GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Quiz No. 3
March 4, 1998

---

**Name:** ____________________________________________

1. The exam is closed book. You may use one 8.5" by 11" sheet of notes (both sides). You are permitted to use a calculator. **I have given you a sheet of Fourier transform formulas as the last page. Tear it off and use it!**
2. Do all work in the space provided. If you need more room, use the *back* of the *previous* page and indicate that you have done so.
3. Indicate your answer clearly by circling it or drawing a box around it.
4. Think carefully about the the problem before you begin to write. Move on to the next problem if you cannot come up with a plan for the solution.
5. If you want to receive partial credit, you should clearly indicate your reasoning and method of attack on the problem.

| **Problem** | **Points** | **Score** |
|---|---|---|
| 1 | 40 | |
| 2 | 30 | |
| 3 | 30 | |
| TOTAL | 100 | |

**Problem W98.Q2.1 (40 %)**

A linear time-invariant system with input $x(t)$ and output $y(t)$ has system function

$$H(s) = \frac{s-2}{s+2}$$

(a) If it is known that the system is causal, determine the impulse response $h(t)$.

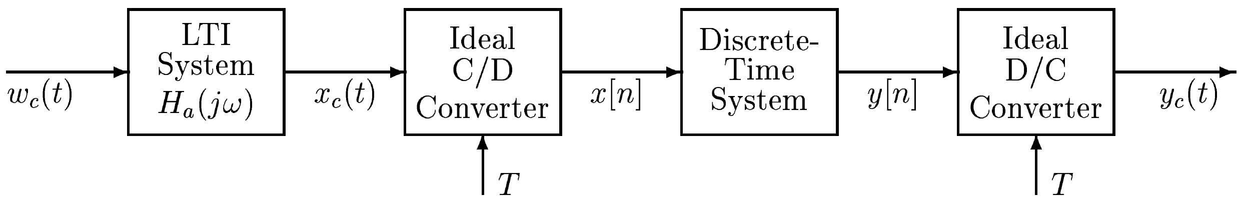
(b) Is the system stable? *yes* *no*
How do you know?

(c) Suppose that the input to the system is $x(t) = u(-t)$. Determine the corresponding output $y(t)$.

(d) Determine the Laplace transform $X(s)$ of the input $x(t)$ so that $y(t) = \delta(t-5)$.
If it is known that $\int_{-\infty}^{\infty} |x(t)| dt < \infty$, what is the region of convergence of $X(s)$?

**Problem W98.Q2.2: (30 %)**

All parts of this problem are concerned with the following system.




$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t - nT)}{\frac{\pi}{T}(t - nT)}$$

(a) If $T = 1/1000$, what condition on $H_a(j\omega)$ will guarantee that $y_c(t) = x_c(t)$ if the discrete-time system is defined by the difference equation $y[n] = x[n]$?

(b) If $T = 1/1000$ and the system $H_a(j\omega)$ is as chosen in part (a), determine $y_c(t)$ in terms of $x_c(t)$ if the discrete-time system is defined by the difference equation $y[n] = x[n - 100]$.

(c) If the filter $H_a(j\omega)$ satisfies the condition determined in part (a) and the discrete-time system now is defined by $y[n] = x[n] + x[n-1]$, what is the frequency response $H_c(j\omega)$ of the overall system from $w_c(t)$ to $y_c(t)$; i.e., what is $H_c(j\omega)$ in the equation

$$Y_c(j\omega) = H_c(j\omega) W_c(j\omega)$$

Be as specific as you can.

**Problem W98.Q2.3: (30 %)**

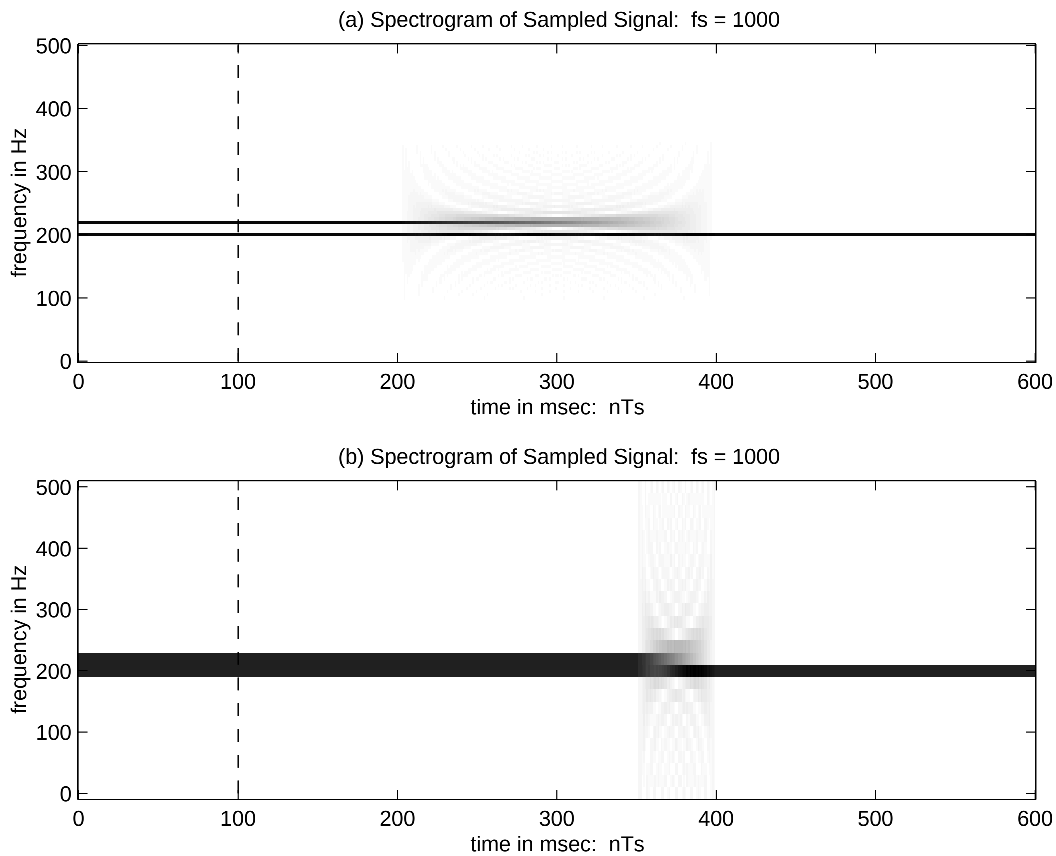



Figure 1: Spectrum slices for two different values of $N$.

Figure 1 shows the spectrogram defined as

$$X[k,n] = \sum_{m=n}^{n+N-1} x[m]e^{-j(2\pi k/N)m} \qquad k = 0,1,\ldots,N/2 \text{ and } n = 0,1,\ldots,L \qquad (1)$$

In the figure, different values of $N$ are used in parts (a) and (b). Also, the $k$ axis is labelled in terms of continuous-time frequency and the $n$ axis is labelled in terms of continuous-time. The sampling rate of the signal is $f_s = 1000$ Hz.

(i) From the figure and the given sampling rate, estimate the value of $N$ for both spectrograms.

**Problem W98.Q2.3 (continued): (30 %)**

The dotted vertical lines in Figure 1 correspond to the particular value of $n = 100$ samples in Equation (1). The plots in Figure 2 show $|X[k, 100]|$ as a function of $k$ for $0 \le k \le N/2$ for both case (a) and case (b).

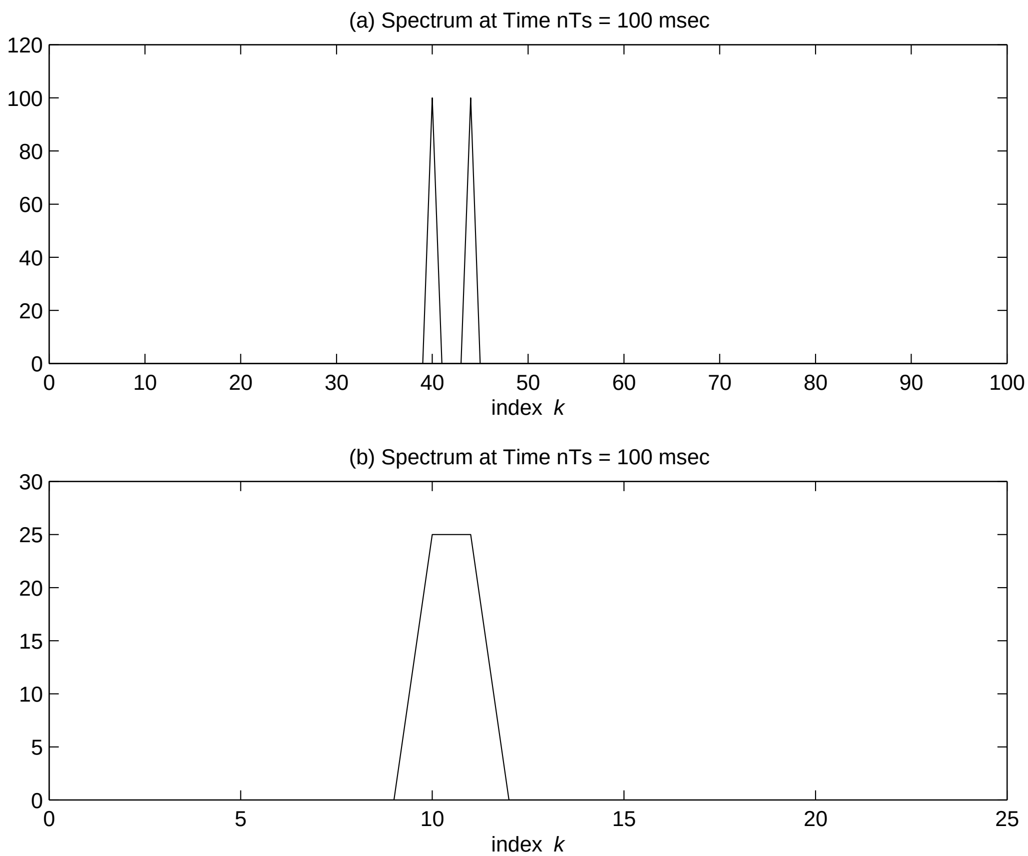



Figure 2: Spectrum slices for two different values of $N$.

(ii) Label the $k$ axis in both of the above plots. In particular, label the far right point of the plot and label the exact index of the locations of the peaks.

(iii) At what time does the signal $x[n]$ change from having 2 frequencies to having only one frequency? *Answer:* $n =$ __________
Give an equation that describes $x[n]$ as completely as you can.

## FORMULAS FOR SECOND EXAM

### Fourier Transform Pairs

$$x(t) = e^{-at}u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{1}{a + j\omega}$$

$$x(t) = \begin{cases} 1 & |t| < T_p/2 \\ 0 & |t| > T_p/2 \end{cases} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{2\sin(\omega T_p/2)}{\omega}$$

$$x(t) = \frac{\sin \omega_b t}{\pi t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

$$x(t) = \delta(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 1$$

$$x(t) = u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$x(t) = u_1(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = j\omega$$

$$x(t) = 1 \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega)$$

$$x(t) = e^{j\omega_0 t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega - \omega_0)$$

$$x(t) = \cos \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$x(t) = \sin \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = -j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$$

$$\left.\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t} \\ a_k &= \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt \end{aligned}\right\} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

### Fourier Transform Theorems

$$ax_1(t) + bx_2(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad aX_1(j\omega) + bX_2(j\omega)$$

$$x(t - t_0) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad e^{-j\omega t_0}X(j\omega)$$

$$e^{j\omega_0 t}x(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j(\omega - \omega_0))$$

$$\frac{dx(t)}{dt} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad j\omega X(j\omega)$$

$$-jtx(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{dX(j\omega)}{d\omega}$$

$$y(t) = x(t) * h(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad Y(j\omega) = H(j\omega)X(j\omega)$$

$$x(t)p(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{2\pi}X(j\omega) * P(j\omega)$$

$$x(at) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$$

$$X(jt) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad 2\pi x(-\omega)$$