GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Quiz No. 2
February 18, 1998

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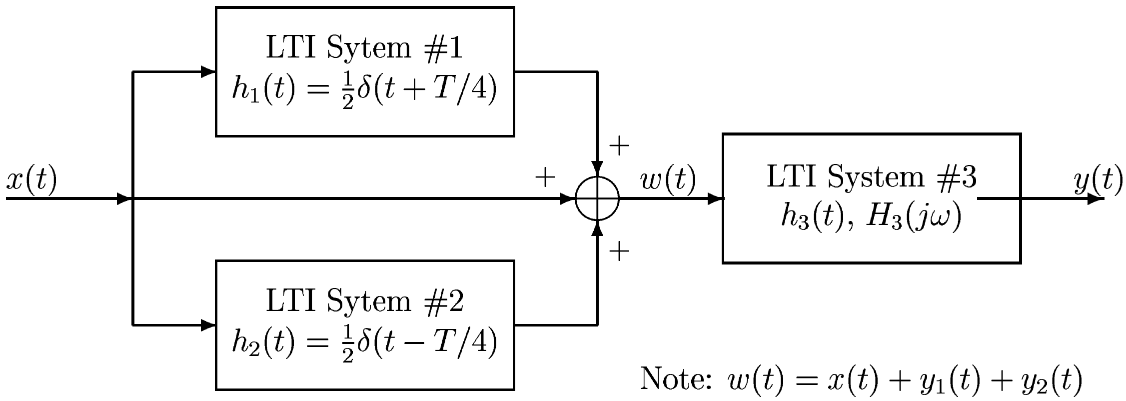
**Name:** ______________________________________

1. The exam is closed book. You may use one 8.5" by 11" sheet of notes (both sides). You are permitted to use a calculator. **I have given you a sheet of Fourier transform formulas as the last page. Tear it off and use it!**
2. Do all work in the space provided. If you need more room, use the *back* of the *previous* page and indicate that you have done so.
3. Indicate your answer clearly by circling it or drawing a box around it.
4. Think carefully about the the problem before you begin to write. Move on to the next problem if you cannot come up with a plan for the solution.
5. If you want to receive partial credit, you should clearly indicate your reasoning and method of attack on the problem.

| **Problem** | **Points** | **Score** |
|---|---|---|
| 1 | 25 | |
| 2 | 25 | |
| 3 | 30 | |
| 4 | 20 | |
| TOTAL | 100 | |

**Problem W98.Q2.1 (25 %)**

The following system is a LTI system.




The frequency response of LTI System #3 is: $H_3(j\omega) = \begin{cases} T/4 & |\omega| < 4\pi/T \\ 0 & |\omega| > 4\pi/T \end{cases}$

(a) Use the given table to determine the impulse response, $h_3(t)$, of LTI System #3.

$h_3(t) =$

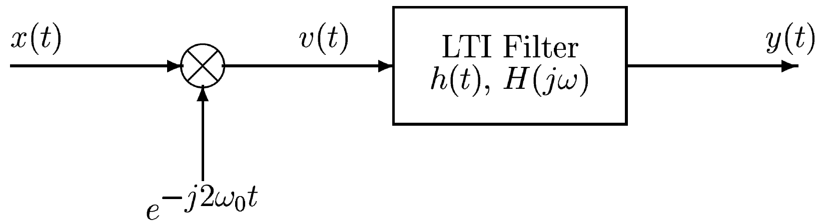
(b) First, give an expression in terms of $h_3(t)$ for the impulse response $h(t)$ of the overall system. Then use your result from part (a) to find an equation for the overall impulse response $h(t)$. Sketch your answer below showing the value of $h(0)$ and the times at which $h(t) = 0$.

$h(t) =$ ______________________________ (only in terms of $h_3(t)$)

(c) Determine the frequency response, $H(j\omega)$, for the overall system. Express your answer in terms of $H_3(j\omega)$ and manipulate it into a simple form so that you can easily plot it below. **Plot it.**

**Problem W98.Q2.2: (25 %)**

Consider the following modulation/filtering system:




The impulse response of the LTI system is: $h(t) = \begin{cases} 1/T & |t| < T/2 \\ 0 & |t| > T/2 \end{cases}$

(a) Determine the frequency response of the LTI system and plot it below.

(b) Suppose that $\omega_0 = 2\pi/T$ and the input signal is the periodic function

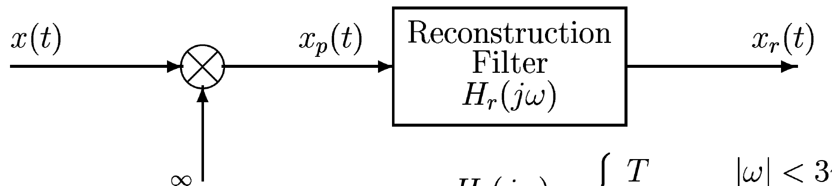
$$x(t) = 1 + 2\cos(\omega_0 t + \pi/2) + \cos(2\omega_0 t)$$

Determine expressions for the Fourier transforms of $x(t)$ and $v(t)$.

(c) Determine the output $y(t)$ for the input $x(t)$ in part (b).

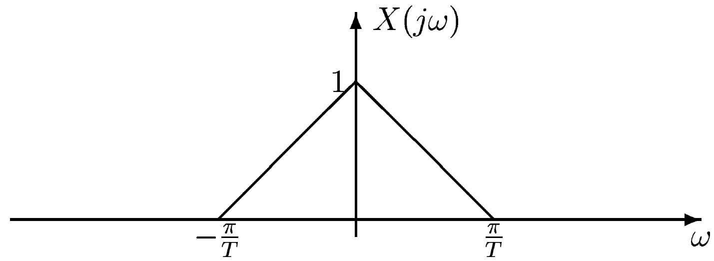
**Problem W98.Q2.3: (30 %)**

Consider the following sampling and reconstruction system:




$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \qquad H_r(j\omega) = \begin{cases} T & |\omega| < 3\pi/T \\ 0 & |\omega| > 3\pi/T \end{cases}$$

Assume that the input signal $x(t)$ has a Fourier transform as depicted below




(a) Give an equation for $X_p(j\omega)$ in terms of $X(j\omega)$.

$X_p(j\omega) =$

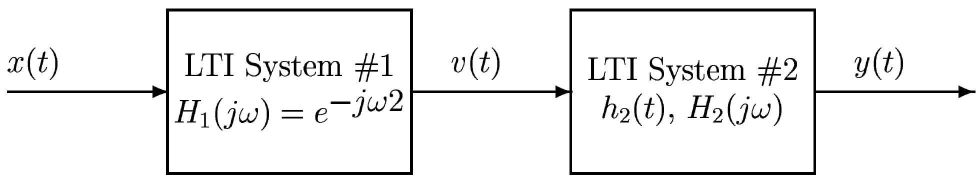
(b) For the Fourier transform $X(j\omega)$ given in the graph above, plot $X_p(j\omega)$.

(c) If the frequency response of the reconstruction filter is as given above, sketch the Fourier transform $X_r(j\omega)$ of the output signal $x_r(t)$ for $X_p(j\omega)$ as determined in (b).

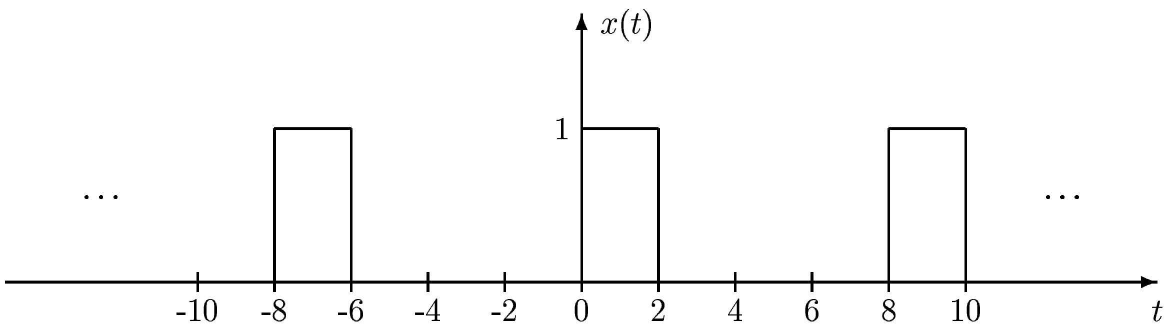
(d) Using your result in part (c) as a guide, write down an expression for $x_r(t)$ that holds for any input such that $X(j\omega) = 0$ for $|\omega| \geq \pi/T$. Your answer should be in terms of $x(t)$.

**Problem W98.Q2.4: (20 %)**

Consider the following cascade of two LTI systems:




System #1 is defined by its frequency response as above, and System #2 is to be determined or given. The input to the cascade system is the periodic pulse wave $x(t)$ depicted below:




*Note Carefully: You do not need to do any complicated algebra to solve this problem. You only need to* **think**.

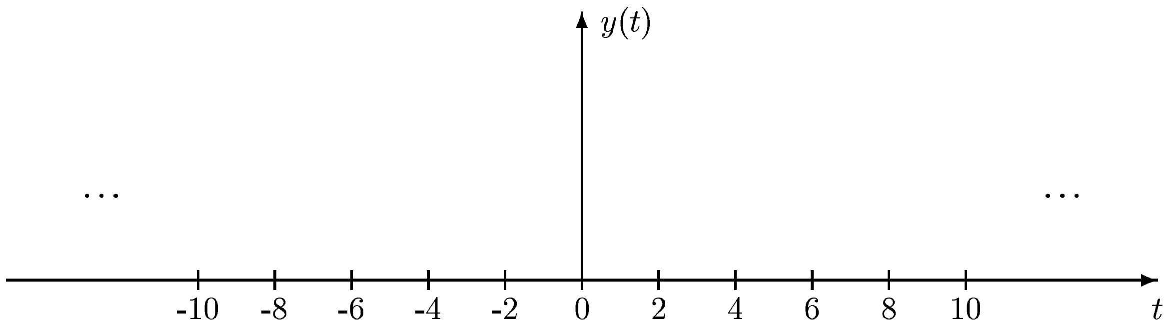
(a) Give an equation or a plot of the frequency response of the second system, $H_2(j\omega)$, such that the output is

$$y(t) = 1 \quad \text{for} \quad -\infty < t < \infty$$

Is your answer unique? *yes* *no*

**Problem W98.Q2.4 (continued): (20 %)**

(b) Suppose that $H_2(\omega) = j\omega$, plot the output waveform $y(t)$.



# FORMULAS FOR SECOND EXAM

## Fourier Transform Pairs

$$x(t) = e^{-at}u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{1}{a + j\omega}$$

$$x(t) = \begin{cases} 1 & |t| < T_p/2 \\ 0 & |t| > T_p/2 \end{cases} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{2\sin(\omega T_p/2)}{\omega}$$

$$x(t) = \frac{\sin \omega_b t}{\pi t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

$$x(t) = \delta(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 1$$

$$x(t) = u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$x(t) = u_1(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = j\omega$$

$$x(t) = 1 \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega)$$

$$x(t) = e^{j\omega_0 t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega - \omega_0)$$

$$x(t) = \cos \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$x(t) = \sin \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = -j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$$

$$\left.\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt} \\ a_k &= \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt \end{aligned}\right\} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

## Fourier Transform Theorems

$$ax_1(t) + bx_2(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad aX_1(j\omega) + bX_2(j\omega)$$

$$x(t - t_0) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad e^{-j\omega t_0}X(j\omega)$$

$$e^{j\omega_0 t}x(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j(\omega - \omega_0))$$

$$\frac{dx(t)}{dt} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad j\omega X(j\omega)$$

$$-jtx(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{dX(j\omega)}{d\omega}$$

$$y(t) = x(t) * h(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad Y(j\omega) = H(j\omega)X(j\omega)$$

$$x(t)p(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{2\pi}X(j\omega) * P(j\omega)$$

$$x(at) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$$

$$X(jt) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad 2\pi x(-\omega)$$