

PROBLEM 4.4*:



(a) A continuous-time signal $x(t)$ is defined by the following formula: $x(t) = \sum_{k=-8}^{8} \frac{j\pi k}{k^2+1} e^{j20\pi kt}$

Determine the Nyquist rat (in Hz) for sampling $x(t)$.

(b) If the input to the ideal C/D converter is a sinusoid with frequency of 180 Hz, and the output is the discrete-time sinusoid: $x[n] = 3\cos(\frac{1}{2}\pi n)$, then determine all possible value(s) of the sampling frequency f_s.

(c) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(400\pi t^2) \qquad \text{for } 0 \le t \le 5 \text{ sec.}$$

If the sampling rate is $f_s = 800$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`.

PROBLEM 4.5:

In the rotating disk and strobe demo described in Chapter 4 of *DSP First*, we observed that different flashing rates of the strobe light would make the spot on the disk stand still.

(a) Assume that the disk is rotating in the clockwise direction at a constant speed of 900 rpm (revolutions per minute). Express the movement of the spot on the disk as a rotating complex phasor.

(b) If the strobe light can be flashed at a rate of n flashes *per second* where n is an integer greater than zero, determine all possible flashing rates such that the disk can be made to stand still.
NOTE: the only possible flashing rates are 1 per second, 2 per second, 3 per second, etc.

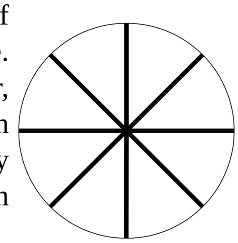
(c) If the flashing rate is 13 times per second, explain how the spot will move and write a complex phasor that gives the position of the spot at each flash.

(d) Draw a spectrum plot of the discrete-time signal in part (c) to explain your answer.

PROBLEM 4.6*:

When watching old TV movies, all of us have seen the phenomenon where a wagon wheel appears to move backwards. The same illusion can also be seen in automobile commercials, when the car's hubcaps have a spoked pattern. Both of these are due to the 30 frames/sec sampling used in transmitting TV images.

In the figure to the right, an eight-spoked wheel is shown. Assume that the diameter of this wheel is two feet, which is almost exactly the tire diameter of a typical automobile. In addition, assume that the wheel is actually rotating CCW, so that if attached to a car, the car would be traveling to the viewer's left *at a constant speed.* However, when seen on TV the spoke pattern of the car wheel appears to rotate *clockwise* once every 4 seconds. How fast is the car traveling (in miles per hour)? Derive a general equation that will make it easy to give all possible answers.



PROBLEM 4.7:

A non-ideal D-to-C converter takes a sequence $y[n]$ as input and produces a continuous-time output $y(t)$ according to the relation

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

where $T_s = 0.001 = 10^{-3}$ second. The input sequence is given by the formula

$$y[n] = \begin{cases} \frac{1}{5}(n+1) & 0 \le n \le 4 \\ (0.5)^{(n-4)} & 5 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $y[n]$ versus n.

(b) For the pulse shape

$$p(t) = \begin{cases} 1 & -0.0005 \le t \le 0.0005 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

(c) For the pulse shape

$$p(t) = \begin{cases} 1 - 1000|t| & -0.001 \le t \le 0.001 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.