GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Winter 1999
Problem Set #5

Assigned: 12 Feb 99
Due Date: 19 Feb 99 (FRIDAY)

Quiz #2 will be held on 1-March-99. Closed book, calculators permitted, and one page of hand-written formulas ($8\frac{1}{2}''\times 11''$). It will cover material from Chapters 3, 4, 5, and 6, as represented in Problem Sets #4, #5 and #6.

Reading: In *DSP First*, Chapter 5 on *FIR Filters.*

A lab quiz is planned for the labs on 16 & 18 Feb

$\Longrightarrow$ The six (6) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the "unstarred" problems.

PROBLEM 5.1:

A linear time-invariant system is described by the difference equation

$$y[n]=\sum_{k=0}^{5}x[n-k]$$

The input to this system is *unit step* signal, denoted by $u[n]$:

$$x[n]=u[n]=\begin{cases}0 & n<0\\ 1 & n\geq 0\end{cases}$$

Compute $y[n]$, over the range $-5\leq n\leq\infty$. Make a plot of $y[n]$ vs. n.

PROBLEM 5.2*:

Consider a system defined by

$$y[n]=\sum_{k=0}^{13}b_k x[n-k]$$

(a) What is the filter length?

(b) Suppose that the input $x[n]$ is non-zero only for $0\leq n\leq 33$. Show that $y[n]$ is non-zero at most over a finite interval of the form $0\leq n\leq P-1$ and determine P.

(c) Suppose that the input $x[n]$ is non-zero only for $242\leq n\leq 942$. Show that $y[n]$ is non-zero at most over a finite interval of the form $N_3\leq n\leq N_4$. Determine N_3 and N_4.

Hint: Draw a sketch similar to Fig. 5.5 to illustrate the zero regions of the output signal.

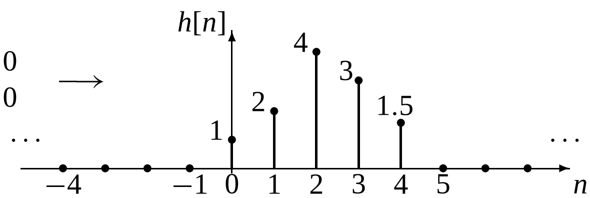
PROBLEM 5.3*:

Answer the following questions about the time-domain response of FIR digital filters:

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

(a) When tested with an input signal that is an impulse, $x[n] = \delta[n]$, the observed output from the filter is the signal $h[n]$ shown below:

$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases} \longrightarrow$$



Determine the filter coefficients $\{b_k\}$ of the difference equation for the FIR filter.

(b) Is the filter *causal?*

(c) If the input signal is

$$x[n] = \begin{cases} 0 & \text{for } n < -2 \\ 1 & \text{for } n = -2, -1, 0, 1, 2 \\ 0 & \text{for } n > 2 \end{cases}$$

use convolution to determine the output signal $y[n]$ for all n. Give your answer as either a plot or a table of values.

PROBLEM 5.4*:

Suppose that an FIR filter is specified by the filter coefficients $\{b_k\} = \{0,\ 0,\ 2,\ 0,\ -1,\ 0,\ 2\}$.

(a) If the input signal to the filter is $x[n] = -7\delta[n-3]$, determine the output, $y[n]$, and make a plot of the output signal.

(b) Write a short MATLAB program (just a few lines) that will solve this problem and make the plot.

PROBLEM 5.5*:

Use linearity and time-invariance to solve the following problem: For a particular LTI system, when the input is a delayed *unit impulse* signal: $x_1[n] = \delta[n-3]$, the corresponding output is

$$y_1[n] = \delta[n] - 2\delta[n-2] = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ 0 & n = 1 \\ -2 & n = 2 \\ 0 & n \geq 3 \end{cases}$$

Determine the output when the input to the LTI system is $x_2[n] = \delta[n] - 2\delta[n-4] - \delta[n-8]$. Give your answer as a plot of $y_2[n]$ versus n, or a list of values for $-\infty < n < \infty$.

PROBLEM 5.6*:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] - \alpha x[n-1]$$

(a) When the input to this system is

$$x[n] = \begin{cases} 0 & n < 0 \\ \alpha^n & n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \\ 0 & n \geq 10 \end{cases}$$

Use convolution to compute the values of $y[n]$, over the range $0 \leq n \leq 10$. Give a general formula in terms of α, and also show that most of the output values are equal to zero.

(b) Use the results from the previous part and plot both $x[n]$ and $y[n]$ for the case where $\alpha = 0.8$.

PROBLEM 5.7*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

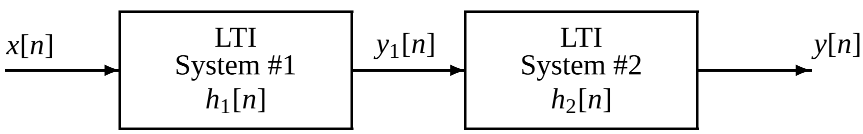


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a blurring filter described by the impulse response:

$$h_1[n] = \begin{cases} 0 & n < 0 \\ \alpha^n & n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \\ 0 & n \geq 10 \end{cases}$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n] - \alpha y_1[n-1]$$

Determine the impulse response function of the overall cascade system.

(b) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1.